

Reviewer #2 Response

AMT manuscript: *Resolving heterogeneous structure and finite scattering in clouds and precipitation with ultra-high-resolution lidar*

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The authors would like to thank the reviewer for their thoughtful consideration and feedback on the manuscript. Their input has been valuable as the resulting revisions have improved the quality of the research article.

We respond to the reviewer's comments in this document. The comments are restated below in **orange** text, our responses in **black** text, and the revised sections in the updated manuscript in **blue** and strikethrough text. In this document, the term "original lines" refers to the line numbers from the manuscript draft prior to revisions, while "updated lines" refers to the ones in the revised version.

Reviewer #2

1. There is an inconsistency in the treatment being presented. That is, the stochastic nature of the backscatter is treated, but the transmission term remains unchanged from the standard treatment. Treating this would likely be a non-trivial complication. Further, I can well believe that in most circumstances that this term (which will depend on the fluctuations of the particle number in all the bins between the lidar and the target range) will approach the large number limit faster than the backscatter term. However, there should be some acknowledgment of this issue added to the paper.

The section addressed is restated here:

Following the single-scattering treatment of particle scattering and standard elastic lidar equation geometry, the received power at range R for a given particle configuration is given by

$$P(R|\Xi = \xi) = \left[\sum_{i=1}^N I(x_i, y_i, z_i) \frac{d\sigma_b}{d\Omega}(s_i) \right] A_{rx} R^{-2} O(R) \eta T(R)^2,$$

where $I(x, y, z)$ is the range-filtered irradiance field evaluated at the particle location, $d\sigma_b/d\Omega$ is the differential backscatter cross section, A_{rx} is the receiver aperture area, $O(R)$ is the geometric overlap function, η is the receiver efficiency, and $T(R)^2$

is the two-way transmission loss. In this derivation, we assume that the received signal is given by linear superposition of single-particle scattering contributions, such that multiple scattering, coherent interference, particle-scattering dependence, and inter-bin correlations are treated as negligible. These effects may be considered non-negligible under specific conditions, like optically dense clouds, which would require a more comprehensive assessment.

The intent behind this study was not to create a comprehensive stochastic lidar scattering model. Instead, the theoretical framework developed here was intended to isolate one mechanism during analysis: random scattering behavior within the interrogated sample volume and its effect on the lidar return.

We believe that this decision was justified because, outside of very specific scattering conditions, the backscatter process will be more sensitive to stochastic effects than the transmission process. We base this on two reasons: first, in agreement with the reviewer, transmission is an integrated quantity, meaning that it approaches the high-occupancy limit more quickly than backscatter because of the large range integration contributing to it; and second, diffractive interference and multiple scattering effects represent higher-order scattering processes (i.e., involve two or more particle scattering contributions) which occur less frequently and thus have an overall lesser impact on the observed signal. However, these assumptions would require more rigorous analysis.

We have revised the following section in original lines 149-153 to reflect this explicitly. First, once the stochastic model is derived (updated lines 149-155):

It is important to note that the single-scattering treatment is only applied to the backscatter term. This was done to isolate one mechanism during analysis: random scattering behavior within the interrogated sample volume and its effect on the observed lidar return. ~~In this derivation, we assume that the received signal is strictly given by linear superposition of single-particle backscattering contributions, such that higher order scattering effects like multiple scattering, coherent interference, and inter-bin correlations are treated as negligible.~~ Thus, we intentionally omit extending the stochastic treatment to scattering processes associated with transmission, such as multiple scattering and diffractive interference. These effects constitute higher-order scattering that are presumably less sensitive to stochastic processes. In addition, as a range-integrated term, the transmission approaches the high-occupancy limit faster than backscatter, further reducing sensitivity to sparse scattering processes during beam propagation. These assumptions though require more rigorous analysis if a comprehensive stochastic lidar model was desired.

~~These effects may be considered non-negligible under specific conditions, like optically dense clouds, which would require a more comprehensive assessment.~~

We revised the end of Sec. 3 (original lines 130-133) to improve the transition into the theoretical formulation (updated lines 129-132):

*In the following section, we **begin to** address this gap by deriving the volume backscatter lidar equation from discrete scattering statistics **applied to the sample volume**, providing the underlying theory for identification of **backscatter** regimes in which the continuum approximation ceases to be valid. This provides a stochastic framework **to begin to interpret the resolution dependence** ~~for resolution-dependent interpretation~~ of cloud lidar observations.*

We also revised statements throughout the paper to narrow the focus of the numerical study and make its scope more explicit. In the opening paragraph of Sec. 4.1 (original lines 137-138, updated lines 136-137):

*This derivation makes these assumptions explicit and clarifies how the classical lidar equation emerges as a well-constrained limit of the underlying discrete **backscattering** process.*

And also, in original line 139 (updated line 138):

*To begin, the **sampl**ed cloud particle field is modeled as a point process [...]*

2. I think that in terms of the effect on the linear superposition that the coupling between the transmission and backscatter (which is ignored in the treatment presented here) may be more important than MS effects.

