

Dear Professor Claussen,

It is a privilege to have your insightful review. I am sure that our discussion below, inspired by it, will be helpful to our readers.

First, you provide a concise synopsis of the work (for clarity, your text is marked **bold blue**).

Mikhail Verbitsky explores the idea that the dynamics of Pleistocene ice ages is an initial value problem. This idea was proposed by Verbitsky and Volobuev (2025) who developed a long-memory ice-climate system of reduced complexity in the sense of Saltzman (2002). In their model, the dynamics of large ice sheets is defined by the advection of mass and temperature. Because of the ice-climate system's nonlinearity, the response of the surface mass balance of ice sheets to the orbital forcing is not symmetrical. This asymmetry may lead to a dynamics of the ice-climate system with a much longer memory of its initial conditions than previously supposed.

Mikhail Verbitsky applies this ice-climate model to three experiments. In a first experiment, the ice-climate model is forced by a pure obliquity period of 41 kyr to reproduce the results of Verbitsky and Volobuev (2025). The resulting time series of the area of glaciation is pretty close to the reconstruction by Lisiecki and Raymo (2005) with a pronounced 100 kyr peak and secondary, smaller 41 kyr and 23 kyr peaks. In a second experiment, the ice-climate model is driven by an additional forcing with a period of 5 kyr and an amplitude twice as large as the original orbital forcing, everything else being unchanged. Consistent with the initial-value hypothesis, the time series of the simulated glaciation area changes considerably with roughly equally strong peaks at periods of 80 kyr and 41 kyr. The later is reminiscent of the new reconstruction by Clark et al. (2025) – if one accepts that the Clark et al reconstruction captures the changes in global ice-sheet area. In a third experiment, the ice-climate model is run over the last 500 kyr into the future, but it is perturbed by a strong reduction in the glaciation area at $t = 0$ kyr. Interestingly the resultant time series of future glaciations shows a spectrum similar to the one of the second experiment, but with an even stronger 41 kyr signal. These results point at an intransitive response to orbital forcing which is at variance with other long-term paleoclimate simulations with more complex models by Ganopolski (2024), for example, who concludes that the pacing of orbitally driven glaciations is predictable.

...and then you formulate the main question:

Comment:

Predictable vs chaotic, transitive vs intransitive? Can the hypothesis of intransitivity be validated, or better: be invalidated, in any way? Looking at the glaciation reconstructions by Lisiecki and Raymo (2005) and Clark et al.(2025), I find peaks at 41 kyr and 100 kyr in both time series, only the ratio of the amplitude of these peaks differ. Wouldn't this speak against an initial-value problem, but in favor of predictable periods with, perhaps, only unpredictable amplitudes?

Response:

Let us start from the very general and undisputed facts: The dynamics of large ice sheets is defined by the advection of mass and temperature. The timescale of the advection is the ratio of ice thickness and mass (snow) influx rate. During the climate history, the snow influx rate changed dramatically and sometimes in a very unintuitive way when, for example, colder climate caused less snow (continentality effect) and warm climate caused increase of snow. This means that **the memory of the ice-climate system and its sensitivity to initial values is not an intrinsic, permanent property of nature; instead, it may have varied throughout climate history**. Indeed, the “butterfly” similarity parameter (2) contains initial values, but it also includes other physical parameters, and therefore multiple physical phenomena these parameters represent, orbital and terrestrial, fast and slow, may make this similarity parameter bigger or smaller and consequently engage or disengage initial values.

Certainly, the orbital periods will always be present in the ice volume historical spectra, but it does not make the **dominant-period trajectories** predictable. For example, Fig. 1(A) shows dominant periodicity fluctuating from 80 to 120 kyr (giving 100 kyr in the spectrum), but Fig. 1(C) (all parameters are the same as in the experiment shown in Fig. 1(A) but initial area of glaciation is different) presents steady 40-kyr periodicity and later bifurcation to steady 80-kyr periodicity.

The amplitude of the fluctuations is a function of the dominant periodicity. We will now derive a scaling law connecting these two major variables.

