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Unsupervised neural network for dynamics control under chaotic regime

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Abstract. Some natural phenomena have strong impacts on society, implying the loss of human lives and a huge amount of financial costs. Therefore, the possibility of controlling severe natural phenomena is of practical interest, preserving lives and mitigating costs. The simulation of dynamical systems could help us to predict severe events, allowing the deployment of control actions to avoid or to mitigate the severity associated with the predicted phenomenon. Similar to the numerical experiments of Miyoshi and Sun (2022), here, ensemble predictions are employed to drive the control action in a chaotic dynamical system. The Observing Systems Simulation Experiment (OSSE) is performed using the Cellular Neural Network (CeNN) as the data assimilation operator with the Lorenz-63 system. The proposed CeNN-based assimilation achieved control performance comparable to or better than a perturbed observation ensemble Kalman filter, particularly for the more frequent assimilation interval.

1 Introduction

The "butterfly effect" was discovered by Lorenz at the beginning of the 1960s (Lorenz, 1963). Through his model, he perceived that two initial states, with very small differences, could evolve quite differently in the future. For this reason, the prediction of this kind of system becomes a difficult problem, as even when given a perfect model, the prediction is critically sensitive to the initial condition. Later works (Ott et al., 1990) elaborated on this characteristic behavior with a different consideration – that, for this kind of dynamics, setting the initial condition appropriately could be seen as changing the regime of the dynamics to a desirable outcome with very little intervention. In other words, making an insightful use of the chaotic behavior and its attractors. Whereas in other kinds of dynamics a great amount of energy may be required for effective future changes to take place, in chaotic systems, by using very little input, it should be possible to change or adapt the regime to a desirable outcome. In the case of the weather, a small early intervention could be used to prevent or mitigate, for example, a forecasted destructive extreme meteorological event.

Miyoshi and Sun (2022), inspired by the aim of controlling the weather (i.e. localized, short duration changes, not climate engineering), described a set of Control Simulation Experiments (CSE) using the Lorenz-63 system of three-variables as a proof of concept, therefore focusing on the essence of controlling one chaotic dynamical system while, at the same time, avoiding

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Preprint. Discussion started: 4 August 2026

the greater complexities of NWP models. In this study, we reproduce the control simulation framework of Miyoshi and Sun and evaluate a Cellular Neural Network (CeNN) as an alternative data assimilation method to EnKF. The main objective is to assess whether CeNN-based assimilation can support the same control strategy with comparable or improved performance. As such, this paper follows, as best as possible, the same setup of 40 CSE presented by them.

A CSE can be thought of as four connected pieces. The first piece is the Nature Run (NR), which represents the natural phenomena being observed. Given an initial state, the unperturbed Nature Run is the result of the time evolution of the Lorenz-63 system from that initial state. The second piece is the Observing Systems Simulation Experiment (OSSE), which takes readings from the NR and adds noise to represent the observation error of a real sensor. For the CSE, that noise is given by the uncertainty of a Gaussian distribution with variance 2.0. The full observation of Lorenz-63 system variables $[x(t), y(t), z(t)]$ is made to keep the experiments simple and focused on the control aspects. The third piece contains the background evolution of the Lorenz-63 system using a set of initial conditions, an *ensemble prediction* with three members, and the data assimilation (DA) technique. The objective of the DA process is to bring the background state closer to the NR state, with readings from the OSSE. The need for control deployment is also established here, through the evaluation of the ensemble's forecasts, with appropriate control parameters generated from them. The fourth piece is the activation of control, whose strength is determined by the experiment parameter D, representing the total amount of intervention in all controlled axes, per step, and will nudge the NR accordingly, in the direction defined by the received parameters. The objective of each CSE is to keep the x variable of the Lorenz-63 system positive at all times. This is done by the deployment of control, when a regime shift is detected, using the knowledge provided by the background ensemble members, whose states will depend on the data assimilation techniques used to assimilate the sensor's imperfect observations. The CSE framework has been mathematically formalized recently (Miyoshi, 2026).

The Lorenz-63 dynamical system has a *strange attractor* with two regions – like butterfly wings. In other words, the control action consists of maintaining the dynamics evolving in the region of just one part of the Lorenz's strange attractor, where $x(t) > 0$ – for our designed experiment. If, during prediction, one (or more) ensemble member enters a region where $x(t) < 0$, this triggers the application of control actions.

Artificial intelligence (AI) algorithms have been used in many related applications. A neural network was employed by Tang and Hsieh as an alternative to one missing equation of the Lorenz-63 system (Tang and Hsieh, 2001), where the 4D-Var technique was used for the data assimilation procedure. A survey of AI applications to data assimilation was done by Ghorbani et al. (2023). In Ghorbani's survey, all the AI approaches used for DA were supervised algorithms. A recent paper has proposed the use of Cellular Neural Networks (CeNN) as a new method for data assimilation (de Oliveira Júnior et al., 2025). The CeNN is an unsupervised AI algorithm and the DA procedure applied in our control experiments. It is worth noting that, besides using alternative DA approaches, there are other approaches for control, such as assimilating a pseudo-observation (Sawada, 2024).

A comparison with two techniques for data assimilation was carried out to activate the control process, using CeNN (de Oliveira Júnior et al., 2025) and an ensemble Kalman filter (EnKF) (Miyoshi and Sun, 2022). The CeNN application showed better or similar overall results when compared to the EnKF approach, particularly in the more frequent assimilation scenarios.

https://doi.org/10.5194/egusphere-2026-3775
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2 The Dynamical System

The dynamical system used in this experiment is the Lorenz-63 system with typical parameters, shown as

$$\begin{aligned}\frac{dx}{dt} &= \sigma(y-x),\\ \frac{dy}{dt} &= x(\rho-z)-y,\\ \frac{dz}{dt} &= xy-\beta z,\end{aligned} \tag{1}$$

with $\sigma = 10$, $\rho = 28$, and $\beta = 8/3$.

For the discrete implementation and numerical integration of the Lorenz-63 system, a 4th order Runge-Kutta method is used, with a time step $\Delta t = 0.01$, the same as that adopted in Miyoshi and Sun (2022), and the default value everywhere a step is mentioned. Data assimilation was tested at every 8th and 25th step ($T_a = 8$ and $T_a = 25$).

