Response to Reviewer 1

We thank the reviewer for carefully evaluating our manuscript and for providing thoughtful and constructive comments. We have considered all of the points raised and have revised the manuscript accordingly. Below, we have listed the reviewer's major and minor comments (in italic font), followed by our responses (in regular font).

Major comments

In my opinion, section 2 should be reorganized in a manner as it was done in KS22. There should be a subsection about the temperature distribution used (equations (6) and (2)), from this the diurnal heating rate (5) can be derived and finally the initial values T and the pressure field can be given.

In another subsection it should be described how to determine the constant pressure upper boundary surface. Some parts are mentioned e.g. in lines 98-103. This seems to be slightly different from KS22 (?).

In a third subsection one should describe what is necessary on the model side, to successfully perform this test (Rayleigh damping, ...)

Finally, in an own section the numerical experiments should be placed. This would e.g. contain l. 114-120 and l. 125-...

We agree that the description of this global diurnal heating test case was not well organized and thank the reviewer for the helpful suggestions on how to improve it. We have restructured this section accordingly and believe that these modifications have significantly improved the clarity of the presentation for this case. We have included the revised text for the first part of this section below:

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2. Diurnal heating test case

2.1 Analytic temperature field and internal diabatic heating rate

The 2-D diurnal heating test case described in KS22 is based on a linear analytic solution for a steady periodic wave driven by a propagating internal heating function. To modify this test case for a global domain, we specify an initial temperature field and a time dependent diurnal heating rate that are essentially the same as that in KS22 at the equator, but that tapers the initial temperature perturbation and the heating rate to zero at the north and south poles. The analytic temperature field is specified in a form that permits the hydrostatic equation to be integrated vertically to yield a closed form analytic expression for pressure. For this implementation on the globe, the analytic

temperature T_a maintains the same form as in the 2-D case:

$$T_a = T_t \left[\frac{T_t}{T_b}(1 - \tanh \tilde{z}) + \tanh \tilde{z} \right]^{-1}. \tag{1}$$

Here, T_b is the constant temperature at the fixed height z_b of the lower boundary of the simulation domain, z_d is the scale height for the lower level vertical temperature gradients, with $\tilde{z} = (z - z_b)/z_d$, and T_t is the temperature at $z \to \infty$, given by

$$T_t = \overline{T} + \Delta T_d \cdot \sin(\lambda + 2\pi\omega t) \cos\gamma, \tag{2}$$

where $\overline{T}$ is the initial mean temperature at the top boundary, and ΔT_d defines the amplitude of the initial sinusoidal diurnal variation along the upper boundary. In specifying the horizontal variation of T_t, γ and λ denote the latitude and longitude, respectively, and $\omega = \text{day}^{-1}$. For this test case, T_t also accurately represents the temperature along the constant pressure upper boundary of the simulation domain. Here, equations (1) and (2) are the same as KS22 (14) and (16) except for the addition of the $\cos\gamma$ dependence in (2). Since the linear analytic solution derived in KS2 does not include the influences of the horizontal winds, the global analytic solution is the same as in the 2D case except for the latitudinal dependence included in (2).

The diurnal heating rate Q_d governs the evolution of the flow as a forcing term in the thermodynamic equation. Taking the time derivative of (1), Q_d becomes

$$Q_d = \frac{\partial T_a}{\partial t} = \frac{\partial T_t}{\partial t} \tanh \tilde{z} \left[\frac{T_t}{T_b} + \left(1 - \frac{T_t}{T_b}\right) \tanh \tilde{z} \right]^{-2}. \tag{3}$$

where

$$\frac{\partial T_t}{\partial t} = 2\pi\omega\Delta T_d \cdot \cos(\lambda + 2\pi\omega t) \cos\gamma. \tag{4}$$

This expression for Q_d is the same as in KS22 except that the second term in the numerator of KS22 (15) has been dropped as it is negligible for this case.

2.2 Height of the constant pressure upper boundary

To achieve a constant pressure upper boundary condition in MPAS, KS22 implemented a coordinate transformation in the upper portion of the model domain to allow the hybrid coordinate surfaces to transition adaptively from a constant height surface at a specified height z_p to one at height H that follows a specified constant pressure surface at the model top:

$$z = \left(\frac{\zeta - z_p}{H_0 - z_p} \right)^2 (H - H_0) + \zeta, \quad \text{for} \quad z > z_p \tag{5}$$

such that the transformed coordinate $\zeta = H_0$ at $z = H$. For this test case, we follow KS22 and set $z_p = 200$ km. We specify the desired constant pressure

