Dear Reviewer,

Thank you so much for taking care of our manuscript and we appreciate the opportunity to revise it with comprehensive comments. We highly appreciate the detailed valuable comments from the referees on our manuscript of "A Multi-Angle and Polarization-Based Retrieval Algorithm for Aerosol Layer Height of Smoke and Dust". The suggestions are quite helpful and we have incorporated them in the revised version of manuscript. And we hope that the reviewers and the editors will be satisfied with our responses to the 'comments' and the revisions for this manuscript. Please find our detailed responses below to all these comments/suggestions.

Yours sincerely,

Yong Xue

This manuscript investigates theoretically the information content of multi-angle polarimetric measurements for ALH retrievals in the UV, using a Degrees of Freedom for Signal (DFS) analysis. It demonstrates that polarisation measurements adds information, especially at geometries that are relevant for satellite observations. Subsequently, an ALH retrieval algorithm is applied to multi-angle polarimetric PACE/HARP2 observations at 441 nm and compared to independent ALH retrievals from EarthCARE/ATLID, using active lidar measurements, and TROPOMI ALH using the O2-A band.

The paper is relevant and interesting, and the analysis is generally of good quality. However, for publication a number of issues need to be addressed, mainly to better explain what has been done and provide concise information.

Response:

We sincerely thank the reviewer for the positive evaluation and constructive comments. Following the reviewer's suggestions, we have carefully revised the manuscript to improve the clarity and consistency of the theoretical description, refine several statements regarding previous studies, provide more detailed descriptions of the simulation settings and figures, and reorganize some theoretical discussions to improve the overall structure of the manuscript.

Details are listed below:

l64-67

"Specifically, more photons are scattered back to space at higher altitudes, thereby reducing their probability of penetrating into the lower atmosphere and being absorbed by O_2. As a result, higher aerosol layers lead to stronger Top-of-Atmosphere (TOA) reflectance within the absorption bands."

suggest to reformulate something like:

Specifically, for scattering layers at higher altitudes, the probability for photons to penetrate into the lower atmosphere and be absorbed by O_2 reduces, leading to shallower absorption lines in within the absorption bands for higher altitude aerosol layers.

Response:

We thank the reviewer for this suggestion. We have revised this sentence to better describe the photon transport process and the resulting variation of absorption features in the manuscript.

Lines 93-95: Specifically, for scattering layers at higher altitudes, the probability for photons to penetrate into the lower atmosphere and be absorbed by O_2 reduces, leading to shallower absorption lines in within the absorption bands for higher altitude aerosol layers.

l77

It has shown -> It has been shown

Response:

Thank you for pointing out this grammatical error. We have corrected "It has shown" to "It has been shown" in the revised manuscript.

l107-108

Moreover, the quantitative contributions of multi-dimensional observations to ALH retrieval have not yet been systematically assessed, highlighting the need for further investigation.

I do not agree here. There is a large body in literature, some of which are already listed. The GRASP consortium has shown quite systematically that they can use multi-dimensional observation to improve the ALH and other aerosol parameters.

Another reference is

Hasekamp, O. P., P. Litvinov, and A. Butz (2011), Aerosol properties over the ocean from PARASOL multiangle photopolarimetric measurements, J. Geophys. Res., 116, D14204, doi:10.1029/2010JD015469.

Response:

We sincerely thank the reviewer for this important comment. We agree that GRASP-based retrieval studies and Hasekamp et al. (2011) have systematically demonstrated that the combination of spectral, angular, and polarimetric measurements can provide additional constraints on aerosol properties, including aerosol vertical information.

Following the reviewer's suggestion, we have revised the related statement in the Introduction to avoid overstatement, added the relevant references, and clarified the specific scientific objective of this study.

Revision:

Lines 145-150: Previous studies, including GRASP-based retrieval frameworks and multi-angle polarimetric retrieval approaches (Hasekamp et al., 2011), have demonstrated the potential of combining spectral, angular, and polarization measurements to improve aerosol property retrievals. Nevertheless, uncertainties remain in quantifying the specific information contribution of individual observation dimensions to ALH retrieval, particularly for multi-angle polarimetric observations under different viewing geometries.

l170

"which further improves the reliability of the data."

This is a rather empty statement. The ALH product is for cloud-screened pixels only. Period.

Response:

We thank the reviewer for this comment. Since the ALH product is already provided only for cloud-screened pixels, describing cloud screening as further improving the reliability of the product is not precise. We have removed the relevant statements from the manuscript.

Section 3.

The theoretical treatment needs improvement. The relationship between eq (1) and (2) is not given. S_epsilon (3) is diagonal, which means that the errors are assumed to be independent, this needs to be mentioned.

The Stokes parameters were mentioned in section 2, but only here treated formally. Better to remove the introduction in section 2, and treat it here properly. Same for DoLP (change all DOLP to DoLP).