3 Data Assimilation Methods

This paper compares two data assimilation (DA) approaches. The first is the ensemble Kalman filter (EnKF) and the second is the Cellular Neural Network (CeNN), with both making use of three ensemble members. The ensemble members are used for assimilation purposes, as usual, but their current states are also individually forecast into the future and used to make the control activation decision and to infer the control parameters to be applied to the NR (Section 4). The two DA approaches used are detailed below.

3.1 Ensemble Kalman filter

This paper uses a simple stochastic implementation of the ensemble Kalman filter (EnKF), the perturbed observation approach (Houtekamer and Zhang, 2016). The approach makes it straightforward to use the alternative data assimilation method while keeping all other parts of the algorithm unchanged. A general assimilation formula can be written as

$$\boldsymbol{x}^a = \boldsymbol{x}^b + W\left[\boldsymbol{y}^o - \mathcal{H}(\boldsymbol{x}^b)\right] \tag{2}$$

where $\boldsymbol{x}^b$ is the model array (background), $\boldsymbol{y}^o$ is the observation array, and $\mathcal{H}(\cdot)$ is the observation operator that transforms from model variables to the observation space. The difference within the brackets is called innovation, and W is a weight representing how much of the innovation should be added to the background, resulting in $\boldsymbol{x}^a$, the assimilated result (analysis). Note that a, b, and o are not exponents, but labels for respectively analysis, background, and observation.

The ensemble Kalman filter (EnKF) can be formulated similarly as:

$$\boldsymbol{x}^{a,(n)} = \boldsymbol{x}^{b,(n)} + K\left[\boldsymbol{y}^{o,(n)} - \mathcal{H}(\boldsymbol{x}^{b,(n)})\right] \tag{3}$$

https://doi.org/10.5194/egusphere-2026-3775
Preprint. Discussion started: 4 August 2026

where the weights K are called the *Kalman gain* and n is a label that represents the n^{th} ensemble member from a total of N ensemble members (i.e. for the used ensemble $N = 3$, and $n = 1, 2, 3$). The perturbed observations are created as

$$\{\boldsymbol{y}^{o,(n)}(t)\}_{n=1}^{N} = \boldsymbol{y}^{o}(t) + \{\boldsymbol{\varepsilon}^{(n)}(t)\}_{n=1}^{N}\ , \quad \boldsymbol{\varepsilon}^{(n)} \sim \mathcal{G}(0, \sigma^2) \tag{4}$$

where each perturbed observation $\boldsymbol{y}^{o,(n)}$ is generated from the OSSE observation $\boldsymbol{y}^{o}$ with an added Gaussian ($\mathcal{G}(\cdot,\cdot)$) noise $\boldsymbol{\varepsilon}$ of variance $\sigma^2 = 2.0$, the same variance of the OSSE. The background states for the ensembles $\{\boldsymbol{x}^{b,(n)}(t)\}_{n=1}^{N}$ are evolved in each step using the Lorenz-63 system, and a small process noise of variance 0.1 is included in that evolution, if the EnKF is to be used. For the particular step 0, the states from the ensembles are initialized directly with the perturbed observation of each ensemble as

$$\boldsymbol{x}^{b,(n)} = \boldsymbol{y}^{o,(n)}, \qquad n = 1, .., N\ . \tag{5}$$

The ensemble mean $\bar{\boldsymbol{x}}^{\boldsymbol{b}}$ is given by the average of the ensembles

$$\bar{\boldsymbol{x}}^{b} = \frac{1}{N}\sum_{n=1}^{N}\boldsymbol{x}^{b,(n)}, \qquad n = 1, .., N. \tag{6}$$

and with it we can calculate the background deviations X_d^b, while considering an inflation factor γ, as

$$\begin{aligned} X_d^b = \gamma\,[\boldsymbol{x}^{b,(n)} - \bar{\boldsymbol{x}}^{b}], \qquad & n = 1, .., N \ \text{(and)} \\ & \gamma = 1.05\ . \end{aligned} \tag{7}$$

The background covariance P^b is

$$P^b = \left(\frac{1}{N-1}\right) X_d^b\ [X_d^b]^T \tag{8}$$

and the Kalman gain can be calculated as

$$K = P^b H^T\,(HP^bH^T + R)^{-1}\ , \tag{9}$$

with R being the covariance noise matrix, the matrix $\boldsymbol{H}$ is a linear representation to the observation operator $\mathcal{H}(\cdot)$ in Eqs. (2) and (3), and K the weight matrix to be applied to the innovation – the Kalman gain.

3.2 Cellular Neural Network

The Cellular Neural Network (CeNN) is a type of unsupervised machine learning algorithm. The implementation used for this work is based on de Oliveira Júnior et al. (2025). The CeNN is adaptable, and it is possible, for example, to use the CeNN technique for the time integration of the Lorenz-63 dynamics, instead of 4th order Runge-Kutta, as shown in that reference. For this study, we used only the DA application of the CeNN, adapted for the Lorenz-63 system.

The general CeNN-DA formulations are presented below. It must be noted that the procedures are described for a single ensemble member assimilation, for simplicity. For other ensemble members, the procedure is the same, but changing the inputs

https://doi.org/10.5194/egusphere-2026-3775
Preprint. Discussion started: 4 August 2026

to the cell (i.e., the current background state of the ensemble members and the observation to be assimilated). For the N ensemble members needed by the algorithm, each ensemble member n uses the same approach, assimilating its corresponding perturbed observation.

