https://doi.org/10.5194/egusphere-2026-3507
Preprint. Discussion started: 26 June 2026

To tip or not to tip

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Abstract. Tipping points have become a buzzword in Earth sciences, but ambiguous or overly narrow definitions of tipping are causing confusion around the concept. Agreeing on what tipping means, and whether a system tips or not, is important for robust science and communicating tipping risk. Based on a critical evaluation of existing tipping definitions, we propose a revised, general definition that characterizes a tipping event as a persistent nonlinear transition in forced systems. Our definition emphasizes both the phenomenology (observed time series) and cause (feedback mechanism) of a tipping event. Inspired by response theory, our proposition is compatible with more specific mathematical formulations while avoiding challenging notions such as bifurcations, equilibrium states, abruptness and irreversibility – making the definition testable also on transient dynamics in diverse complex systems under time-varying forcing. We showcase its practical use and limitations in a toy model and a case study of Earth system model data.

1 Introduction

Fifty years ago, René Thom's catastrophe theory was popularized as a mathematical framework capable of explaining sudden, abrupt changes in a wide range of real-world contexts (Thom, 1974; Zeeman, 1976). The theory describes such changes, or "catastrophes", in terms of singularities where a control parameter reaches a critical point. Seemingly a universal and powerful tool, the theory received great attention in science and beyond, finding application in diverse fields from biology (Thom, 1969) to economics (Zeeman, 1974), psychology (Stewart and Peregoy, 1983) and even linguistics.

However, soon after the initial rise, criticism grew regarding the theory's lack of justification and predictive power in applications (Zahler and Sussmann, 1977; Kolata, 1977). Dissonance emerged between the rather narrow mathematical assumptions underlying catastrophe theory and the complexity of systems under study. Arguably, loose applications of the theory and the difficulty of testing its hypotheses with empirical evidence eventually led to its demise (Rosser, 2007).

In recent years, a theory of tipping behavior has been developed to describe and understand critical change, centered around the concept of tipping points. Much of the mathematical formalism is rooted in bifurcation theory of dynamical systems (Ashwin et al., 2012; Kuehn, 2011), which is closely related to concepts of catastrophe theory (Arnol'd and Gamkrelidze, 1994). Popularized by Gladwell (2000), the term 'tipping point' has been adopted in various scientific disciplines – sometimes referring to a well-defined mathematical scenario while at other times invoked rather as a metaphor for large, rapid change. Also here, concerns about overuse, ambiguity and limited applicability of the concept are increasingly being raised (Kopp

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et al., 2025a, b; Milkoreit, 2023; Shaw and Stevens, 2025; Russill, 2015). If not sufficiently clarified, there is a danger that tipping theory will suffer the same fate as catastrophe theory.

Since their introduction to Earth system science (Lenton et al., 2008), 'climate tipping points' have been a fast growing research topic as well as a buzzword in climate change communication to the public (Armstrong McKay et al., 2022; Lenton et al., 2019). The Global Tipping Points (GTP) Report 2025 discusses tipping points in the physical climate system and in society, extending the scope to both natural and socio-economic or technological systems (Global Tipping Points, 2025). However, also here a lack of consensus persists about a general definition of tipping points and tipping behavior (Abrams et al., 2026). In light of the ongoing seventh assessment cycle of the Intergovernmental Panel on Climate Change (IPCC), we see a need and opportunity to revisit current tipping definitions, with the aim of establishing a terminology that accommodates both the mathematical formalism and the real-world complexity of tipping phenomena (Smith et al., 2025). Agreeing on what tipping means, and whether a system tips or not, is important both for robust science as well as for communicating about nonlinear change.

This paper seeks to provide the discussion of tipping behavior with a clear, practical foundation. While we focus here on the domain of Earth system science, our proposition is more general and intended to be meaningful also in other contexts. Our goal is to define a tipping event – rather than a tipping point – in a way that is empirically testable and avoids overly restrictive assumptions, yet remains compatible with the existing mathematics and intuition of tipping phenomena. In section 2, we critically evaluate current tipping definitions, highlighting key issues. Based on this, we propose a revised definition of tipping and discuss its implications in section 3. We illustrate its connection with bifurcation-, noise-, and rate-induced tipping and apply the definition to model simulations of a climate tipping system (section 4). A summary and discussion conclude the paper, where we also reflect on wider applicability and implications of our tipping definition.

2 Problems with existing definitions

The latest Assessment Report of the IPCC defines a tipping point as "a critical threshold beyond which a system reorganizes, often abruptly and/or irreversibly" (Intergovernmental Panel On Climate Change, 2023). Other versions have been proposed by Lenton et al. (2008) and updated in the GTP Reports (Global Tipping Points, 2023, 2025) (see also Nes et al. (2016); Milkoreit et al. (2018); Brovkin et al. (2021); Armstrong McKay et al. (2022); Winkelmann et al. (2025); Abrams et al. (2026)), but we argue in the following that they share common challenges causing confusion and misperceptions. These are mainly a lack of specificity, uniqueness and testability.

2.1 Threshold or event?

Originally, the term 'tipping point' was predominantly used to refer to a threshold, either in an external forcing parameter (control parameter) or in the system's state (Lenton et al., 2008; Nes et al., 2016). A single, tiny perturbation past the threshold would cause a drastic change in the long-term behavior of the system. In case of a forcing threshold, this can be mathematically described as a bifurcation where the stability properties of the system change instantly and qualitatively. In particular, the fold

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(or saddle-node) bifurcation has become a paradigm of a tipping point, and bifurcation-induced tipping (Ashwin et al., 2012) shapes much of the thinking about tipping dynamics.

The problem is that such bifurcations characterize the equilibrium properties of the system. If the forcing changes slowly compared to internal system timescales, the equilibrium description usually yields an adequate approximation of the dynamics: the tipping transition occurs immediately after crossing the tipping point, and the system change is abrupt and irreversible relative to the forcing change. However, in most situations of interest (including anthropogenic climate change), the system and forcing timescales are not cleanly separated, meaning that the system can be driven far from equilibrium and transient dynamics become important (Caraballo and Han, 2016; Hastings et al., 2018). A common misunderstanding is that crossing a tipping threshold always coincides with observing a tipping event. On the contrary, owing to multiscale dynamics, inertia and chaos, tipping can be significantly delayed after crossing a threshold, avoided under temporary threshold exceedance, or happen even without crossing a forcing threshold (Bastiaansen et al., 2023; Ritchie et al., 2021).

The concepts of rate-induced (Wieczorek et al., 2023; Ritchie et al., 2023) and noise-induced (Freidlin and Wentzell, 1998) tipping deal with scenarios where the system is pushed out of equilibrium by time-dependent forcing or random variability (Ashwin et al., 2012). Here, the relevant threshold (in multistable systems (Feudel, 2008)) is a basin boundary that separates competing basins of attraction (Feudel, 2023). The basin boundary can be a complicated geometrical object partitioning the state space – a tipping surface[1] rather than a tipping point in systems with more than one dimension. This tipping boundary generally moves in state space as a function of system parameters. Tipping occurs when the basin boundary is crossed without returning. Also bifurcation-induced tipping may be viewed as a special case of crossing a basin boundary, since at a (fold) bifurcation the stable equilibrium itself collides with its basin boundary and thereby disappears.

Thus, from a dynamical systems perspective, it seems reasonable to generally define tipping as the persistent crossing of a state space threshold partitioning different domains of stability. However, applying this definition in practice requires knowledge of the global stability landscape of the system. This is often not known and the location of basin boundaries cannot simply be derived from observed trajectories of the system.

Given the challenges of identifying critical thresholds in complex systems, the term ‘tipping point’ is increasingly used to refer to an event, not a threshold. For example, the GTP Report 2025 defines: a tipping point “occurs when change in part of the system becomes self-perpetuating beyond some threshold [...]” (Global Tipping Points, 2025). This shift from threshold to occurrence is widely reflected in current tipping point language, where many use ‘tipping point’ as a synonym for transition. However, this blurs the important distinction between the stability properties of the system and its transient dynamics, causing misunderstanding about the timing and predictability of tipping. For example, a change point detected in a timeseries generally does not indicate the location of a critical threshold but is often labeled as the tipping point.