The most powerful scaling laws are always based on simplest and purest physical considerations. Verbitsky and Crucifix (2020) derived such scaling law for the initial-values-independent climate system. Their major hypothesis was as simple as the following: The amplitude of the glaciation area A (m^2) depends on only three major parameters of the orbital forcing and terrestrial dynamics. The amplitude of the orbital forcing ε (translated into the mass influx, m/s) is the major orbital parameter, the ratio of terrestrial positive-to-negative feedbacks V (adimensional) defines all the complexities of the terrestrial dynamics, and the dominant period P (s) is the result of the orbital-terrestrial interaction. Thus:

$$A = \varphi(P, \varepsilon, V) \quad (AC1-1)$$

Taking P and ε as parameters with independent dimensions and applying π -theorem, we have

$$\frac{A}{\varepsilon^2 P^2} = \Phi(V) \quad (AC1-2)$$

$$\text{or } A \sim P^2$$

It means that the **amplitude spectrum of the initial-values-independent climate system evolution should have slope $n \sim 2$** . Multiple experiments with ice-climate system confirmed this finding (i.e., $n = 1.8$).

Obviously, if we add additional parameters into the right side of the equation AC1-1, such as initial area (measured in m^2) or amplitudes and periods of the millennial forcing (measured in m/s and s correspondingly) the scaling law will deviate from AC1-2.

As we already know the spectral slopes of both Lisiecki and Raymo (2005) and Clark et al.(2025) reconstructions are far-off from $n \sim 2$. For Lisiecki and Raymo (2005) $n \sim 0.5$ (Huybers and Curry, 2006) and for Clark et al.(2025) $n \sim 0$.

Verbitsky and Crucifix (2020) demonstrated that the observed spectrum of climate variability with reduced spectral slope could be due to the millennial forcing propagated into the orbital domain. We now understand that the millennium forcing and sensitivity to initial values are not independent physical phenomena but they are parts of the conglomerate “butterfly” phenomenon combining multiple physical effects. Therefore we consider **the reduced spectral slopes as an indication in favor of the initial-value dependence.**

Action: To the extent allowed by the editor, I will include this discussion in the text

Comment:

Besides addressing the issue of validation, the following comments should be considered in a revision: The idea that anthropogenic greenhouse gas emissions could push future glaciation dynamics away from its Pleistocene path was already proposed by Loutre and Berger (2000) or Ganopolski et al. (2016). In a keynote lecture at an ESF conference in Stockholm in early 2002, André Berger even speaks of the end of Quaternary and the start of a new geological period, the Quaternary, with much less ice than the Quaternary. This change in dynamics is, however, not a consequence of intransitive ice sheet dynamics, but of a drastic change in the type of forcing which dominates over the next at least 10 or 20 millennia when the orbital forcing becomes negligibly small.

Response:

Thank you for bringing this important work to my attention.

Action: All these papers will be appropriately quoted.

Comment:

The ice-climate system model is forced by the 41 kyr obliquity period. But the spectrum of the insolation changes during the caloric half-year also shows a sizable contribution from the 23 kyr precession period. In the maximum summer insolation, the latter period is even the dominant one. Would this multi-period forcing have any effect on the question of whether the ice-climate system is an initial value problem?

Response:

This has been investigated in Verbitsky and Volobuev (2025, paragraph 2.3) – see Figures 5 and 6. Specifically: “When the system is forced by **a combination of eccentricity-**

modulated precession and obliquity, both with intermediate values of their amplitudes, the system response, depending on initial conditions, may be dominated either by the eccentricity period of ~100 kyr, or by the doubled obliquity period of ~80 kyr, or by a combination of both.”

Also in Verbitsky and Volobuev (2025, paragraph 4) we observe: “The conglomerate similarity parameter, the C-number, that defines the memory-duration sensitivity to initial values, is the ratio of the timescale of vertical advection and the period of orbital forcing. It implies that though we have discovered this phenomenon using the obliquity periods, there is nothing unique about it. Therefore the response to the eccentricity-modulated precession should not be immune from the initial-value sensitivity either. Indeed, since the eccentricity period is longer than the obliquity period, it takes a consistently longer advection timescale to obtain the initial-value-sensitive memory. In Fig. 8, we present the dynamical system response to the **pure eccentricity-modulated precession forcing**. It can be observed that the memory duration M and the dominant-period trajectories in this case are also sensitive to initial conditions.”

Action: I will mention this in the revised text.

Thank you again for your review.

Mikhail Verbitsky

References:

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