REFEREE REPORT FOR "SUNLIT AND UNLIT: THE LIMITATIONS OF ALBEDO PREDICTION IN ENERGY BALANCE MODELS" BY G. BLAIR, M. KAZMA, A. TAHA, R. BENNARTZ, S. MAHADEVAN

1. Summary

The work studies the prediction of top-of-atmosphere albedo for a 1D EBM. Rather than prescribing the albedo through a fixed parametrisation, the authors consider it as a dynamical variable whose evolution is inferred from reanalysis data (ERA5). The authors compares many different regression, including polynomial regression and Gaussian kernel ridge regression, to classical parametrisation from EBM literature. They also studies the temporal resolution, spatial information, and the interaction between temperature and albedo prediction errors.

2. Evaluation

In my opinion, the manuscript is well written and enjoyable to read. Moreover, the topic fits the scope of *Geoscientific Model Development*. In particular, I appreciate the authors' effort to study relatively simple climate models. Besides their computational efficiency and physical interpretability, these models are allows a rigorous mathematical analysis and provide a valuable framework for understanding fundamental mechanisms and feedbacks in the climate system. Therefore, the strategy of the authors to combine classical EBMs with data-driven approaches to albedo prediction is particularly valuable.

However, during the review process, I found some typos and points that should be clarified, listed in the next section. I therefore recommend **minor revisions**, and encourage the authors to address the provided comments.

3. Comments

(1) I think there is an inconsistency in the temperature convention. In eq. (4), the outgoing longwave radiation is parameterised as

$$A(t) + B(u - 273.15),$$

where u is expressed in Kelvin. However, eq. (9) contains the term

$$A(t) + Bu.$$

I suspect that, switching between the two formulations, the authors are implicitly redefining the coefficients $A(t)$. This could be ambiguous for some readers and make not clear which choice is adopted in the numerical implementation.

(2) I have a question regarding the Crank-Nicolson scheme in eq. (14). For me, the Crank-Nicolson method for an ODE of the form $\dot{u}(t) = f(t, u(t))$ is given by

$$u^{k+1} = u^k + \frac{\Delta t}{2}\big(f(t_k, u^k) + f(t_{k+1}, u^{k+1})\big).$$

In the notations of the authors, writing $F^k := \omega^k + g^k$, the ODE for the temperature u is

$$\dot{u}(t) = \mathrm{diag}(c(t))^{-1} F(t, u(t)).$$

Thus, the Crank-Nicolson discretisation gives

$$u^{k+1} = u^k + \frac{\Delta t}{2}\left[\mathrm{diag}(c^k)^{-1} F^k + \mathrm{diag}(c^{k+1})^{-1} F^{k+1}\right].$$

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Instead, eq. (14) uses

$$u^{k+1} = u^k + \frac{\Delta t}{2} \operatorname{diag}(c^k)^{-1} \left(F^k + F^{k+1}\right),$$

which effectively freezes the heat capacity over each time step. Since c depends on time (see Eq. (35)), I am not sure that this modification preserves the usual second-order accuracy of the Crank-Nicolson method. I have no objection to the discretisation in eq. (14) itself, but I would appreciate a justification or reference for the claimed second-order accuracy (line 226) in the present setting.

(3) In Section 2.2.2, the authors assume that $\Delta\mu = \mu_{i+1} - \mu_i$ is constant (lines 238–240). However, immediately afterwards (lines 241-244), they state that temperature and radiative data are evaluated on a uniformly spaced latitude grid, with constant $\Delta\varphi = \varphi_{i+1} - \varphi_i$. Since $\mu = \sin(\varphi)$, these two assumptions cannot hold simultaneously on the same grid. Can you please clarify this point?

(4) I think that only $D_0\eta_n$ can be identified from temperature observations. Indeed, the diffusion coefficient D is defined in eq. (13) as

$$D(\mu, t_k) = D_0 \left(\sum_{n=1}^{N_d} \eta_n \chi_n(\mu)\right) \left(1 + \frac{\mu}{2} \cos \nu^k\right).$$

I think that, at the moment, D_0 and η_n are not uniquely identifiable. Indeed, for any $a > 0$, the transformation

$$D_0 \mapsto aD_0, \qquad \eta_n \mapsto \frac{\eta_n}{a}, \quad n = 1, \ldots, N_d,$$

keeps $D(\mu, t_k)$ unchanged, and shows that it is not possible to identify D_0 and η_n uniquely. I thin this is possible, provided that some additional conditions (constraints or normalisation) are added.

(5) Related to the previous point, how are the coefficients η_n and D_0 determined in Table 1 and Figure 4?

(6) Typo in eq. (24). The last component of the vector on the RHS contains $\hat{y}_1$, but I think it should be$\hat{y}_N$.

(7) Possible incosistency in the construction of the regional KRR predictors in Section 2.3.4 and Algorithm 1. Let $R_j = \{i : a(i) = j\}$ denote the set of latitudes belonging to the j-th region. Algorithm 1 constructs the regional input vectors by averaging the individual inputs, so that

$$v_j^k = \frac{1}{|R_j|} \sum_{i \in R_j} v_{\mathrm{inf},i}^k. \tag{1}$$

However, in eq. (23) it is written that

$$v_{\mathrm{inf},1}^k, v_{\mathrm{inf},N}^k \in \mathbb{R}^5, \qquad v_{\mathrm{inf},i}^k \in \mathbb{R}^7, \quad 2 \le i \le N-1.$$

Thus, if a region contains both a polar grid point and an interior grid point, the average in (1) is not well defined, since the vectors have different dimensions. This seems relevant, for instance, when $N_r = 72$ and $N = 721$, so that each region contains approximately ten latitudes.

Moreover, the subsequent kernel evaluation in eqs. (18)-(21) requires the local input vector to have the same dimension as the corresponding regional training vectors.

Could the authors clarify how the polar grid points are handled in the regional averaging and kernel evaluation?

(8) I have a question regarding the definition of the albedo error in Section 3.1, immediately after eq. (36). The MAE sums over all N latitudes. However, Section 2.4.3 explains that observed albedo is undefined during polar night and that α_{PN} is only a numerical placeholder.

Are these undefined observations excluded when computing the reported errors? For instance, if I_k denotes the set of latitudes with valid albedo observations at time t_k, is the intended definition

$$\mathrm{MAE}(t_k) = \frac{1}{|I_k|} \sum_{i \in I_k} \left| \alpha^k_{\mathrm{true},i} - \alpha^k_i \right| ?$$

Since the number of valid albedo observations varies with time, could the authors clarify how these missing observations are treated in the computation of MAE, and whether the normalisation $\frac{1}{|I_k|}$ accounts for the number of valid observations?