[1]Note that the tipping surface in *state* space discussed here is different from a tipping surface in *parameter* space (or forcing space) discussed by Abrams et al. (2026).

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2.2 Abruptness and/or irreversibility

Tipping behavior is considered important because it is associated with large, abrupt, nonlinear change that cannot easily be reversed. The notions of abruptness and irreversibility are thus central to many tipping definitions (Intergovernmental Panel On Climate Change, 2023; Global Tipping Points, 2025). However, it is often not useful to define a concept via its relevance. In fact, the two attributes are neither necessary nor sufficient criteria for tipping, as is evident in the IPCC formulation, "often abrupt and/or irreversible" (Intergovernmental Panel On Climate Change, 2023).

Abruptness implies a rapid change relative to a given timescale, but the choice of reference timescale is unclear. Is it the forcing timescale (Global Tipping Points, 2025), the typical rate of change in the system's history (Intergovernmental Panel On Climate Change, 2023), or a timescale of policy relevance (Lenton et al., 2008)? For example, present-day mass loss of ice sheets may be considered fast compared to glaciological (multi-millennial) timescales but not compared to the current forcing timescale, and is arguably not perceived as abrupt in society. Some tipping events may thus appear slow and gradual, especially relative to changing external influences. Nonetheless, it is possible to distinguish a slow tipping event from other gradual changes, as we will discuss in section 3. On the other hand, arguably not all abrupt changes classify as tipping events; they could be driven by an abrupt forcing change or represent an extreme event without a 'reorganization' (Intergovernmental Panel On Climate Change, 2023) of the system.

Defining irreversibility is an open problem in itself. While some call a change irreversible if it is not reversed under reversal of the forcing that caused it (Global Tipping Points, 2025), other definitions are related to the recovery rate of a system after a perturbation (Intergovernmental Panel On Climate Change, 2023). These are two very different aspects in dynamical systems theory, and like abruptness require a clarification of timescales. One major problem with linking tipping to irreversibility is that even a linear system (which cannot have tipping points) can be irreversible under forcing reversal if it has inertia. This can result in spurious hysteresis effects under forcing rates faster than or similar to the internal damping timescale. Conversely, there is controversy about whether nonlinear but reversible shifts should be considered tipping events. This question relates to the debate about including Arctic sea ice and permafrost in the list of Earth system tipping elements (Notz, 2009; Li et al., 2013; Levermann et al., 2012; Brovkin et al., 2025; Armstrong McKay et al., 2022). As demonstrated in simple, monostable dynamical systems without bifurcations, a nonlinear sensitivity to a control parameter can give rise to tipping-like behavior in the absence of multistability and hysteresis (Scheffer et al., 2001). Lastly, irreversibility with respect to forcing reversal can only be empirically verified if the forcing is actually reversed, making it difficult to test in practice.

In summary, while abruptness and irreversibility can be important features of tipping behavior, they are poorly suited to define tipping dynamics and to distinguish tipping from other forms of change.

2.3 Metastability

The coexistence of multiple stable states (multistability) is central to many tipping definitions (Nes et al., 2016; Milkoreit et al., 2018). The mathematical concepts of bifurcation-, rate-, and noise-induced tipping all require multistability (Ashwin et al., 2012). At the same time, the IPCC and GTP definitions are more vague, requiring merely a "reorganization" (Intergovernmental

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Panel On Climate Change, 2023) of the system or "substantial and widespread impacts" (Global Tipping Points, 2025), without mention of an alternative stable state.

Multistability is an important concept for understanding tipping dynamics, but it describes the equilibrium properties of the system – like bifurcations. The term 'metastability' has been introduced to account also for transient dynamics, such as transient chaos or nonautonomous dynamics (Rossi et al., 2025). Metastability does not rely on equilibrium states but more generally refers to distinct, relatively long-lived dynamical regimes and shifts between them.

We argue that defining tipping as a transition between different stable equilibrium states makes it difficult to apply the definition to the full variety of tipping phenomena observed in complex systems. As mentioned above, the global stability properties are often not known. In adaptive systems and systems exposed to continuous time-dependent forcing, the notion of equilibrium can become less clear since the system may never settle in an equilibrium. A general tipping definition should accommodate aspects of metastable, transient dynamics and clarify what form of 'reorganization' occurs.

3 Tipping definition

Besides threshold behavior, abruptness, and irreversibility, a common aspect of existing tipping definitions is the self-amplifying, self-sustaining or accelerating nature of the change (Abrams et al., 2026; Armstrong McKay et al., 2022). This is typically linked to a positive (i.e. destabilizing) feedback mechanism becoming dominant in the system, leading to a nonlinear self-sustained response. We propose to focus on this aspect, taking both the observed behavior (phenomenology) and knowledge of the underlying mechanisms (cause) into account. Phenomenologically, a tipping event is a transition process in the system's state. The cause aspect of tipping requires system-specific knowledge about internal feedback processes and the mechanistic connection between forcing and response.

3.1 Definition

In short, we define a tipping event as a persistent nonlinear transition driven by a self-amplifying feedback process. Persistence means that the system settles in an alternative dynamical regime following the transition. By nonlinear we mean that the system observable responds disproportionately to a driver, which could be an external forcing or internal fluctuations. The change of the observable is thus not merely a linear function of a nonlinear forcing. Self-amplification means that a change in the observable leads to further change of the observable in the same direction.

To make this more precise, let $\tau = [t_0, t_1]$ denote a time horizon over which the time series of the system observable $\psi(t)$ and the forcing $F(t)$ are known. This time horizon sets the reference timescale on which tipping is assessed. If tipping occurs (once) during τ, we can conceptually divide τ into a pre-tipping interval $\tau_A = [t_0, t_s]$, a transition phase $\tau_{\text{tr}} = [t_s, t_e]$, and a post-tipping phase $\tau_B = [t_e, t_1]$, where $t_0 < t_s < t_e < t_1$. The exact choice of t_s and t_e is not important but τ_{tr} should roughly reflect the transition suspected in the time series of ψ.

We define that tipping occurs during τ if and only if there exists t_s, t_e such that

1. A persistent nonlinear transition occurs, i.e. the observable ψ

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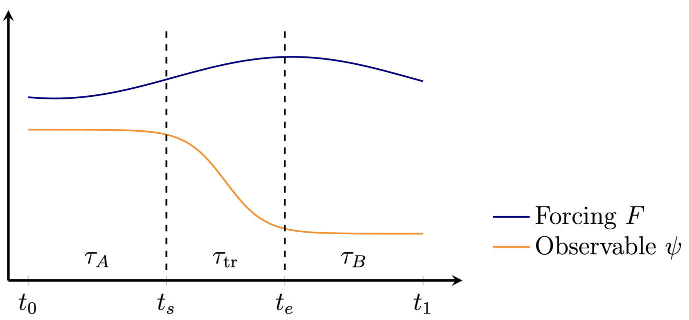

Figure 1. Schematic of the problem setting. During the time interval $\tau = [t_0, t_1]$, is the system tipping in the observable ψ under the forcing F?

(a) evolves in a regime of approximately linear response during τ_A;

(b) increasingly deviates from linear response during τ_{tr};

(c) evolves in a dynamically distinct (compared to τ_A) regime of approximately linear response during τ_B, where τ_B lasts at least as long as τ_{tr} (i.e. $t_1 - t_e > P(t_e - t_s)$ for some persistence $P \geq 1$).

2. The deviation from linear response during τ_{tr} is attributed to a positive feedback process that operates on the timescale τ_{tr}.

Criterion 1 (phenomenology) and criterion 2 (cause) are necessary conditions for tipping: phenomenology and system understanding must come together. In combination, these criteria are sufficient.

A *tipping system* is a dynamical system in which tipping can be shown to occur. The *tipping duration*, i.e. the duration of the tipping event, can be inferred from the duration of the amplifying nonlinear response. The *tipping onset* marks the point in time where the identified destabilizing feedback becomes dominant relative to stabilizing processes. This point in time does not necessarily coincide with an abrupt change point in the timeseries of the observable.

On a given forcing timescale, a tipping system is a fast (slow) tipping system if τ_{tr} is short (long) compared to the timescale on which the forcing varies. Tipping events can be bifurcation-induced, rate-induced or noise-induced as defined in Refs. (Ashwin et al., 2012; Wieczorek et al., 2023), or a combination of these. Furthermore, tipping can be induced by a singular shock (Halekotte and Feudel, 2020) or occur spontaneously following a long transient (Lai and Tél, 2011; Börner et al., 2026).