The section of interest is restated here:

Another important implicit condition is the linear superposition of single-particle scattering contributions, such that multiple-scattering effects are negligible. Multiple scattering violates this condition by making the received signal depend on coupled photon paths rather than independent single-particle contributions, inducing statistical dependence in particle-scattering events. It can also increase effective optical path lengths and spatially redistribute scattered energy, thereby smoothing the measured return from heterogeneous cloud features. This condition is especially important in optically thick media, such as dense liquid-water clouds, where the high particle occupancy required for the volumetric approximation may be satisfied even as the single-scattering assumption breaks down.

We agree that coupling between transmission and backscatter represents an additional departure from strict linear superposition, and that its relative importance in comparison to multiple scattering depends on scattering regime. In the present formulation, this coupling is intentionally omitted for the reasons stated in Response #1.

Thus, we narrow the scope by revising the manuscript (original lines 208-229) as follows (updated lines 210-231):

*Another important implicit condition is the linear superposition of single-particle scattering contributions, such that ~~multiple-scattering effects~~ **higher-order scattering effects** are negligible. Multiple scattering violates the single-scattering condition by making the received signal depend on coupled photon paths rather than independent single-particle contributions, inducing statistical dependence in particle-scattering events. ~~It can also increase effective optical path lengths and spatially redistribute scattered energy, thereby smoothing the measured return from heterogeneous cloud features.~~ This condition is especially important in optically thick media, such as dense liquid-water clouds, where the high particle occupancy required for the volumetric approximation may be satisfied even as the single-scattering assumption breaks down. **Diffraction interference is also omitted in this formulation. Those contributions can be important depending on the scattering regime within clouds and precipitation, especially those consisting of particles that perturb the wavefront considerably.***

*While the breakdown of linear superposition must be considered in ~~these~~ **more rigorous studies**, it does not undercut the **benefit of this formulation**. ~~results in this study.~~ Rather, the significance of this discrete-to-continuum formulation **for scattering isolated to the sample volume** is that it provides a foundation for understanding and characterizing **the impact of discrete scattering on the lidar return**. ~~lidar measurements in clouds, even when the scattering regime is not easily classified as single or multiple.~~ Moreover, the ultra-high-resolution architecture provides a pathway for investigating **higher-order scattering effects (e.g., multiple scattering and coherent interference)** ~~multiple-scattering effects~~ in clouds at scales that would otherwise be obscured by conventional averaging.*

*Together, these assumptions define a regime in which the **backscatter component** of the lidar equation can be interpreted as a homogeneous, continuum approximation of an underlying discrete **backscattering** process. In this regime, the effects of individual particle configurations **within the sample volume** on the measurement (i.e., finite scattering and heterogeneity) are suppressed. Outside of this regime, deviations arise, leading to departures from the ensemble-mean*

description. However, while the preceding discussion identifies the physical conditions under which the continuum approximation fails, it does not quantify the resulting error introduced when the volume backscatter formulation is applied to *finite backscattering regimes*. ~~outside its validity regime.~~ This motivates a statistical formulation of the variability induced by finite-particle occupancy *within the sample volume*.

4.2 Finite-sampling variability and the continuum limit

*In this section, we derive finite-sampling variability: an expression that quantifies how strongly *the backscatter contribution to* a lidar measurement is influenced by the discreteness of the underlying particle field.*

Later, in original lines 257-259, we better qualify the statement (updated lines 260-266):

Although finite-sampling variability is not intended as a comprehensive uncertainty metric, it can be interpreted as an estimate of the uncertainty associated with applying the volume approximation to backscatter in finite-particle regimes. [...] This assumption may not hold in all regimes when diffractive effects from the propagated beam significantly perturb the irradiated field.

The conclusion section (original lines 362-365) has also been refined to better encapsulate the focus of the study (updated lines 374-377):

*In this study, we interrogated the conditions under which *interpretations of* cloud-lidar measurements *can deviate from* ~~be interpreted through~~ the conventional volumetric framework. First, we used ultra-high-resolution lidar observations of clouds to reveal fine-scale discrete scattering features, i.e., heterogeneous structures and finite-particle scattering. This motivated a stochastic description of *scattering within the cloud-lidar sample volume*, ~~cloud-lidar measurements~~, framing *lidar* returns as the cumulative backscatter from individual hydrometeors.*

3. In other words.. 1. Multiple-scattering is not important. "Directives effects" on the beams shape imply forward scatter events giving rise to photons that are later detected by a backscatter event. 2. The transmission term can be treated as being constant (see my earlier comments).