along the top boundary to be $p_{top} = 2.18\text{x}10^{-8}$ Pa, which is the value obtained by vertically integrating the hydrostatic equation for the sounding with $T_t = \overline{T}$ in (1) and $z_d = 50$ km from z_b to H_0. We then adjust the height H of the top boundary at the end of each time step based on the deviations of the top pressure from the target value p_{top} using the hydrostatic vertical extrapolation based on $\phi^* = \ln(p^*/p_0)$ as defined in KS22 eq. (5):

$$H = H^* + \alpha \frac{R_d T^*}{g}(\phi^* - \phi_{top}), \tag{6}$$

where the asterisk superscript denotes values along the upper surface at the current time step or iteration and $\phi_{top} = \ln(p_{top}/p_0)$. The dimensionless height adjustment coefficient α is unity for a full hydrostatic extrapolation, but can be adjusted to smaller values to remove any oscillations in the upper boundary height, if necessary.

We initialize the thermodynamic fields and the height of the constant pressure upper boundary by utilizing the iterative technique described in KS22. With this approach, we first compute the hydrostatically balanced thermodynamic fields using (1) and (2) based on a rigid-lid upper boundary at the nominal height of the top boundary H_0. A new height H for the top boundary is computed from (6) with the adjustment coefficient α set at its full value $\alpha = 1.0$ for this test case. With this new height, we adjust the height of the coordinate surfaces using (5), reinitialize the thermodynamic fields on the modified coordinate surfaces, and then continue the iterative process. This approach yields a well converged initialization with a constant pressure along the model upper boundary in three iterations. The initial temperature field is displayed in the vertical cross section along the equator in Fig. 1, which is similar to Fig. 2 in KS22.

[Figure 1]

2.3 Test case parameters and model configuration

Following KS22, we place the lower boundary at $z_b = 100$ km, where we specify a constant temperature $T_b = 200$ K and pressure $p_b = 0.03$ Pa. At the top boundary of the simulation domain, we specify an initial mean height $H_0 = 600$ km, and define the initial top temperature $T_t(t = 0)$ from (2) using $\overline{T} = 850$ K and $\Delta T_d = 150$ K for this test case. Although the thermodynamic coefficients and gravity vary throughout the thermosphere, for consistency with the analytic solution, we specify constant values for the gas constant ($R = 500$ m^{2}s^{-2}K^{-1}), the heat capacity ($c_p = 1300$ m^{2}s^{-2}K^{-1}), and the gravitational acceleration ($g = 9.81$ ms^{-2}), as in KS22.

As mentioned above, we seek qualitative guidance in evaluating the results from this idealized test case through comparisons with a simulation from the WACCM-X model (Liu et al. 2018). For this purpose, we have chosen a WACCM-X simulation valid at 00Z on 21 March 2000 for comparison because

at the spring equinox the global flow fields are most symmetrical about the equator, based on climatology. The parameters for our idealized diurnal heating test case are the same as those used in KS22 except that $\overline{T}$ has a lower value for better consistency with this WACCM-X simulation

In simulating this test case on the globe, we employ the same configuration of MPAS-A as in the 2-D simulations described in KS22 except that the 2-D mesh configuration is replaced by the 10242 cell global mesh (∼240 km horizontal grid size). We employ the transformed vertical coordinate described in KS22 that follows the height of the time dependent constant pressure upper boundary as described in KS22 with a constant $\Delta\zeta = 5$ km nominal vertical grid spacing on the transformed grid.

After initializing the model fields as described above, the dynamical equations are stepped forward in time using the split-explicit integration numerics in MPAS-A, as described in Skamarock et al. 2012, with an overall time step of $\Delta t = 600$ s and a smaller acoustic time step $\Delta\tau = 100$ s. A diurnal heating function based on (3) and (4) is included in the prognostic equation for the potential temperature to provide the periodic thermodynamic forcing. As explained in KS22, the linear analytic solution produces unrealistically large horizontal velocities (even exceeding the speed of sound). Consequently, in lieu of realistic model physics, a dissipation mechanism is required to limit the unrealistic growth of the horizontal velocity and to produce a proper phase relationship between the temperature and the zonal wind. Thus, we include a Rayleigh damping term in the horizontal velocity equation with a coefficient of $K = 0.001$ s^{-1}.

In conducting simulations over several diurnal periods with the above described configuration, we found that the longitudinal and meridional winds developed an essentially steady periodic structure, but the temperature field at upper levels continued to increase over time. This temperature rise appears to be associated with a net positive change in the internal energy as the simulation evolves, and may occur because appropriate cooling mechanisms, such as molecular thermal conduction, are not considered in the model. The heating function (3) we derived in KS22 for the 2-D diurnal heating case was based on the linear equations in which the horizontal winds are neglected as they are higher order. Thus, to restrict the continuing increase in temperature due to nonlinear effects, we have modified the heating function by adding a Newtonian cooling term that acts on the deviation of the temperature from what it would be if the solution was linear and the horizontal winds were ignored. The modified heating rate Q'_d then can be written as:

$$Q'_d = \frac{\partial T_a}{\partial t} - \beta(T - T_a) \tag{7}$$

where $\partial T_a/\partial t$ is as defined in (3) and T_a is the analytic temperature based on (1). The Rayleigh damping coefficient is set at $\beta = 2 \times 10^{-5}$ s^{-1} as this was

about the smallest value that still allowed the simulated temperature field to converge to a nearly steady periodic structure.