3.1.1 Remarks

– Constant forcing, $\mathrm{d}F/\mathrm{d}t = 0$ for all $t \in \tau$, represents a special case where any nonlinearity in the observable is necessarily a deviation from linear response. Transitions are then a result of internal variability, e.g. a noise-induced transition between metastable states or a chaotic transient.

– Our definition permits both irreversible and reversible tipping. For example, a monostable system with a nonlinear sensitivity to a forcing parameter could produce tipping dynamics in the sense of our definition.

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- We strongly recommend to distinguish between abrupt shifts and tipping. A tipping event can occur as an abrupt shift but may also appear more gradual in slow tipping systems. Conversely, not every abrupt shift is a tipping event: it can be a linear response to an abrupt change in the forcing, for example, or a non-persistent deviation from linear response. The persistence of a distinct dynamical regime after τ_{tr} also clarifies the distinction between a tipping event (persistent change) and an extreme event (non-persistent change).
- Our tipping definition assumes that tipping can be asserted based on the evolution of the forcing and observable during the interval τ, without knowledge of the system's history before τ. This is only the case if the system has sufficiently lost its memory at the beginning of τ. In multiscale systems with a slow timescale relative to τ, fast-slow dynamics such as relaxation oscillations could be classified as tipping events when considering only the dynamics during τ, whereas on longer timescales they would be interpreted as a mode of internal variability associated with a single dynamical regime.

3.2 Tipping test

To be useful, a tipping definition should be testable in the sense that applying the definition to empirical data yields a consistent assertion of whether tipping has occurred. In the following, we present a generic tipping test inspired by response theory (Ruelle, 2009; Ghil and Lucarini, 2020) that aligns with our tipping definition in section 3.

Given a timeseries pair of an observable $\psi(t)$ and forcing $F(t)$, the general idea is to test the null hypothesis: the evolution of $\psi(t)$ can be explained by a linear response model (null model) subjected to $F(t)$. Rejection of this null hypothesis is a necessary condition for tipping. If the deviation from the null model is then shown to be persistent and attributed to a plausible feedback mechanism, this signifies tipping.

3.2.1 Deviation from linear response

Consider a nonlinear dynamical system described by[2] $\dot{u} = f(u, F(t))$. The state u maps to the observable ψ via the projection $\psi = Mu$. Linearizing around an unperturbed reference state u_* for reference forcing $F = F_*$ (let $F_* = 0$ without loss of generality), we obtain a counterfactual linear system, $\dot{y} = Ay + BF(t)$ with $y(t) = u(t) - u_*$. The null model predicts the solution

$$x(t) = x_* + e^{-t/m}(x_0 - x_*) + \int_{t_0}^{t} G(t,t')F(t')\mathrm{d}t' , \quad t \geq t_0 , \quad x_0 = x(t_0) , \tag{1}$$

for the observable $x = My$ of the linear system. Here m is a damping timescale of the unperturbed system (with projected reference state $x_* = Mu_*$) related to the system's inertia. The linear response to the forcing F is encoded in the (first-order) Green's function G (Kubo, 1966; Ghil and Lucarini, 2020; Ragone et al., 2016; Lucarini and Chekroun, 2024). Although the approximation (1) is exact only if the system is linear, it often remains valid also in nonlinear systems for a relatively wide range of forced perturbations (Leith, 1975; Hairer and Majda, 2010; Ragone et al., 2016). However, linear response breaks down near tipping thresholds (among other possible reasons, such as large perturbations) (Lucarini, 2025).

[2] Here $\dot{x}$ is typical short-hand notation for $\mathrm{d}x/\mathrm{d}t$.

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We introduce the absolute residual,

$$\eta(t) = |\psi(t) - x(t)|, \quad t \in \tau, \tag{2}$$

which quantifies the deviation from linear response. The timeseries of η during τ can be analyzed to identify tipping. The null hypothesis is rejected if η diverges significantly from zero, i.e. $\eta(t) > \eta_c$ for some tolerance η_c and $t \in \tau$. In practice, tipping requires that:

- The null model explains the behavior during τ_A (i.e. $\eta(t) \leq \eta_c$ for almost all $t \in \tau_A$).
- The same null model persistently fails to predict the behavior during τ_B (i.e. $\eta(t) > \eta_c$ during most of τ_B), but a recalibrated null model (e.g. with a different reference state) captures the behavior during τ_B.
- The increase of $\eta(t)$ during τ_{tr} is attributed in magnitude and rate to a positive feedback process.

This practical test highlights several implications of our tipping definition. First, if the system does not fulfill approximately linear response conditions before and after the transition, tipping cannot be asserted. This seems a reasonable restriction, since tipping should lead from one well-defined dynamical regime to another; it is less restrictive than considering transitions between equilibrium states. Essentially, this reflects that tipping is difficult to detect if the system is strongly forced or features strong nonlinear variability on the timescale τ. Secondly, tipping can only be determined retrospectively, after observing the system during the persistence time $[t_e, t_e + P(t_s - t_e)]$. Of course, it may be possible to monitor tipping indicators throughout τ, such as an increase in nonlinear response ($\mathrm{d}\eta/\mathrm{d}t > 0$), or to potentially predict tipping based on knowledge of the system's stability properties and feedback processes.

3.2.2 Elementary null models

In applications where the Green's function G is not known for the specific system, a simple Green's function can be fitted to the empirical data. The simplest choice is an exponential memory kernel,

$$G(t,t') = \frac{\beta}{m} e^{-(t-t')/m}, \tag{3}$$

where β is the sensitivity to the forcing F and the timescale m represents the system's inertia. This Green's function corresponds to the null model,

$$m\dot{x} = -(x - x_*) + \beta F(t), \tag{4}$$

with free parameters m, β and x_*. This null model captures the inertial relaxation to a moving reference equilibrium $x_* + \beta F(t)$ (provided that the system has sufficiently lost memory of its history before t_0; otherwise the pullback attractor must be considered (Ghil and Lucarini, 2020)). However, it cannot describe internal oscillations of the system. For systems featuring oscillatory dynamics, a more appropriate null model is given by the damped, driven harmonic oscillator (see Appendix A). Equation (4) may be viewed as a harmonic oscillator in the overdamped limit.

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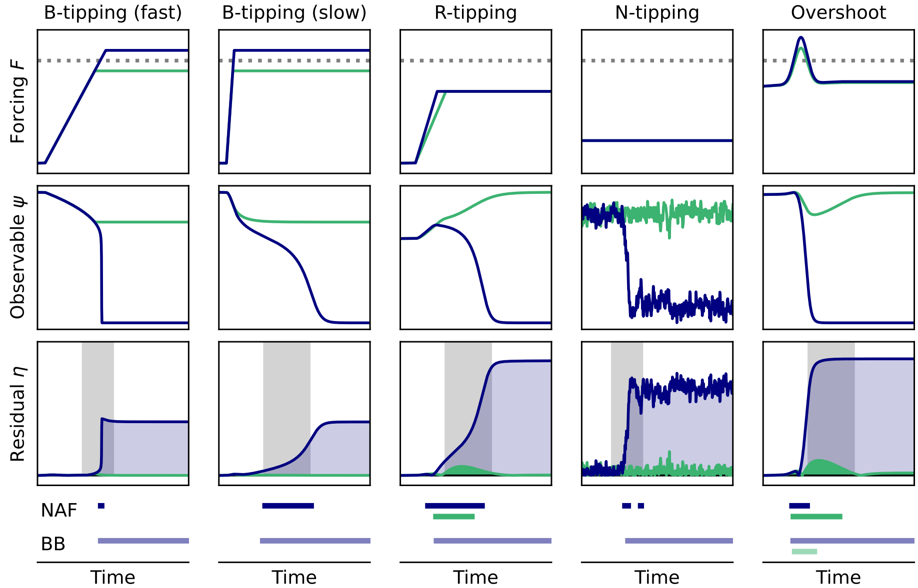

Figure 2. Different types of tipping demonstrated in the simple tipping system (5). Each scenario (column) shows a tipping case (blue) and a non-tipping case (green). Top row: Forcing protocols; the gray dotted line marks the fold bifurcation (i.e. the 'classical' tipping point). Row 2: Trajectories of the system determined under the respective forcing. Row 3: Nonlinear residual of the response η (see Eq. (2)). The null model (Eq. (4)) is fitted to each non-tipping case outside of the gray shaded area marking τ_{tr} (see Appendix B). Bottom: Horizontal bars mark the time periods during which the system evolves in a region of net accelerating feedback (NAF, defined by a downward concave potential landscape) or has crossed the basin boundary (BB) to the competing stable state.