When using the DA by CeNN, the best initial condition (*analysis*) is computed by taking a perturbed observation and combining it with the prediction (*background*) of the Lorenz-63 system. This is done iteratively by solving the following differential equation:

$$\frac{d\mu_{ij}(\tau)}{d\tau} = -\mu_{ij}(\tau) + \sum_{(k,l)\in\{N_i,N_j\}} A_{ij,kl}\, v_{kl}(\tau) + \sum_{(k,l)\in\{N_i,N_j\}} B(\tau)_{ij,kl}\, u_{kl}(\tau) \tag{10}$$

with $i = 1,2,...,N_i$, and $j = 1,2,...N_j$ – for the Lorenz-63 system, $N_i = 3$ and $N_j = 3$. The state matrix $\mu_{ij}(\tau)$, feedback template A, and control template B are given by

$$\mu_{ij}(\tau) = \begin{bmatrix} x(\tau) & x(\tau) & x(\tau) \\ y(\tau) & y(\tau) & y(\tau) \\ z(\tau) & z(\tau) & z(\tau) \end{bmatrix}, \tag{11}$$

$$A = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix}, \tag{12}$$

with $\{B_{mn}\}_{m,n=1}^{N=3} = \dfrac{1}{w_b}\,\dfrac{\mu^o_{ij}}{\mu_{ij}(\tau)+\epsilon_r}$.

The elements of the diagonal matrix A represent the weighting assigned to the innovation term v (see Eq. (13)). Here, $\alpha = 0.1$ was adopted for the contribution of innovation to the assimilation process by the CeNN. The parameter w_b acts as a normalization factor based on the local neighborhood topology of the cells. $w_b = 3$ was used, since the set of cells $\{X, Y, Z\} \equiv \{x(\tau), y(\tau), z(\tau)\}$ has total connectivity, where each cell has two adjacent neighbors (de Oliveira Júnior et al., 2025), and $\epsilon_r = 10^{-4}$ is a small parameter. Definitions for $v_{kl}(\tau)$ and $u_{kl}(\tau)$ in Eq. (10) are

$$v_{kl}(\tau) = \begin{cases} -1, & \text{if } (\mu^o_{ij} - \mathcal{H}\{\mu_{ij}\}) < -1 \\ \mu^o_{ij} - \mathcal{H}\{\mu_{ij}(\tau)\}, & \text{if } -1 \le (\mu^o_{ij} - \mathcal{H}\{\mu_{ij}(\tau)\}) \le 1 \\ 1, & \text{if } (\mu^o_{ij} - \mathcal{H}\{\mu_{ij}\}) > 1 \end{cases} \tag{13}$$

$$u_{kl}(\tau) = \mu_{ij}(\tau)\big|_{\tau=0} = \boldsymbol{x}^b\ . \tag{14}$$

where $\boldsymbol{x}^b$ in the above equation is the same background vector as in Eq. (3).

The variable τ defines an evolution integration domain of the network, where the CeNN dynamics evolve until the influence of observation and background balances in the analysis state. The data assimilation process via CeNN is initialized at $\tau = 0$,

https://doi.org/10.5194/egusphere-2026-3775
Preprint. Discussion started: 4 August 2026

using the model background ($\boldsymbol{x}^b$) as the initial condition – see Eq. (14). The central objective is to determine this analysis state at the moment the network reaches equilibrium or saturates, which can be established through a deliberate choice of the integration time τ or by applying a specific stopping criterion. The maximum number of integration steps was fixed at 20, but the integration process may be stopped early if: (a) the difference between the previous calculated state and the current state begins to diverge, or (b) if the state starts to diverge from the observation target. Therefore, if (a) or (b) happens, the assimilation uses the prior innovation, and at least one integration step is always calculated.

In the CeNN assimilation process, the observations μ^o_{ij} follow the same perturbation routine defined in Eq. (4). For three ensemble members, the CeNN formulations as presented are applied three times, with each ensemble member assimilating its own perturbed observation. Therefore, there is no shared information among the ensemble members, unlike the EnKF, where they are coupled by the Kalman gain calculation.

4 Strategy for Dynamical System Control

The algorithm for the control strategy follows the strategy presented by Miyoshi and Sun (2022), now with a choice of DA.

1. Perform a DA update using the observations at time t, using CeNN or EnKF as a choice for this CSE run.

2. Run an ensemble forecast for T steps from time t to $t+T$.

3. If at least one ensemble member shows the regime shift, activate the control (step 4); otherwise, go to step 1 for the next DA cycle at time $t+T_a$.

4. Add perturbations computed from the difference

$$\delta\xi_k = \xi^{\text{Shift}}(t+k) - \xi^{\text{NoShift}}(t+k), \qquad k = 1,..,T \tag{15}$$

where ξ represents one variable from system (1) – for instance: $\xi = x$. NoShift is a label for an ensemble member whose ξ (for instance: $\xi = x$) system variable remains on the same attractor region (i.e., x is always positive), and Shift is a label for an ensemble member that shows regime shift (i.e., x becomes negative within T steps). Differences are calculated for all Lorenz-63 variables. The Euclidean norm D represents the intervention strength. The calculated δx_k, δy_k, and δz_k – hereafter called *deltas* – are added to the NR at every step from $(t+1)$ up to $(t+T_a-1)$. More precisely, at time $t+k$ ($k = 1,...,T_a-1$), the NR state is evolved from the previous NR state at time $t+k-1$ and is perturbed by adding (δx_k, δy_k, δz_k), where

$$D \equiv \sqrt{\delta x_k^2 + \delta y_k^2 + \delta z_k^2}, \qquad k = 1,..,T. \tag{16}$$

5. At time $t+T_a$, the new NR is used to simulate the observations; go to step 1 for the next DA at time $t+T_a$.

Figure 1 shows one assimilation step in which two ensemble members show a regime shift before $T = \lceil 4T_0 \rceil$, where $\lceil\rceil$ indicates rounding up to the closest integer since discrete integration is carried out. Here, T_0 is a constant value of 75.1 steps,

https://doi.org/10.5194/egusphere-2026-3775
Preprint. Discussion started: 4 August 2026

the average of a full orbit around the attractor (Miyoshi and Sun (2022)), and T_a is the interval between two assimilation cycles. At each assimilation cycle, the ensemble members' current states are forecast into the future, and the decision (item-3 of the algorithm) must be made to apply control or not. The selection of the no-shift (N) and shift (S) ensemble members to calculate