The section of interest is restated:

Note that separating the irradiance field from the particle backscatter contribution assumes that the field is largely insensitive to the presence of particles. This assumption may not hold in all regimes when diffractive effects significantly perturb the irradiated field.

The effects described here are outside the scope of the present formulation, which was designed specifically to isolate stochastic backscattering behavior within the interrogated sample volume. We have already revised the relevant text accordingly.

4. I think the authors should expand their conclusion and discussion! It seems to me that by appropriately aggregating their measurements it should be possible to infer meaningful estimates of the range-time dependent total number density (N) corresponding to traditional scales. This by itself would be very broadly relevant information for cloud-precipitation-radiation studies etc.. Moreover, it looks like particle backscatter pdfs (i.e. $dN/d\sigma_b$) can be estimated. From there, by applying scattering models .e.g. Mie theory for spheres) then $dN/dr(r)$ could be estimated. To do this the attenuated backscatter signals would have to be inverted/calibrated to produce estimates of Particle Backscatter (and not just relative attenuated backscatter or what the author's call flux (e.g. range corrected photon counts)).

We amended this paragraph (original lines 375-379) to better highlight the implications of these results and point towards future directions of work (updated line 384-391):

While the results of this work suggest that lidar measurements of clouds and precipitation may not be as straightforward as previously thought, ~~but~~ they also open the door to new possibilities for lidar design and retrievals ~~that were~~ previously out of reach. For instance, the results in Sec. 5 strongly suggest that direct particle-resolved estimates of both number concentration and backscatter intensity for sparse features are possible (like in light rain and ice clouds), which would help constrain our understanding of the radiative properties of clouds and precipitation. Furthermore, the potential of ultra-high-resolution lidar extends beyond the work presented here. Because the observed scattering regime depends on both the cloud microphysical properties and instrument configuration, future lidar designs may be able to leverage this relationship to isolate variable- or finite-discrete-scattering effects and extract new information about the environment.

In the spirit of expanding the discussion, we believe that another important implication of these results should be highlighted. Specifically, variable- and finite-scattering (like those observed in this paper) should impact the accuracy of conventional lidar retrievals, which are generally predicated on the continuum-scattering assumption underlying the volume

backscatter lidar equation. In its current form, the manuscript does not immediately communicate this important consequence. By expanding on this, we hope that it challenges cloud-lidar (and other particulate-lidar) users to consider how non-continuum scattering may impact the interpretation of lidar signals in clouds. It also motivates future instrument-configuration developments that interrogate discrete scattering more directly. We insert this section and revise the conclusion (updated lines 392-404):

It is also worth highlighting that this work exposes two limitations in conventional cloud-lidar retrievals: (1) atmospheric structure can evolve more rapidly than the lidar sampling scale, and (2) in sparse particle regimes, even a non-evolving atmosphere may not be well represented as a continuum. Careful attention must therefore be given to identifying variable or sparse scattering regimes in clouds to reduce ambiguity during signal interpretation. These results ~~motivates a~~ motivate the need for a more unified framework for lidar measurements in sparse or transitional cloud regimes, where neither purely continuous-media assumptions nor coarse averaging are strictly valid. In such cases, observables must be interpreted through models that explicitly account for individual particle-scattering contributions and their evolving statistical nature. Furthermore, although the present work focuses on cloud lidar, the same implications extend to sensing other sparse or heterogeneous particulate media, such as wildfire smoke, dust layers, and sea spray, as well as other remote-sensing modalities, like radar and passive imagers.

Conventional lidar resolutions will nevertheless remain widely used in cloud sensing, making it important to understand the conditions under which the volumetric interpretations remain valid. The ultra-high-resolution approach presented here provides a means to probe those limits directly, characterize the uncertainties that emerge as those assumptions break down, and motivate new approaches for measurements in sparse, variable, and transitional scattering regimes. ~~Conventional lidar resolutions will continue to be used in cloud sensing. The ultra-high-resolution approach presented here complements those measurements by providing a mechanism to better understand the uncertainties and validity of volumetric lidar regimes while motivating new approaches. Although the present work focuses on illustrative cloud examples, the same principles extend to other sparse or heterogeneous particulate media, like smoke plumes and aerosol layers.~~