4 Examples

To illustrate our tipping definition, we apply the tipping test to simulated tipping events in a simple toy model and in an Earth system model.

4.1 Simple tipping system

Consider a simple dynamical system that features a bistable regime with basin instability (O'Keeffe and Wieczorek, 2020), bounded by a fold bifurcation (based on Ritchie et al. (2023)):

$$m\,\mathrm{d}x = -x\left[(x - A - s(F(t)-1))^2 + F(t) - 1\right]\mathrm{d}t + \sigma \mathrm{d}W_t\,, \tag{5}$$

where m denotes inertia, A and s are parameters and W_t represents a Wiener process (Gaussian white noise) scaled by the noise strength $\sigma \geq 0$. This system is susceptible to bifurcation-, rate-, and noise-induced tipping. We can thus test our definition on these different cases as well as an overshoot case where the bifurcation threshold is crossed temporarily (Ritchie et al., 2021).

As a null model, we use the linear system in Eq. (4), with x_* set to the upper stable fixed point at reference forcing $F_0 = F(t_0)$. For each tipping scenario, we generate one trajectory that tracks the initial fixed point (non-tipping) and one trajectory

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that tips to the alternative stable fixed point (Fig. 2; details in Appendix B). In the noise-induced scenario, we add Gaussian white noise to the system under otherwise constant forcing. We consider bifurcation-induced tipping both in a fast system (small inertia) and a slow system (large inertia). In the fast case, tipping is abrupt relative to the forcing. In the slow case, both the tipping and non-tipping trajectories continue to change long after the forcing has stabilized, and the transition in the tipping case is delayed compared to the crossing time of the bifurcation.

In all non-tipping cases, the fitted null model accurately describes the evolution of the observable during τ_A and τ_B, even though the system is nonlinear (Fig. 2). During τ_{tr}, a transient nonlinear response signal is sometimes visible, for example under overshoot forcing. This is because the system temporarily crosses the fold bifurcation; the positive feedback amplifies the response but is subsequently reversed. The key point is that the residual η decays back to zero in the non-tipping cases, whereas it grows significantly larger and never recovers in the tipping cases. After the transition, the residual stabilizes in the tipping cases, showing a linear response again but now with the alternative stable fixed point as reference state.

The positive feedback arises from the presence of an unstable equilibrium that generates a basin boundary and, locally, a downward concave stability landscape (see Appendix B). Within the concave state space region, the linearized dynamics are destabilizing and accelerating, which we propose to interpret as a net positive feedback in an out-of-equilibrium context. Indeed, major increases of the nonlinear residual coincide with periods in which the system evolves within the region of net positive feedback. Additionally, tipping occurs when the system crosses the unstable equilibrium (basin boundary) permanently. We can thus attribute this nonlinear response to the action of a positive feedback.

4.2 AMOC tipping in an Earth system model

We next consider simulations with the Community Earth System Model (CESM) version 1, a comprehensive climate model with around 10^7 degrees of freedom (Hurrell et al., 2013). This model has been used extensively to study the tipping behavior of the Atlantic Meridional Overturning Circulation (AMOC), a major ocean current system, under changes in atmospheric CO_2 concentrations (van Westen et al., 2025, 2026). The AMOC is a widely studied climate tipping system and has been shown to feature bistability in CESM, where the two stable states correspond to a strong and a collapsed circulation, respectively (Dijkstra et al., 2026).

The AMOC response to radiative forcing depends strongly on the forcing path, meaning that not only the magnitude but also the rate of radiative forcing change is crucial. In CESM, the AMOC tips under certain CO_2 forcing scenarios while returning to the stable strong state in others, sometimes after a transient weakening. Since radiative forcing is related to the logarithm of CO_2 concentration, we use $\log[CO_2]$ as the forcing parameter F. The AMOC state is also affected by the surface freshwater forcing $\bar{F}_H$, which is set to a constant background value in the simulations (van Westen et al., 2026). The observable ψ is the AMOC strength (defined in Appendix C), measured as volume transport in units of Sv ($1\,\text{Sv} = 10^6\,\text{m}^3\,\text{s}^{-1}$) (Johns et al., 2023; Dijkstra et al., 2026).

One complication with the CESM simulations is that the AMOC strength can feature oscillatory dynamics in response to the forcing. The appropriate elementary null model is thus the damped, driven harmonic oscillator. With a background freshwater forcing of $\bar{F}_H = 0.18\,\text{Sv}$, the AMOC tips under the high-emissions Representative Concentration Pathways (RCP)

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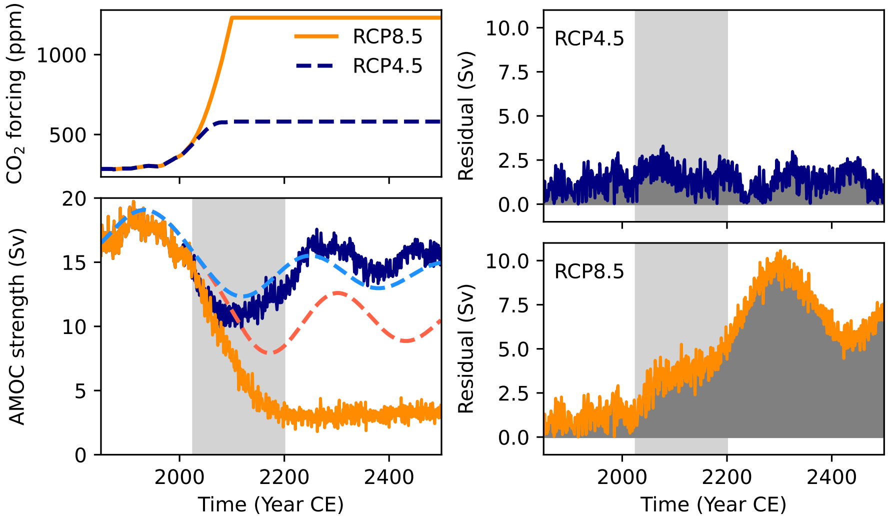

Figure 3. Tipping analysis of CESM simulations under the RCP4.5 (blue) and RCP8.5 (orange) scenarios for constant $\bar{F}_H = 0.18$ Sv (van Westen et al., 2025). The light gray shaded interval indicates τ_{tr}. The null model (light blue dashed) is fitted to the RCP4.5 case using data outside of τ_{tr}, with $\log[CO_2]$ as forcing (see Appendix C). The same null model is applied to the RCP8.5 case (red dashed).

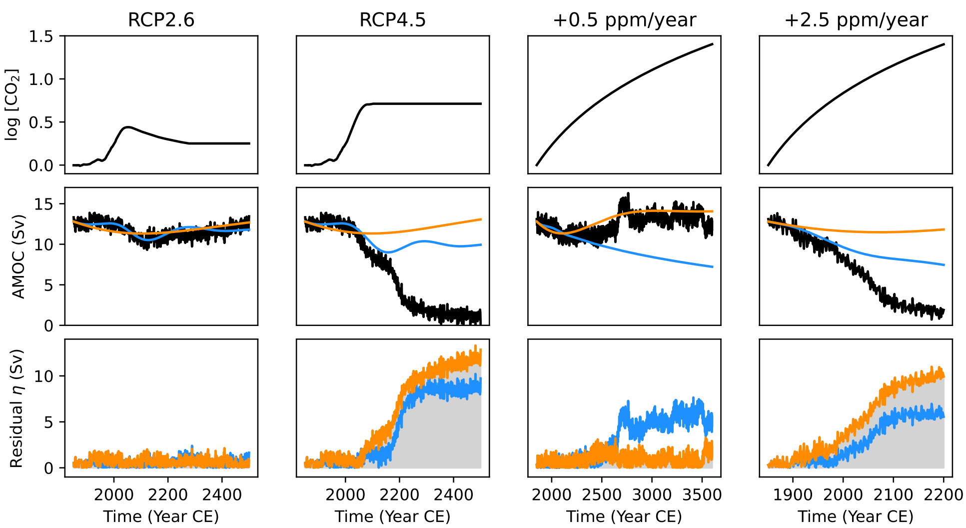

Figure 4. Tipping analysis of CESM simulations with constant $\bar{F}_H = 0.45$ Sv for two RCP scenarios (from 1850 CE extended to 2500 CE) and two linear CO_2 ramp protocols with different rates (van Westen et al., 2026). All trajectories are initialized from the same statistical equilibrium state. Predictions of a null model with multidecadal (blue) and multicentennial (orange) oscillations are shown for each case (see Appendix C). Tipping cases (RCP4.5 and +2.5 ppm/year) and non-tipping cases (RCP2.6 and +0.5 ppm/year) are accurately classified by the multicentennial null model but not by the multidecadal null model.