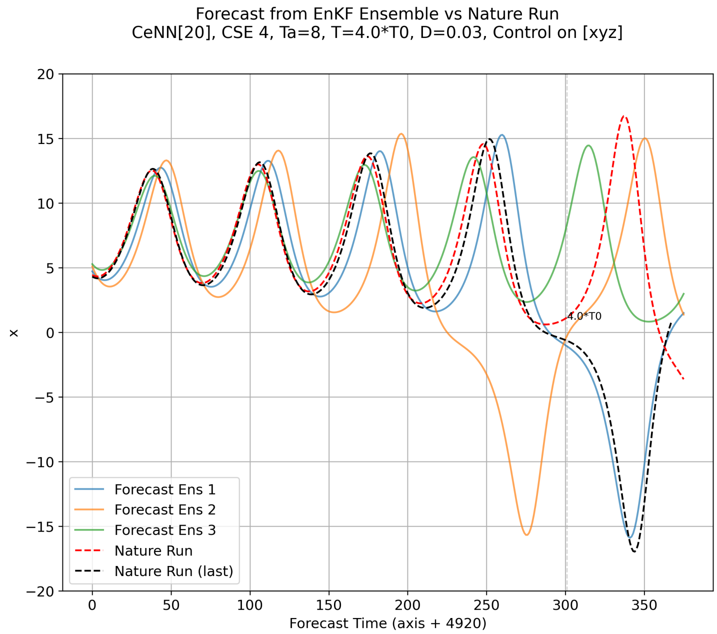

Figure 1. A successful earlier application of control. The original nature would evolve as shown in black, but was changed to red. In this current situation, control will be applied again, because ensemble member-1 (blue) and ensemble member-2 (orange) show a regime shift before $\lceil 4.0 * T0 \rceil$. New deltas are calculated from the differences between ensemble member-1 and ensemble member-3.

the δx, δy, δz, from the pool of two options when using three ensemble members, is to always take the lowest index ensemble member S or N. The three *deltas* are then calculated by the difference: $\mathrm{S} - \mathrm{N}$, for the whole duration of the forecast. One situation that has to be carefully considered is when all ensemble members show a regime shift (S), and there is no ensemble member of type (N) to be taken as the reference for calculating (δx, δy, δz). In that situation, a two-fold specific exceptional treatment is applied. In treatment (I), where the earlier detection had at least one or more (N) ensemble member forecasts, the one with the lowest index is always kept as a backup. Using data from that backup forecast, forwarded by the appropriate number of steps between two assimilation cycles, the *deltas* can be calculated from it and one current (S) ensemble member forecast, albeit with some reduced depth. In treatment (II), where during the assimilation all ensemble members show a regime shift (S), and the earlier assimilation was also one where all ensemble members were (S), then the earlier calculated deltas will be used up to when (IIa) a (N) condition ensemble member reappears, permitting a new calculation of *deltas*; or (IIb) to exhaustion, from where control is not applied any longer.

https://doi.org/10.5194/egusphere-2026-3775
Preprint. Discussion started: 4 August 2026

5 Results for Lorenz-63 system control

The main results are organized and presented at the end in two dedicated pages (13–14). They contain tables where each slot represents the control success for the 40 CSE, in various configurations. EnKF and CeNN with 20 integration steps – from Eq. (10): τ, with $\tau = 1, 2, \ldots, 20$ – as assimilation schemes are shown side by side for comparison of the same configurations (Table A1 and Table A2, Table A3 and Table A4). In page 13, Table A1 illustrates EnKF, and Table A2 illustrates the CeNN with 20 integration steps for variable τ, for all tested configurations, with $T_a = 8$. Next, the same arrangement follows on page 14, but with $T_a = 25$, where Table A3 shows EnKF results and Table A4 shows the CeNN with 20 integration steps.

Tables A1–A4 relate how long the control action (parameter T) is applied, and the strength of the intervention (parameter D in Eq. (16)). Table cell contents were made bold when over 50% of effective control was achieved to indicate the effectiveness of the control action.

In our tests, EnKF parameters such as the inflation factor or other adjustments do not necessarily make the overall performance in all configurations better. Therefore, instead of making individual adjustments, fixed parameters were adopted (e.g., inflation factor 1.05 as shown on Eq. (7)).

Figure 2 displays the 3D plot of the uncontrolled and controlled trajectory of a CSE using CeNN. For the CeNN specifically, it was noted that more integration steps for variable τ were generally associated with better performance.

Figure 3 shows the CeNN assimilation method, progressively with more iterations: 5 on the top, 10 in the middle, and 20 at the bottom. Although in all cases control is successful, one can note that, for the worked example, more integration steps can produce better control, with the distance from the attractor becoming smaller. Figure 4 shows the 3D equivalents of Fig. 3 going from 5 iterations (top) to 10 (middle) and 20 (bottom).

On the 3D graphs, NR change by control means that the nature had a regime change within the forecasted window and, after the application of control, it does not have it anymore, so control was successful. A false alarm occurs when nature does not exhibit a regime shift within the forecasted window, yet control is applied anyway. No NR change by control means that control was not sufficient to change the regime back to the right wing in the strange attractor within the forecasted window. And no control means that no need for control was detected at the assimilation step, and no control is applied between that assimilation step and the next.

Our results agree that false alarms do not seem to hurt (i.e., those applications of control usually make the nature go towards the attractor anyway), as reported by Miyoshi and Sun (2022). This situation is actually depicted in Fig. 1, where the application of control for the current configuration is determined by ensemble member-1 (blue) and member-3 (green), and will be applied to the current NR (red line). The current NR (red) would not actually change regime within the forecasted window, but it is possible to see that it would change regime in the future time later. In our findings, this is the main reason false alarms seem to be helpful.

https://doi.org/10.5194/egusphere-2026-3775
Preprint. Discussion started: 4 August 2026

