Review Comments

Overview of the preprint

The core idea of this manuscript is to introduce fractional kinematics to modeling the spread of wildfires, based on the observation that ignition waiting time should have memory and the distribution of ignition waiting time should have a heavy tail. This preprint is not the first paper that introduces fractional kinematics to wildfire modeling. But in my opinion, the motivation behind the mathematics in this manuscript is closer to the nature of wildfires than the previous work. Starting from this initial idea, instead of directly modeling the fire front based on the continuous-time random walk (CTRW) framework, the author chose to describe the propagation of fire front using Huygens' principle. The author derived a new wind factor, which is used to define the normal propagation rate of the fire front. The author then heuristically modified the Hamilton-Jacobi equation by replacing the temporal partial derivative $\frac{\partial u}{\partial t}$ with the Riemann-Liouville derivative ${}^{RL}D_t^{\alpha}u$, and by adding a tempered Riesz operator to represent the long-range spotting. An interesting consequence of the new wind factor is that the curvature at the fire head would start to decrease at high wind speed, thus preventing the persistent collapse which occurs at high wind speed in the classical (projected-wind) Rothermel model. The author provided a mathematical demonstration to support this claim.

General comment

While the initial motivation of this paper is indeed insightful and it is probably true that the new wind speed factor could mathematically prevent persistent collapse, I am not quite sure if I should take this manuscript as a new theoretical foundation for a wildfire spread model, or treat this manuscript as an introduction of a new empirical model. If it is a theoretical foundational paper, the reasoning in this paper has several gaps, including the introduction of hierarchical structure. If it is an introduction of a new empirical model, this manuscript lacks numerical demonstration using realistic data. I list the logical gaps (in my opinion) that I found in the following. In case the author can not fix these gaps rigorously, I suggest that the author consider a more precise way to state his argument. In sum, I recommend major revision of this manuscript.

Major comments

1, The author claims in several places (abstract, line 205, 445, etc.) of the manuscript that lemma 1 and theorem 1 are proved for every $\alpha \in [0, 1]$. What I find is that the statement in lemma 1 (iv) is only validated numerically and only for $\alpha \in [0.3, 1]$. This demonstration is not rigorous. The statement in lemma 1 (iv) is then used in the proof of theorem 1 (a), (b), and (d). I suggest that the author take (iv) out of lemma 1, and write it as a conjecture. Then make an explicit assumption in the manuscript that the conjecture is true.

2, The author introduced hierarchical structure in section 4, citing another paper (Chishtie 2026a) of his own. The author directly states that Eq.(8) is derived in theorem 3 in (Chishtie 2026a). However, theorem 3 is established for particles in a Hamiltonian system. I do not see why the results in theorem 3 of (Chishtie 2026a) can be directly applied to wildfire parcels. My intuition is that parcels in a wildfire are not moving particles. I suggest the author provide a more clear demonstration on why the scenario considered in theorem 3 of (Chishtie 2026a) is similar to the ignition of parcels in wildfire. The hierarchical structure should be removed from the current paper if this connection can not be established in a reasonable way.

3, It might be true (assuming that lemma 1 is indeed true) that the new wind factor could prevent persistent collapse. But ϕ_∞ in the new wind factor is still taken from the classical Rothermel model. It is not clear if ϕ_∞ is a parameter that could be derived from physics or just another empirical parameter. If it is just another empirical parameter, then this is mathematically equivalent to artificially choosing an upper bound for wind speed.

Minor comments

1, In line 317, I believe the author instead wanted to write "Let $u(s,t)$ be the level-set function, so the front level set $\{u = 0\}$, and ...".