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scenario RCP8.5 but recovers to a strong AMOC state in the intermediate-emissions scenario RCP4.5. A null model with a natural frequency of roughly $1/41$ year^{-1} and damping ratio of around 0.1 (see Appendix A) can approximately explain the AMOC evolution under RCP4.5 (Fig. 3), suggesting that no tipping occurs. If the system would be linear, the same null model should also describe the evolution under RCP8.5; however, the null model persistently underestimates the magnitude of AMOC weakening, resulting in a nonlinear residual of more than 5 Sv. This nonlinear response can be linked to the positive salt-advection feedback, which has been shown to govern an AMOC collapse in CESM (Vanderborght et al., 2025). Beyond the year 2200, both the forcing and the AMOC strength are relatively constant under RCP8.5, giving a linear, non-oscillating response with a significantly weaker AMOC as the reference state.

A second complication arises from the timescale-dependent relationship between the AMOC and radiative forcing (van Westen et al., 2026), shown for a background freshwater forcing of $\bar{F}_H = 0.45\,\mathrm{Sv}$ (Fig. 4). Under rapid CO_2 increase, the AMOC strength tends to decrease, implying a negative forcing sensitivity β in Eq. (A1). Conversely, the AMOC can strengthen due to a slow multi-centennial increase in CO_2 concentration. An overdamped null model or multi-decadal oscillator model cannot simultaneously explain the linear response of the AMOC to slow and fast radiative forcing change. However, when allowing for centennial-scale oscillations, we can determine a single null model that correctly distinguishes between tipping and non-tipping cases under various forcing scenarios (Fig. 4; details in Appendix C).

This example demonstrates that our tipping test can be successfully applied to complex tipping phenomena in high-dimensional, multiscale dynamical systems. It also highlights the challenges of determining an appropriate null model based on limited empirical data. A poorly constrained choice of null model can inhibit the robustness of the tipping test and yield false positives or negatives.

5 Discussion and conclusion

Tipping points have become a highly popular concept in Earth sciences, but inconsistent language around tipping points is causing growing confusion. In this paper, we critically evaluated existing tipping definitions, in particular the definition used by the Intergovernmental Panel on Climate Change (IPCC). We identified several key issues: confusion between tipping thresholds versus tipping events, a problematic focus on abruptness and irreversibility, and a reliance on equilibrium concepts that have limited applicability to transient dynamics.

We proposed a revised definition that characterizes tipping as a persistent nonlinear transition of a system observable in relation to an external forcing. This effectively shifts the focus from tipping points to tipping events, acknowledging the increasingly common and sometimes confusing use of the term ‘tipping point’ as a synonym for transition. The definition avoids notions of abruptness, irreversibility, bifurcations and equilibria, instead emphasizing the nonlinearity and persistence of the change driven by a self-amplifying feedback process. Our goal was to define tipping in a general way that applies to diverse complex systems, minimizing restrictive assumptions while still formulating clear, testable, necessary and sufficient criteria for tipping.

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Our tipping definition is inspired by the idea that linear response breaks down near tipping points. While we did not aim to make this rigorous here, the relationship between response theory, tipping behavior and climate prediction is an active research field (Ghil and Lucarini, 2020; Santos Gutiérrez and Lucarini, 2022; Lucarini, 2025; Galatolo and Lucarini, 2026). Moreover, there has been great interest recently in data-driven approaches for estimating Green's functions (Aengenheyster et al., 2018; Lucarini and Chekroun, 2024). This holds potential for expanding the mathematical basis of our tipping definition, and for developing more advanced null models. Conceptualizing tipping as a deviation from a counterfactual linear world frames the problem in terms of detection and attribution (Lucarini and Chekroun, 2024), which can offer a useful complement to the prediction problem posed by early warning methods (Dakos et al., 2008; Rietkerk et al., 2026). How our definition applies to adaptive systems, such as socio-technical or biological systems, is an interesting question for future work.

Overly simplistic pictures of tipping dynamics have led to a common belief that tipping is restricted to simple systems (Shaw and Stevens, 2025; Kopp et al., 2025a). However, the mathematical theory of tipping goes far beyond fold bifurcations and quasi-equilibrium dynamics. Methods from global stability analysis, nonautonomous dynamics and chaos theory have been successfully applied to high-dimensional systems, and concepts such as rate-induced tipping enable a more nuanced view that incorporates transient, metastable dynamics. What has been missing is a practical and overarching tipping definition that bridges between specific mathematical scenarios and purely qualitative notions that often remain vague.

Carefully defining what tipping means is important because it shapes research questions and the public perception of tipping risk. A strong emphasis on abruptness has possibly caused a misunderstanding of climate tipping events occurring instantly once a threshold is crossed, conveying a misleadingly binary picture of "safe" versus "catastrophic" climate change. We argue that from an impact perspective, the essential aspect is the internally driven nonlinearity of the change because the change is then no longer externally controlled, making it difficult to stop or reverse. Prediction models based on linearity would then fail and lead to false risk assessments.

Code and data availability. The TippingTest.jl package (https://github.com/reykboerner/TippingTest.jl) provides a generic implementation of our tipping test in the Julia language. The CESM model simulation data used in this work are archived on Zenodo under the DOIs 10.5281/zenodo.15088540 (Fig. 3) and 10.5281/zenodo.18599844 (Fig. 4).

Appendix A: Damped oscillator null model

A simple linear null model for oscillatory dynamics is given by the damped, driven harmonic oscillator,

$$m^2\ddot{x} = -(x - x_* - \beta F(t)) - \gamma\dot{x} \tag{A1}$$

Here $\ddot{x} = \mathrm{d}^2x/\mathrm{d}t^2$ denotes the second time derivative (acceleration) of the state x, which relaxes to the time-dependent equilibrium $x_* + \beta F(t)$ with inertia m. Displacements out of equilibrium lead to damped oscillations with eigenfrequency $\omega_0 = 1/m$ and damping ratio $\xi = \gamma/(2m)$. The oscillator is overdamped (underdamped) if $\xi > 1$ ($\xi < 1$).

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Eq. (A1) can equivalently be written as a pair of first-order ordinary differential equations,

$$\dot{x} = v$$
$$\dot{v} = -\frac{1}{m^2}\left[(x - x_* - \beta F(t)) - \gamma v\right], \qquad \text{(A2)}$$

which can be solved given an initial condition (x_0, v_0) comprising the initial displacement $x_0 = x(t_0)$ and velocity $v_0 = v(t_0)$.

Appendix B: Simple tipping system

Trajectories of the simple tipping system (Eq. (5)) shown in Fig. 2 were generated using the parameter settings given in Tab. B1. The fold bifurcation occurs at $F = 1$. In the B-tipping cases, the forcing F is linearly increased up to a final constant value of $F = 0.9$ (no tipping) or $F = 1.1$ (tipping). In the R-tipping case, the linear increase is stopped at $F = 0.7$; the durations of increase are $T/5$ (no tipping) and $T/7$ (tipping), where $T = t_1 - t_0$. In the N-tipping case, $F = 0.22$ is held constant. In the overshoot case, the forcing profile is given by $F(t) = h[\exp(-70(t-(t_0+t_1)/2)^2/(t_1-t_0)) + 0.1t]$ with $h = 0.355$ (no tipping) and $h = 0.455$ (tipping).