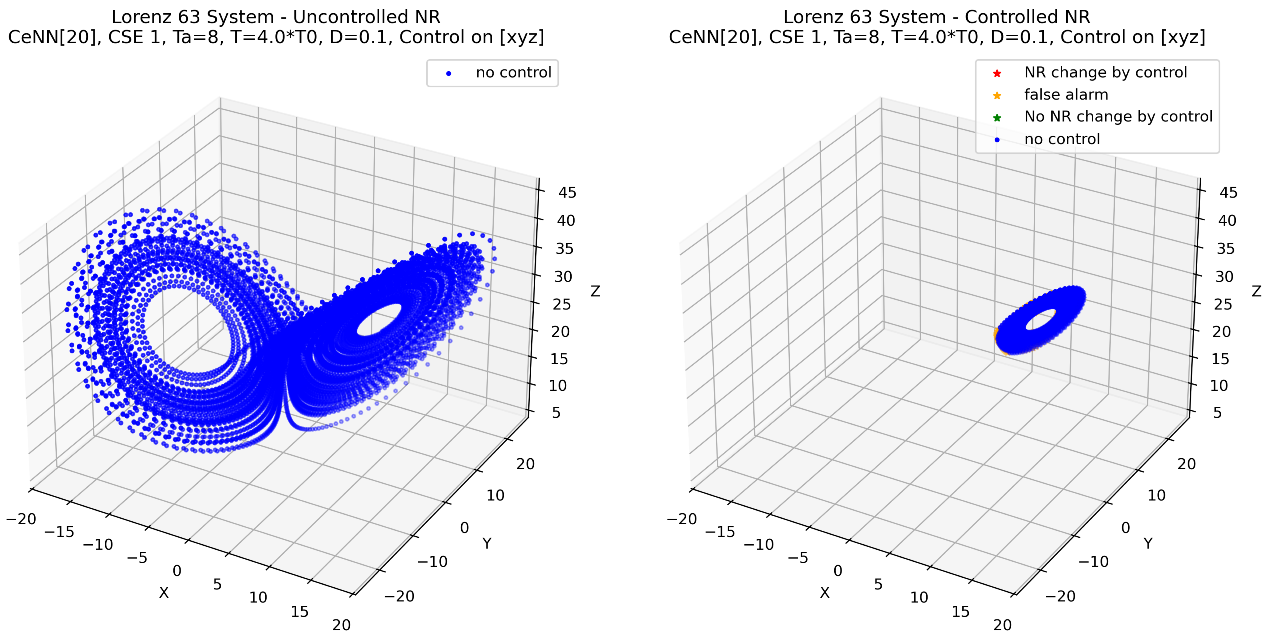

Figure 2. Uncontrolled NR (left) and controlled NR using CeNN (right).

https://doi.org/10.5194/egusphere-2026-3775
Preprint. Discussion started: 4 August 2026

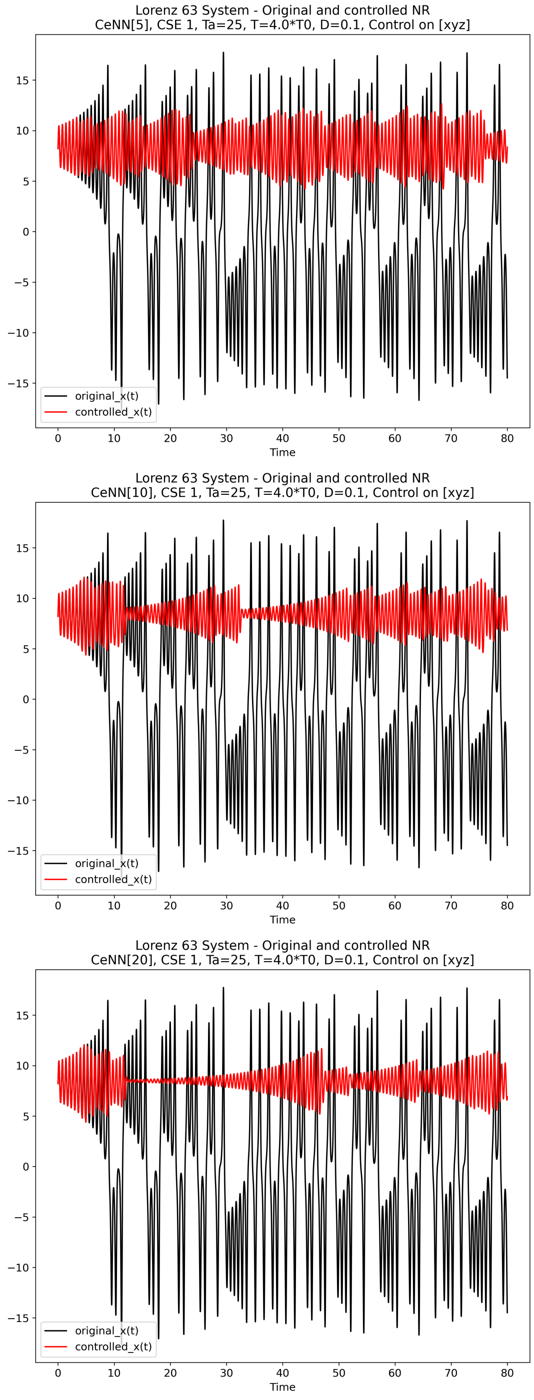

Figure 3. Uncontrolled (black) and controlled (red) NR using the CeNN; 5 (top), 10 (middle), and 20 (bottom) iterations.