2.4 Simulation results

[Remainder of section 2]

Minor comments

• *author list: I think it is 'William' instead of 'Willam' Skamarock.*

Fixed

• *l 37: 'although we no longer have a linear analytic solution for comparison ...'. Why is it not possible to do similar derivations as in KS22? As far as I can see, mainly the hydrostatic equation to derive the pressure field was used and a simplification to derive the velocity field was made. The additional* $\cos\gamma$*-factor shouldn't be a too hard obstacle, isnt it? There seems to be a remark in l. 130 which I unfortunately don't understand. Or are there other obstacles that prohibit the derivation of a similar analytic solution? (no shallow atmosphere approximation?).*

Upon further reflection, we agree that the 2-D linear analytic solution derived in KS22 should remain valid for the global configuration with the addition of the $\cos\gamma$-factor in the expression for T_t. The influence of the Coriolis terms would complicate this solution, but as we have acknowledged, the feedback of the horizontal winds are not included in this analytic solution. We have revised the discussion of this global diurnal heating test case (see the revised section 2.1 above) to utilize the validity of this analytic solution in explaining the initial temperature field and the diurnal heating rate used in the simulations. We thank the reviewer for pointing this out as it helps further clarify the description of this test case.

• *l 65: '..* T_{t_0} *at the constant pressure upper boundary ...'. If I insert eq. (1) into eq. (2) at* $z = H$ *then I don't get* T_{t_0}*. Isn't* T_{t_0} *rather the temperature for* $z \to \infty$*?*

Yes, T_{t_0} is the temperature for $z \to \infty$. However, for the sounding used in this test case, the temperature is virtually constant in the thermosphere, so T_{t_0} is also the initial temperature at $z = H$. We've clarified this in the revised section 2.1 above.

• *eq. (1): this is just the specialized version of eq. (6) for t=0. I see the need for an initial condition, but again: the analogous presentation in KS22 starting from the full time dependent T-field is somehow much clearer to*

understand. By the way, the denotation with a double lower index 0 is misleading, t is obviously the denotation for 'top', t_0 gives the false impression of an initial time. I would rather write $T_{t,0}$ or even better $T_t(t = 0)$.

We agree and have removed that eq. 1 and begin with the full temperature expression as indicated in the revised text above. We have also changed the notation for the initial top temperature to $T_t(t = 0)$ as suggested by the reviewer.

• *l. 74: 'Here, T_L is the temperature from the analytic solution ...'. What does this sentence mean? Do I have to insert the analytic solution from KS22 in T_L? I guess not. Eq. (2) is just the eq. (14) from KS22 and I think this is what the authors wanted to express.*

Eq. (2) in the manuscript is just the analytic expression eq. (14) from KS22 as the reviewer points out, and we have stated that more directly in the revised text above. We have also changed T_L to T_a to help emphasize that. We only use a subscript here for T_a because we need to reference that analytic value in our expression for the modified heating rate in eq. (7) in the revised text above.

• *l. 77-84: Also this paragraph is not wrong, but simply misleading. The reader expects the description of an idealized test case, therefore he/she wants to know which values e.g. for zb, Tb, Pb, Tbar, ... must be specified. A sentence like 'For this purpose, we have chosen a WACCM-X simulation valid at ...' gives the impression that the modeler has to dig into this data set first. I think one could improve this text passage by just swapping lines 77-80 at the end of the paragraph with the note 'To motivate these values ...' (or one could could even put it into a footnote).*

The parameters z_b, T_b, P_b, H_0, and ΔT_d are all the same as those used in the 2D test case presented in KS22. The only modification in the parameters is that we reduced the value of $\bar{T}$ to a value more consistent with the mean temperature at the top of the domain in the WACCM-X simulation we chose for comparison. We have moved the reference to the WACCM-X simulation to the end of the paragraph as suggested by the reviewer to help clarify that there is little tailoring of the parameters of the parameters to the specifics of the WACCM-X simulation.

• *l. 125: what means 'initial simulations' ?*

We have removed "initial" from that sentence; there is no need for it.

• *l. 205: I would add 'parameterization' after '(YSU) gravity wave drag'*

Done