The location of the saddle point x_S is given by $x_S(F) = A + (F-1)s - \sqrt{1-F}$ for $F < 1$. The potential V of the system is defined as $f(x) = -\mathrm{d}V/\mathrm{d}x$, where $f(x) = -x[(x - A - s(F-1))^2 + F - 1]$. The inflection points of the potential (where its curvature changes sign) are located at $x_\pm = [2(A - s + Fs) \pm \sqrt{A^2 + 2As(F-1) + (F-1)(s^2(F-1)-3)}]/3$.

Table B1. Parameter settings for the toy model trajectories shown in Fig. 2. Here m is the inertia, s is the tilt parameter, $F(t_0)$ is the initial forcing value at t_0, σ is the noise strength and $T = t_1 - t_0$ (in system time units) is the time intercal shown in each panel of Fig. 2.

	m	A	s	$F(t_0)$	σ	T
B-tipping (fast)	1	2	0	0	0	1000
B-tipping (slow)	120	2	0	0	0	1000
R-tipping	400	5	3	0	0	1000
N-tipping	1	5	3	0.22	0.75	50
Overshoot	300	5	3	0.75	0	1000

Appendix C: CESM simulations

In all CESM simulations, the atmospheric CO_2 concentration is prescribed as a function of time. The AMOC strength ψ is defined as the maximum of the Atlantic meridional overturning streamfunction at the latitude $y_0 = 26°$N and below $z_0 = 1000$ m depth, $\psi(t) = \max_{z \geq z_0} \Psi(y = y_0, z, t)$, where

$$\Psi(y,z,t) = \int_z^0 \int_{x_W}^{x_E} v(x,y,z',t)\,\mathrm{d}x\,\mathrm{d}z'. \qquad \text{(C1)}$$

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Here v is the meridional velocity field, which is a function of longitude x, latitude y, depth z and time t. The inner integral is over the Atlantic basin from its western (x_W) to eastern (x_E) boundary. The observable ψ is thus a nonlinear one-dimensional mapping of the full state vector including ocean variables (velocity, temperature, salinity, ...), atmospheric variables (air temperature, pressure, humidity, ...) and all other model variables such as sea ice, soil moisture etc. at all model grid points.

365 The null model trajectories shown in Figs. 3 and 4 are generated with the harmonic oscillator null model (Eq. (A1)) using the parameter values and initial conditions listed in Tab. C1. In the case $\bar{F}_H = 0.18\,\mathrm{Sv}$ (Fig. 3), we fitted the null model to the RCP4.5 scenario using the data in $\tau_A = (1850, 2026)$ and $\tau_B = (2200, 2500)$, where the years refer to the Common Era (CE).

Table C1. Parameter settings of the harmonic oscillator null models () shown in Figs. 3 and 4.

Null model	x_* (Sv)	x_0 (Sv)	v_0 (Sv yr^{-1})	β (Sv)	m (yr)	γ (yr)
Decadal oscillator $\bar{F}_H = 0.18\,\mathrm{Sv}$ (Fig. 3)	17.5	16.5	0.04	-4.76	41.3	5.9
Centennial oscillator $\bar{F}_H = 0.45\,\mathrm{Sv}$ (Fig. 4, orange)	12.8	12.8	-0.012	1.0	223.6	200.0
Decadal oscillator $\bar{F}_H = 0.45\,\mathrm{Sv}$ (Fig. 4, blue)	12.8	12.8	-0.012	-4.0	41.3	20.0

Author contributions. R.B. and H.A.D. conceived the study and developed the tipping definition. R.B. performed the analysis and created the figures. R.B. wrote the manuscript with input from H.A.D.

370 *Competing interests.* The authors declare no competing interests.

Acknowledgements. This work was motivated by conversations at and after the ClimTip/TipESM project meeting in April 2025, involving M. Ghil, T. Stocker, V. Lucarini, C. Kuehn, N. Boers, H.A.D. and others. The authors wish to thank L. Jackson for useful comments on the manuscript, and N. Boers, V. Lucarini, M. Santos Gutierrez, A.K. Klose, A. Romanou and the AMOC group at the Institute for Marine and Atmospheric research Utrecht for insightful discussions. This is ClimTip contribution #XXX; the ClimTip project has received funding from
375 the European Union's Horizon Europe research and innovation programme under grant agreement No. 101137601. H.A.D. was also partially funded by the European Research Council through the ERC-AdG project TAOC (project 101055096).

https://doi.org/10.5194/egusphere-2026-3507
Preprint. Discussion started: 26 June 2026

References

Abrams, J. F., Pereira, L. M., Spaiser, V., Wunderling, N., Villasante, S., Tàbara, J. D., Biggs, R., Armstrong McKay, D. I., Zimm, C., Milkoreit, M., Constantino, S. M., Debnath, R., Dakos, V., Ritchie, P. D. L., Smith, S. R., Rocha, J. C., Kéfi, S., Buxton, J. E., Boulton, C. A., Greenlees, K., Juhola, S., Bhowmik, A., Fesenfeld, L. P., Clarke, J., Dyke, J. G., Laybourn, L., Lade, S. J., Marjanović-Abrams, M., Dickenson Bampton, B., and Lenton, T. M.: Integrating tipping point concepts across diverse systems, Commun. Sustain., 1, 68, https://doi.org/10.1038/s44458-026-00063-5, 2026.

Aengenheyster, M., Feng, Q. Y., van der Ploeg, F., and Dijkstra, H. A.: The point of no return for climate action: effects of climate uncertainty and risk tolerance, Earth System Dynamics, 9, 1085–1095, https://doi.org/10.5194/esd-9-1085-2018, 2018.

Armstrong McKay, D. I., Staal, A., Abrams, J. F., Winkelmann, R., Sakschewski, B., Loriani, S., Fetzer, I., Cornell, S. E., Rockström, J., and Lenton, T. M.: Exceeding 1.5°C global warming could trigger multiple climate tipping points, Science, 377, eabn7950, https://doi.org/10.1126/science.abn7950, 2022.

Arnol'd, V. I. and Gamkrelidze, R. V., eds.: Dynamical Systems V: Bifurcation Theory and Catastrophe Theory, vol. 5 of *Encyclopaedia of Mathematical Sciences*, Springer, Berlin, Heidelberg, ISBN 978-3-540-65379-0 978-3-642-57884-7, https://doi.org/10.1007/978-3-642-57884-7, 1994.

Ashwin, P., Wieczorek, S., Vitolo, R., and Cox, P.: Tipping points in open systems: bifurcation, noise-induced and rate-dependent examples in the climate system, Philos Trans A Math Phys Eng Sci, 370, 1166–1184, https://doi.org/10.1098/rsta.2011.0306, 2012.

Bastiaansen, R., Ashwin, P., and von der Heydt, A. S.: Climate response and sensitivity: time scales and late tipping points, Proc. A, 479, 20220 483, https://doi.org/10.1098/rspa.2022.0483, 2023.

Brovkin, V., Brook, E., Williams, J. W., Bathiany, S., Lenton, T. M., Barton, M., DeConto, R. M., Donges, J. F., Ganopolski, A., McManus, J., Praetorius, S., de Vernal, A., Abe-Ouchi, A., Cheng, H., Claussen, M., Crucifix, M., Gallopín, G., Iglesias, V., Kaufman, D. S., Kleinen, T., Lambert, F., van der Leeuw, S., Liddy, H., Loutre, M.-F., McGee, D., Rehfeld, K., Rhodes, R., Seddon, A. W. R., Trauth, M. H., Vanderveken, L., and Yu, Z.: Past abrupt changes, tipping points and cascading impacts in the Earth system, Nat. Geosci., 14, 550–558, https://doi.org/10.1038/s41561-021-00790-5, 2021.

Brovkin, V., Bartsch, A., Hugelius, G., Calamita, E., Lever, J. J., Goo, E., Kim, H., Stacke, T., and de Vrese, P.: Permafrost and Freshwater Systems in the Arctic as Tipping Elements of the Climate System, Surv Geophys, 46, 303–326, https://doi.org/10.1007/s10712-025-09885-9, 2025.

Börner, R., Mehling, O., Hardenberg, J. v., and Lucarini, V.: Global stability of the Atlantic overturning circulation: Edge state, long transients and boundary crisis under CO$_2$ forcing, https://doi.org/10.1098/rsta.2025.0087, arXiv:2504.20002 [nlin.CD], 2026.