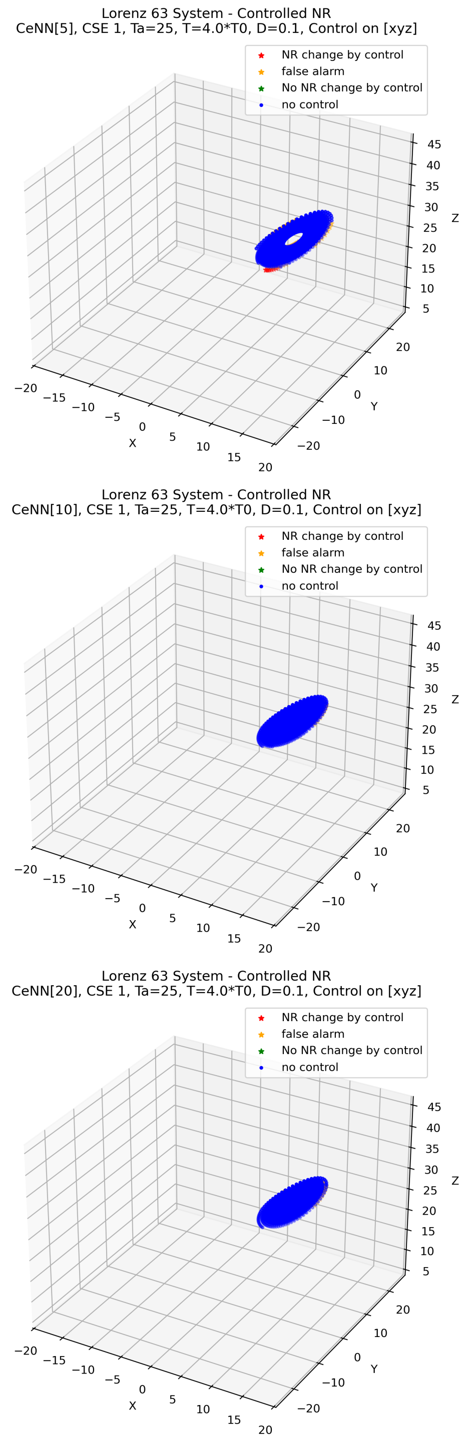

Figure 4. From top to bottom: controlled NR using CeNN with 5, 10, and 20 iterations.

https://doi.org/10.5194/egusphere-2026-3775
Preprint. Discussion started: 4 August 2026

6 Conclusions

This paper applied a Cellular Neural Network as a data assimilation operator, using ensemble prediction to drive a control action for the chaotic dynamics of the Lorenz-63 system. A successful performance was obtained with the approach presented as a strategy for dynamics control, in a manner similar to that proposed by Miyoshi and Sun (2022). From Tables A1 and A2, there are more slots highlighted with bold for CeNN than EnKF, showing a better performance for CeNN. A better performance is also noted for CeNN in Table A3 (EnKF) and A4 (CeNN).

Results indicate that alternative data assimilation methods can improve the performance of a control simulation experiment framework. The DA approach by the CeNN produced better overall results for the Lorenz-63 system than the EnKF, when the time interval between two assimilation cycles was shorter ($T_a = 8$), and showed somewhat more equivalent results with the EnKF when the assimilation interval was longer ($T_a = 25$). This is actually one aspect where the results do not generally agree with Miyoshi and Sun (2022), where they show somewhat better overall results with the larger time interval between two assimilations ($T_a = 25$). One can note that results using CeNN data assimilation can be similar to EnKF with a perfect model simulation, but the CeNN approach offers more options to be included in the dynamics control.

If a device is employed to carry out the control, a controller identification method can be applied (Molter and Cabral (2022)).

https://doi.org/10.5194/egusphere-2026-3775
Preprint. Discussion started: 4 August 2026

220 *Code availability.* The code that supports the findings of this study is available from the corresponding author upon reasonable request.

Video supplement. A video demonstration is available for a single CSE execution with CeNN and parameters at https://doi.org/10.5446/73068

https://doi.org/10.5194/egusphere-2026-3775
Preprint. Discussion started: 4 August 2026

Table A1. EnKF results for $T_a = 8$

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0	0	0	0.100	0.100
$\lceil 2T_0 \rceil$	0	0	0	0	0	0.025	0.025	0.300
$\lceil 2.5T_0 \rceil$	0	0	0	0	0	0.300	**0.550**	**0.800**
$\lceil 3T_0 \rceil$	0	0	0	0	0.200	**0.550**	**0.550**	**0.775**
$\lceil 4T_0 \rceil$	0	0	0	0	**0.875**	**0.950**	**0.900**	**0.900**

EnKF results for Control=x

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0	0.125	0.325	0.250	0.425
$\lceil 2T_0 \rceil$	0	0	0	0	0.150	0.325	0.400	0.400
$\lceil 2.5T_0 \rceil$	0	0	0	0.025	**0.750**	**0.725**	**0.825**	**0.850**
$\lceil 3T_0 \rceil$	0	0	0	0.225	**0.700**	**0.650**	**0.700**	**0.625**
$\lceil 4T_0 \rceil$	0	0	0	**0.875**	**0.950**	**0.825**	**0.825**	**0.825**

EnKF results for Control=y

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0	0	0	0	0.050
$\lceil 2T_0 \rceil$	0	0	0	0	0.075	0.175	0.250	0.225
$\lceil 2.5T_0 \rceil$	0	0	0	0	0.150	0.375	**0.600**	0.475
$\lceil 3T_0 \rceil$	0	0	0	0.025	**0.575**	**0.825**	**0.750**	**0.725**
$\lceil 4T_0 \rceil$	0	0	0	**0.675**	**0.875**	**0.925**	**0.850**	**0.800**

EnKF results for Control=z

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0	0.175	0.300	**0.525**	0.425
$\lceil 2T_0 \rceil$	0	0	0	0	0.200	0.450	0.500	0.425
$\lceil 2.5T_0 \rceil$	0	0	0	0.300	**0.725**	**0.700**	**0.975**	**0.750**
$\lceil 3T_0 \rceil$	0	0	0	0.425	**0.675**	**0.700**	**0.875**	**0.875**
$\lceil 4T_0 \rceil$	0	0.025	0.450	**0.900**	**0.950**	**0.925**	**0.900**	**0.900**

EnKF results for Control=xy

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0	0	0.050	0.200	0.150
$\lceil 2T_0 \rceil$	0	0	0	0	0.200	0.275	**0.550**	0.500
$\lceil 2.5T_0 \rceil$	0	0	0	0.025	**0.575**	**0.825**	**0.625**	**0.825**
$\lceil 3T_0 \rceil$	0	0	0	0.425	**0.750**	**0.900**	**0.925**	**0.950**
$\lceil 4T_0 \rceil$	0	0	0.100	**0.925**	**1.000**	**0.900**	**1.000**	**0.875**

EnKF results for Control=xz

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0	0.100	0.375	0.350	0.400
$\lceil 2T_0 \rceil$	0	0	0	0	0.450	**0.525**	**0.700**	**0.600**
$\lceil 2.5T_0 \rceil$	0	0	0	0.450	**0.775**	**0.800**	**0.850**	**0.925**
$\lceil 3T_0 \rceil$	0	0	0	**0.700**	**0.850**	**0.850**	**0.950**	**0.775**
$\lceil 4T_0 \rceil$	0	0.500	**0.900**	**0.975**	**0.975**	**0.975**	**0.925**	**0.950**

EnKF results for Control=yz

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0	0.125	0.350	0.475	**0.575**
$\lceil 2T_0 \rceil$	0	0	0	0.025	0.275	0.475	**0.525**	0.500
$\lceil 2.5T_0 \rceil$	0	0	0	0.475	**0.775**	**0.675**	**0.775**	**0.800**
$\lceil 3T_0 \rceil$	0	0	0.125	**0.800**	**0.875**	**0.800**	**0.850**	**0.800**
$\lceil 4T_0 \rceil$	0.100	**0.525**	**0.825**	**0.950**	**1.000**	**0.925**	**0.950**	**0.925**

EnKF results for Control=xyz

Table A2. CeNN results for $T_a = 8$

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0	0	0	0.100	0.125
$\lceil 2T_0 \rceil$	0	0	0	0	0	0.125	0.200	0.275
$\lceil 2.5T_0 \rceil$	0	0	0	0	0.025	0.500	**0.675**	**0.700**
$\lceil 3T_0 \rceil$	0	0	0	0	0.300	**0.700**	**0.675**	**0.675**
$\lceil 4T_0 \rceil$	0	0	0	0	**0.875**	**0.925**	**0.975**	**0.850**

CeNN results for Control=x

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0	**0.525**	**0.775**	**0.875**	**0.775**
$\lceil 2T_0 \rceil$	0	0	0	0.350	**0.950**	**0.925**	**0.975**	**0.950**
$\lceil 2.5T_0 \rceil$	0	0	0	**0.900**	**0.975**	**1.000**	**0.975**	**0.925**
$\lceil 3T_0 \rceil$	0	0	0.100	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
$\lceil 4T_0 \rceil$	0	**0.525**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**0.975**

CeNN results for Control=y

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0	0	0	0.050	0.075
$\lceil 2T_0 \rceil$	0	0	0	0	**0.725**	**0.675**	**0.650**	**0.875**
$\lceil 2.5T_0 \rceil$	0	0	0	0.025	**0.825**	**0.875**	**0.925**	**0.625**
$\lceil 3T_0 \rceil$	0	0	0	**0.675**	**0.950**	**1.000**	**0.900**	**0.875**
$\lceil 4T_0 \rceil$	0	0	0	**1.000**	**1.000**	**0.975**	**0.975**	**0.975**

CeNN results for Control=z

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0	**0.625**	**0.775**	**0.800**	**0.875**
$\lceil 2T_0 \rceil$	0	0	0	**0.675**	**0.950**	**0.975**	**0.925**	**0.925**
$\lceil 2.5T_0 \rceil$	0	0	0.025	**0.875**	**0.950**	**1.000**	**1.000**	**0.975**
$\lceil 3T_0 \rceil$	0	0	0.350	**0.975**	**1.000**	**1.000**	**0.950**	**1.000**
$\lceil 4T_0 \rceil$	0	**0.850**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**

CeNN results for Control=xy

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0	0.200	**0.525**	**0.575**	**0.625**
$\lceil 2T_0 \rceil$	0	0	0	0.125	**0.925**	**0.950**	**0.975**	**0.975**
$\lceil 2.5T_0 \rceil$	0	0	0	**0.775**	**0.950**	**0.975**	**0.950**	**0.900**
$\lceil 3T_0 \rceil$	0	0	0	**1.000**	**1.000**	**0.975**	**1.000**	**0.975**
$\lceil 4T_0 \rceil$	0	0	**0.700**	**1.000**	**1.000**	**1.000**	**1.000**	**0.975**

CeNN results for Control=xz

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0.150	**0.575**	**0.800**	**0.775**	**0.800**
$\lceil 2T_0 \rceil$	0	0	0	**0.925**	**1.000**	**1.000**	**1.000**	**0.975**
$\lceil 2.5T_0 \rceil$	0	0	0.500	**1.000**	**1.000**	**1.000**	**0.975**	**0.975**
$\lceil 3T_0 \rceil$	0	**0.675**	**1.000**	**1.000**	**1.000**	**0.975**	**0.975**	**0.975**
$\lceil 4T_0 \rceil$	**0.850**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**

CeNN results for Control=yz

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0.150	**0.700**	**0.775**	**0.850**	**0.850**
$\lceil 2T_0 \rceil$	0	0	0.025	**0.975**	**1.000**	**1.000**	**1.000**	**1.000**
$\lceil 2.5T_0 \rceil$	0	0.100	**0.775**	**0.950**	**0.975**	**0.975**	**0.925**	**0.950**
$\lceil 3T_0 \rceil$	0	**0.800**	**0.950**	**1.000**	**1.000**	**1.000**	**0.975**	**0.975**
$\lceil 4T_0 \rceil$	**0.975**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**

CeNN results for Control=xyz

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Table A3. EnKF results for $T_a = 25$

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0	0	0	0.025	0.100
$\lceil 2T_0 \rceil$	0	0	0	0	0	0	0	0.175
$\lceil 2.5T_0 \rceil$	0	0	0	0	0.075	0.375	0.425	0.400
$\lceil 3T_0 \rceil$	0	0	0	0	0.100	0.325	0.375	0.450
$\lceil 4T_0 \rceil$	0	0	0	0	**0.675**	**0.675**	**0.725**	**0.600**

EnKF results for Control=x

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0	0	0.075	0.150	0.100
$\lceil 2T_0 \rceil$	0	0	0	0	0.075	0.125	0.225	0.075
$\lceil 2.5T_0 \rceil$	0	0	0	0.025	0.150	0.250	0.225	0.200
$\lceil 3T_0 \rceil$	0	0	0	0.125	0.275	0.200	0.075	0.075
$\lceil 4T_0 \rceil$	0	0	0.025	0.225	0.325	0.200	0.050	0.025

EnKF results for Control=y

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0	0	0.075	0	0
$\lceil 2T_0 \rceil$	0	0	0	0	0.100	0.175	0.275	0.200
$\lceil 2.5T_0 \rceil$	0	0	0	0	0.125	0.300	0.175	0.325
$\lceil 3T_0 \rceil$	0	0	0	0.100	**0.700**	**0.575**	0.500	0.375
$\lceil 4T_0 \rceil$	0	0	0	**0.775**	**0.750**	**0.800**	**0.575**	0.400