Caraballo, T. and Han, X.: Applied Nonautonomous and Random Dynamical Systems, SpringerBriefs in Mathematics, Springer International Publishing, Cham, ISBN 978-3-319-49246-9 978-3-319-49247-6, https://doi.org/10.1007/978-3-319-49247-6, 2016.

Dakos, V., Scheffer, M., van Nes, E. H., Brovkin, V., Petoukhov, V., and Held, H.: Slowing down as an early warning signal for abrupt climate change, Proceedings of the National Academy of Sciences, 105, 14 308–14 312, https://doi.org/10.1073/pnas.0802430105, 2008.

Dijkstra, H. A., Krauskopf, B., Börner, R., and van Westen, R. M.: Transitions of the Atlantic Ocean circulation, Nat Rev Phys, pp. 1–15, https://doi.org/10.1038/s42254-026-00945-6, 2026.

Feudel, U.: Complex dynamics in multistable systems, Int. J. Bifurcation Chaos, 18, 1607–1626, https://doi.org/10.1142/S0218127408021233, 2008.

Feudel, U.: Rate-induced tipping in ecosystems and climate: the role of unstable states, basin boundaries and transient dynamics, Nonlinear Processes in Geophysics, 30, 481–502, https://doi.org/10.5194/npg-30-481-2023, 2023.

Freidlin, M. I. and Wentzell, A. D.: Random perturbations of dynamical systems, Springer, 1998.

Galatolo, S. and Lucarini, V.: A Mathematical Framework for Linear Response Theory for Nonautonomous Systems, https://doi.org/10.48550/arXiv.2603.19509, arXiv:2603.19509 [math.DS], 2026.

Ghil, M. and Lucarini, V.: The physics of climate variability and climate change, Rev. Mod. Phys., 92, 035 002, https://doi.org/10.1103/RevModPhys.92.035002, 2020.

Gladwell, M.: The tipping point: how little things can make a big difference, Little, Brown, Boston, ISBN 978-0-316-31696-5, http://archive.org/details/tippingpointhowl00glad, 2000.

Global Tipping Points: The Global Tipping Points Report 2023, Report, University of Exeter, Exeter, UK, 2023.

Global Tipping Points: Global Tipping Points Report 2025, https://publications.pik-potsdam.de/pubman/faces/ViewItemOverviewPage.jsp?itemId=item_34056_2, 2025.

Hairer, M. and Majda, A. J.: A simple framework to justify linear response theory, Nonlinearity, 23, 909, https://doi.org/10.1088/0951-7715/23/4/008, 2010.

Halekotte, L. and Feudel, U.: Minimal fatal shocks in multistable complex networks, Sci Rep, 10, 11 783, https://doi.org/10.1038/s41598-020-68805-6, 2020.

Hastings, A., Abbott, K. C., Cuddington, K., Francis, T., Gellner, G., Lai, Y.-C., Morozov, A., Petrovskii, S., Scranton, K., and Zeeman, M. L.: Transient phenomena in ecology, Science, 361, eaat6412, https://doi.org/10.1126/science.aat6412, 2018.

Hurrell, J. W., Holland, M. M., Gent, P. R., Ghan, S., Kay, J. E., Kushner, P. J., Lamarque, J.-F., Large, W. G., Lawrence, D., Lindsay, K., Lipscomb, W. H., Long, M. C., Mahowald, N., Marsh, D. R., Neale, R. B., Rasch, P., Vavrus, S., Vertenstein, M., Bader, D., Collins, W. D., Hack, J. J., Kiehl, J., and Marshall, S.: The Community Earth System Model: A Framework for Collaborative Research, Bulletin of the American Meteorological Society, 94, 1339–1360, https://doi.org/10.1175/BAMS-D-12-00121.1, 2013.

Intergovernmental Panel On Climate Change: Climate Change 2021 – The Physical Science Basis: Working Group I Contribution to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change, Cambridge University Press, ISBN 978-1-009-15789-6, https://doi.org/10.1017/9781009157896, 2023.

Johns, W. E., Elipot, S., Smeed, D. A., Moat, B., King, B., Volkov, D. L., and Smith, R. H.: Towards two decades of Atlantic Ocean mass and heat transports at 26.5°N, Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences, 381, 20220 188, https://doi.org/10.1098/rsta.2022.0188, 2023.

Kolata, G. B.: Catastrophe Theory: The Emperor Has No Clothes, Science, 196, 287–351, https://doi.org/10.1126/science.196.4287.287, 1977.

Kopp, R., Gilmore, E., and Shwom, R.: Climate change will surprise us, but so-called ‘tipping points’ may lead us astray, Bulletin of the Atomic Scientists, 81, 121–125, https://doi.org/10.1080/00963402.2025.2464445, _eprint: https://doi.org/10.1080/00963402.2025.2464445, 2025a.

Kopp, R. E., Gilmore, E. A., Shwom, R. L., Adams, H., Adler, C., Oppenheimer, M., Patwardhan, A., Russill, C., Schmidt, D. N., and York, R.: ‘Tipping points’ confuse and can distract from urgent climate action, Nat. Clim. Chang., 15, 29–36, https://doi.org/10.1038/s41558-024-02196-8, 2025b.

Kubo, R.: The fluctuation-dissipation theorem, Rep. Prog. Phys., 29, 255, https://doi.org/10.1088/0034-4885/29/1/306, 1966.

https://doi.org/10.5194/egusphere-2026-3507
Preprint. Discussion started: 26 June 2026

Kuehn, C.: A mathematical framework for critical transitions: Bifurcations, fast–slow systems and stochastic dynamics, Physica D: Nonlinear Phenomena, 240, 1020–1035, https://www.sciencedirect.com/science/article/pii/S0167278911000443, 2011.

Lai, Y.-C. and Tél, T.: Transient chaos: complex dynamics on finite time scales, vol. 173, Springer Science & Business Media, https://books.google.com/books?hl=en&lr=&id=rl00oCTsMzEC&oi=fnd&pg=PR3&dq=transient+chaos&ots=ANj1K4ORF3&sig=D_q-0BlUY5lDeEkgRYJE8FtydAg, 2011.

Leith, C. E.: Climate Response and Fluctuation Dissipation, Journal of the Atmospheric Sciences, 32, 2022–2026, https://doi.org/10.1175/1520-0469(1975)032<2022:CRAFD>2.0.CO;2, 1975.

Lenton, T. M., Held, H., Kriegler, E., Hall, J. W., Lucht, W., Rahmstorf, S., and Schellnhuber, H. J.: Tipping elements in the Earth's climate system, Proceedings of the National Academy of Sciences, 105, 1786–1793, https://doi.org/10.1073/pnas.0705414105, 2008.

Lenton, T. M., Rockström, J., Gaffney, O., Rahmstorf, S., Richardson, K., Steffen, W., and Schellnhuber, H. J.: Climate tipping points — too risky to bet against, Nature, 575, 592–595, https://doi.org/10.1038/d41586-019-03595-0, bandiera_abtest: a Cg_type: Comment Subject_term: Climate change, Climate sciences, Environmental sciences, Policy, 2019.

Levermann, A., Bamber, J. L., Drijfhout, S., Ganopolski, A., Haeberli, W., Harris, N. R. P., Huss, M., Krüger, K., Lenton, T. M., Lindsay, R. W., Notz, D., Wadhams, P., and Weber, S.: Potential climatic transitions with profound impact on Europe, Climatic Change, 110, 845–878, https://doi.org/10.1007/s10584-011-0126-5, 2012.

Li, C., Notz, D., Tietsche, S., and Marotzke, J.: The Transient versus the Equilibrium Response of Sea Ice to Global Warming, Journal of Climate, 26, 5624–5636, https://doi.org/10.1175/JCLI-D-12-00492.1, 2013.

Lucarini, V.: Interpretable and Equation-Free Response Theory for Complex Systems, https://doi.org/10.48550/arXiv.2502.07908, arXiv:2502.07908 [cond-mat.stat-mech], 2025.

Lucarini, V. and Chekroun, M. D.: Detecting and Attributing Change in Climate and Complex Systems: Foundations, Green's Functions, and Nonlinear Fingerprints, Phys. Rev. Lett., 133, 244 201, https://doi.org/10.1103/PhysRevLett.133.244201, 2024.

Milkoreit, M.: Social tipping points everywhere?—Patterns and risks of overuse, WIREs Climate Change, 14, e813, https://doi.org/10.1002/wcc.813, _eprint: https://wires.onlinelibrary.wiley.com/doi/pdf/10.1002/wcc.813, 2023.

Milkoreit, M., Hodbod, J., Baggio, J., Benessaiah, K., Calderón-Contreras, R., Donges, J. F., Mathias, J.-D., Rocha, J. C., Schoon, M., and Werners, S. E.: Defining tipping points for social-ecological systems scholarship—an interdisciplinary literature review, Environ. Res. Lett., 13, 033 005, https://doi.org/10.1088/1748-9326/aaaa75, 2018.

Nes, E. H. v., Arani, B. M. S., Staal, A., Bolt, B. v. d., Flores, B. M., Bathiany, S., and Scheffer, M.: What Do You Mean, 'Tipping Point'?, Trends in Ecology & Evolution, 31, 902–904, https://doi.org/10.1016/j.tree.2016.09.011, 2016.

Notz, D.: The future of ice sheets and sea ice: Between reversible retreat and unstoppable loss, Proceedings of the National Academy of Sciences, 106, 20 590–20 595, https://doi.org/10.1073/pnas.0902356106, 2009.

O'Keeffe, P. E. and Wieczorek, S.: Tipping Phenomena and Points of No Return in Ecosystems: Beyond Classical Bifurcations, SIAM J. Appl. Dyn. Syst., 19, 2371–2402, https://doi.org/10.1137/19M1242884, 2020.

Ragone, F., Lucarini, V., and Lunkeit, F.: A new framework for climate sensitivity and prediction: a modelling perspective, Clim Dyn, 46, 1459–1471, https://doi.org/10.1007/s00382-015-2657-3, 2016.

Rietkerk, M., Haasnoot, M., and Vuuren, D. P. v.: Warning for climate tipping without compromising credibility, Climatic Change, 179, 122, https://doi.org/10.1007/s10584-026-04197-2, 2026.

Ritchie, P. D. L., Clarke, J. J., Cox, P. M., and Huntingford, C.: Overshooting tipping point thresholds in a changing climate, Nature, 592, 517–523, https://doi.org/10.1038/s41586-021-03263-2, 2021.

https://doi.org/10.5194/egusphere-2026-3507
Preprint. Discussion started: 26 June 2026

Ritchie, P. D. L., Alkhayuon, H., Cox, P. M., and Wieczorek, S.: Rate-induced tipping in natural and human systems, Earth System Dynamics, 14, 669–683, https://doi.org/10.5194/esd-14-669-2023, 2023.

Rosser, J. B.: The rise and fall of catastrophe theory applications in economics: Was the baby thrown out with the bathwater?, Journal of Economic Dynamics and Control, 31, 3255–3280, https://doi.org/10.1016/j.jedc.2006.09.013, 2007.

Rossi, K. L., Budzinski, R. C., Medeiros, E. S., Boaretto, B. R. R., Muller, L., and Feudel, U.: Dynamical properties and mechanisms of metastability: A perspective in neuroscience, Phys. Rev. E, 111, 021 001, https://doi.org/10.1103/PhysRevE.111.021001, 2025.

Ruelle, D.: A review of linear response theory for general differentiable dynamical systems, Nonlinearity, 22, 855, https://doi.org/10.1088/0951-7715/22/4/009, 2009.

Russill, C.: Climate change tipping points: origins, precursors, and debates, WIREs Climate Change, 6, 427–434, https://doi.org/10.1002/wcc.344, _eprint: https://wires.onlinelibrary.wiley.com/doi/pdf/10.1002/wcc.344, 2015.

Santos Gutiérrez, M. and Lucarini, V.: On some aspects of the response to stochastic and deterministic forcings, Journal of Physics A: Mathematical and Theoretical, 55, 425 002, https://doi.org/10.1088/1751-8121/ac90fd, 2022.

Scheffer, M., Carpenter, S., Foley, J. A., Folke, C., and Walker, B.: Catastrophic shifts in ecosystems, Nature, 413, 591–596, https://doi.org/10.1038/35098000, number: 6856, 2001.

Shaw, T. A. and Stevens, B.: The other climate crisis, Nature, 639, 877–887, https://doi.org/10.1038/s41586-025-08680-1, 2025.

Smith, S. R., Milkoreit, M., Geels, F. W., and Lenton, T. M.: Advancing science, policy and action in tipping points research, Nat. Clim. Chang., 15, 576–577, https://doi.org/10.1038/s41558-025-02335-9, 2025.

Stewart, I. N. and Peregoy, P. L.: Catastrophe theory modeling in psychology, Psychological Bulletin, 94, 336–362, https://doi.org/10.1037/0033-2909.94.2.336, 1983.

Thom, R.: Topological models in biology, Topology, 8, 313–335, https://doi.org/10.1016/0040-9383(69)90018-4, 1969.

Thom, R.: Stabilité structurelle et morphogenèse, Poetics, 3, 7–19, https://doi.org/10.1016/0304-422X(74)90010-2, 1974.

van Westen, R. M., Vanderborght, E., Kliphuis, M., and Dijkstra, H. A.: Physics-Based Indicators for the Onset of an AMOC Collapse Under Climate Change, Journal of Geophysical Research: Oceans, 130, e2025JC022 651, https://doi.org/10.1029/2025JC022651, _eprint: https://agupubs.onlinelibrary.wiley.com/doi/pdf/10.1029/2025JC022651, 2025.

van Westen, R. M., Börner, R., and Dijkstra, H. A.: Failure to track a stable AMOC state under rapid climate change, https://doi.org/10.48550/arXiv.2602.09964, arXiv:2602.09964 [physics.ao-ph], 2026.

Vanderborght, E., Westen, R. M. v., and Dijkstra, H. A.: Feedback Processes Causing an AMOC Collapse in the Community Earth System Model, Journal of Climate, 38, 5083–5102, https://doi.org/10.1175/JCLI-D-24-0570.1, 2025.

Wieczorek, S., Xie, C., and Ashwin, P.: Rate-induced tipping: thresholds, edge states and connecting orbits, Nonlinearity, 36, 3238, https://doi.org/10.1088/1361-6544/accb37, 2023.

Winkelmann, R., Dennis, D. P., Donges, J. F., Loriani, S., Klose, A. K., Abrams, J. F., Alvarez-Solas, J., Albrecht, T., Armstrong McKay, D., Bathiany, S., Blasco Navarro, J., Brovkin, V., Burke, E., Danabasoglu, G., Donner, R. V., Drüke, M., Georgievski, G., Goelzer, H., Harper, A. B., Hegerl, G., Hirota, M., Hu, A., Jackson, L. C., Jones, C., Kim, H., Koenigk, T., Lawrence, P., Lenton, T. M., Liddy, H., Licón-Saláiz, J., Menthon, M., Montoya, M., Nitzbon, J., Nowicki, S., Otto-Bliesner, B., Pausata, F., Rahmstorf, S., Ramin, K., Robinson, A., Rockström, J., Romanou, A., Sakschewski, B., Schädel, C., Sherwood, S., Smith, R. S., Steinert, N. J., Swingedouw, D., Willeit, M., Weijer, W., Wood, R., Wyser, K., and Yang, S.: The Tipping Points Modelling Intercomparison Project (TIPMIP): Assessing tipping point risks in the Earth system, EGUsphere, pp. 1–52, https://doi.org/10.5194/egusphere-2025-1899, 2025.

https://doi.org/10.5194/egusphere-2026-3507
Preprint. Discussion started: 26 June 2026

Zahler, R. S. and Sussmann, H. J.: Claims and accomplishments of applied catastrophe theory, Nature, 269, 759–763, https://doi.org/10.1038/269759a0, 1977.

Zeeman, E. C.: On the unstable behaviour of stock exchanges, Journal of Mathematical Economics, 1, 39–49, https://doi.org/10.1016/0304-4068(74)90034-2, 1974.

Zeeman, E. C.: Catastrophe Theory, Scientific American, 234, 65–83, https://www.jstor.org/stable/24950329, 1976.