EnKF results for Control=z

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0	0.100	0.200	0.275	0.150
$\lceil 2T_0 \rceil$	0	0	0	0	0.125	0.175	0.175	0.175
$\lceil 2.5T_0 \rceil$	0	0	0	0.125	0.425	0.350	0.300	0.175
$\lceil 3T_0 \rceil$	0	0	0	0.150	0.350	0.375	0.275	0.125
$\lceil 4T_0 \rceil$	0	0	0.025	**0.525**	0.500	0.350	0.125	0

EnKF results for Control=xy

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0	0	0.025	0.275	0.225
$\lceil 2T_0 \rceil$	0	0	0	0	0.300	0.350	0.400	0.250
$\lceil 2.5T_0 \rceil$	0	0	0	0.025	**0.575**	**0.600**	0.325	0.350
$\lceil 3T_0 \rceil$	0	0	0	0.225	**0.525**	**0.650**	**0.525**	0.200
$\lceil 4T_0 \rceil$	0	0.025	0.200	**0.800**	**0.900**	**0.675**	**0.525**	0.100

EnKF results for Control=xz

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0	0.150	0.300	0.150	0.075
$\lceil 2T_0 \rceil$	0	0	0	0.050	0.225	0.350	0.325	0.050
$\lceil 2.5T_0 \rceil$	0	0	0	0.225	0.475	0.450	0.275	0.150
$\lceil 3T_0 \rceil$	0	0	0	0.375	**0.650**	**0.600**	0.100	0.050
$\lceil 4T_0 \rceil$	0	0.225	0.400	**0.725**	**0.575**	0.425	0.150	0

EnKF results for Control=yz

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0	0.300	0.175	0.275	0.125
$\lceil 2T_0 \rceil$	0	0	0	0.150	0.400	0.325	0.050	0.050
$\lceil 2.5T_0 \rceil$	0	0	0	0.450	**0.525**	0.450	0.275	0.050
$\lceil 3T_0 \rceil$	0	0	0.025	**0.575**	**0.550**	0.400	0.200	0
$\lceil 4T_0 \rceil$	0.025	0.250	**0.525**	**0.700**	**0.750**	0.225	0.050	0

EnKF results for Control=xyz

Table A4. CeNN results for $T_a = 25$

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0	0.025	0.250	0.325	0.350
$\lceil 2T_0 \rceil$	0	0	0	0	0.075	0.125	0.275	0.375
$\lceil 2.5T_0 \rceil$	0	0	0	0	0.100	0.350	**0.525**	0.475
$\lceil 3T_0 \rceil$	0	0	0	0	0.225	**0.525**	**0.600**	0.500
$\lceil 4T_0 \rceil$	0	0	0	0.025	**0.725**	**0.875**	**0.800**	**0.725**

CeNN results for Control=x

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0.100	0.200	0.375	0.450	0.275
$\lceil 2T_0 \rceil$	0	0	0	0.200	0.325	0.325	0.225	0.175
$\lceil 2.5T_0 \rceil$	0	0	0	0.075	0.450	0.400	0.425	0.225
$\lceil 3T_0 \rceil$	0	0	0	0.325	**0.575**	**0.550**	0.325	0.150
$\lceil 4T_0 \rceil$	0	0	0.325	**0.925**	**0.750**	**0.550**	0.325	0.100

CeNN results for Control=y

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0	0	0	0	0
$\lceil 2T_0 \rceil$	0	0	0	0	0.050	0.125	0.225	0.100
$\lceil 2.5T_0 \rceil$	0	0	0	0	0.175	0.175	0.050	0.025
$\lceil 3T_0 \rceil$	0	0	0	0.250	**0.575**	**0.700**	**0.550**	0.200
$\lceil 4T_0 \rceil$	0	0	0	**0.850**	**0.900**	**0.800**	**0.625**	0.225

CeNN results for Control=z

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0	0.375	0.425	0.475	0.425
$\lceil 2T_0 \rceil$	0	0	0	0.150	0.375	0.450	0.375	0.325
$\lceil 2.5T_0 \rceil$	0	0	0	0.375	**0.600**	0.475	0.500	0.250
$\lceil 3T_0 \rceil$	0	0	0	**0.750**	**0.775**	**0.525**	0.425	0.225
$\lceil 4T_0 \rceil$	0	0.150	0.475	**0.900**	**0.825**	**0.800**	0.300	0.175

CeNN results for Control=xy

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0	0	0.025	0.025	0
$\lceil 2T_0 \rceil$	0	0	0	0	0.125	0.250	0.175	0.125
$\lceil 2.5T_0 \rceil$	0	0	0	0.025	0.300	0.325	0.250	0.025
$\lceil 3T_0 \rceil$	0	0	0	0.450	**0.775**	**0.675**	0.375	0.100
$\lceil 4T_0 \rceil$	0	0.100	0.275	**0.900**	**1.000**	**0.850**	**0.550**	0.150

CeNN results for Control=xz

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0	0.025	0	0	0
$\lceil 2T_0 \rceil$	0	0	0	0	0.225	0.225	0.025	0
$\lceil 2.5T_0 \rceil$	0	0	0	0.225	0.500	0.350	0	0
$\lceil 3T_0 \rceil$	0	0	0.225	**0.825**	**0.750**	0.375	0	0
$\lceil 4T_0 \rceil$	0.100	**0.625**	**0.850**	**0.975**	**0.925**	0.450	0.025	0

CeNN results for Control=yz

T\D	0.03	0.04	0.05	0.1	0.2	0.3	0.4	0.5
$\lceil 1.5T_0 \rceil$	0	0	0	0	0.050	0.025	0	0.025
$\lceil 2T_0 \rceil$	0	0	0	0.050	0.275	0.175	0	0
$\lceil 2.5T_0 \rceil$	0	0	0	0.350	**0.575**	0.425	0.075	0
$\lceil 3T_0 \rceil$	0	0.025	0.225	**0.700**	**0.650**	0.400	0.025	0
$\lceil 4T_0 \rceil$	0.175	**0.700**	**0.850**	**0.950**	**0.850**	0.450	0	0

CeNN results for Control=xyz

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