Response:

We thank the reviewer for pointing out this issue. For the DFS analysis, this nonlinear forward model is linearized around a reference state using the Jacobian matrix K, which represents the sensitivity of the simulated observations to the retrieved parameters. Assuming Gaussian distributions for the measurement and forward model errors, the posterior probability density function can be derived within the Bayesian optimal estimation framework by combining the measurement information with the prior constraint. The posterior error covariance matrix is therefore obtained as Eq. (2). We have added this explanation in the revised manuscript to clarify the theoretical connection between the forward model and the retrieval uncertainty formulation.

Second, we have explicitly stated that the measurement error covariance matrix S_{ε} is assumed to be diagonal, indicating that measurement errors among different observations are considered independent.

Furthermore, we have removed the preliminary introduction of Stokes parameters from Section 2 and provided their formal definitions in Section 3. In addition, all occurrences of "DOLP" have been corrected to the standard notation "DoLP" throughout the manuscript.

Revision:

Lines 300-317: In the framework of optimal estimation theory (Hou et al., 2018), the relationship between the state vector x (containing n parameters) and the observation vector y (with m measurements) can be expressed as:

$$y = F(x) + \in \tag{1}$$

where F is the forward model that describes the physical relationship between satellite observations and atmospheric parameters (such as ALH), x is the state vector to be retrieved, and $\in$ represents the errors arising from forward model simulations and satellite measurements. For the DFS analysis, the nonlinear forward model in Eq. (1) is linearized around a reference state x using the Jacobian matrix $K = \frac{\partial F}{\partial x}$, which represents the sensitivity of the simulated observations to the retrieved parameters. Under the assumption that the measurement and forward model errors follow Gaussian distributions, the posterior probability density function of the retrieved state can be obtained through Bayesian estimation by combining the prior information with the measurement information. The corresponding posterior error covariance matrix S is then expressed as:

$$S^{-1} = K^T S_\varepsilon^{-1} K + S_a^{-1} \tag{2}$$

Where, S_ε denotes the measurement error covariance matrix, and S_a represents the a priori error covariance matrix. In this study, the measurement error covariance matrix S_ε is assumed to be diagonal, indicating that the measurement errors among different observations are independent. It includes uncertainties from both instrument measurements and forward-model simulations.

l244 "radiance III" is unclear.

I also suggest to move eq (13) and its discussion on l632 to section 3. The extinction-weighted height is used in the text before it is defined.

Response:

We thank the reviewer for pointing out these issues.

The expression "radiance III" was a typographical error and has been removes in the revised manuscript.

We also appreciate the reviewer's suggestion regarding the organization of the theoretical framework. Eq. (13) and its related discussion have been moved from the validation section to Section 3, where the theoretical basis and definitions of the retrieval framework are presented. This reorganization improves the logical flow and ensures that the extinction-weighted height is clearly defined before being used in subsequent analyses.

l250 Mie scattering is proper for small and/or spherical particles only.

Table 1: Where are the numbers coming from? The table lists 3 references, the text says Lee et al (2015). Please, be concise and consistent.

Response:

We thank the reviewer for this important comment. In the revised manuscript, we have clarified that the linearized Mie scattering calculations are performed under the spherical particle assumption for constructing the lookup table. This approximation is generally suitable for fine-mode smoke aerosols, while it may introduce additional uncertainties for dust aerosols due to their non-spherical shapes. This limitation has been added to the manuscript.

Regarding Table 1, we appreciate the reviewer for pointing out the inconsistency in describing the parameter sources. We have revised the text and table to clearly distinguish the origins of different aerosol parameters. Specifically, the aerosol size distribution parameters and fine-mode fraction are adopted from Lee et al. (2015), while the complex refractive indices are taken from Chen et al. (2021b) for smoke and Zeng et al. (2008) for dust.

Revision:

Lines 357-365: These microphysical parameters are specified based on previous studies. The aerosol size distribution and fine-mode fraction (FMF) follow Lee et al. (2015), while the complex refractive indices are adopted from Chen et al. (2021b) for smoke and Zeng et al. (2008) for dust, as summarized in Table 1.

Table 1 Microphysical properties of aerosols in forward modeling

Aerosol Type	Complex Refractive Index	$r_{v1}; r_{v2}$ (μm)	$\sigma_1; \sigma_2$ (μm)	FMF	Reference
Smoke	1.52−0.021i	0.15; 3.25	0.40; 0.75	0.85	(Chen et al., 2021b; Lee et al., 2015)
Dust	1.53−0.008i	0.15; 1.90	0.40; 0.63	0.15	(Lee et al., 2015; Zeng et al., 2008)

l271 MCD43A1 product is a MODIS product, which is not described. Please add.

Response:

We thank the reviewer for pointing out this issue. We have added a brief introduction of the MODIS MCD43A1 BRDF/Albedo product in the revised manuscript.

Revision:

Lines 221-228: This study incorporates surface bidirectional reflectance information from the MODIS MCD43A1 BRDF/Albedo product (Schaaf and Wang, 2015). Specifically, the BRDF Albedo Parameters dataset was used to characterize the directional surface reflection properties at the corresponding wavelength. The MCD43A1 product provides the three RossThick–LiSparseReciprocal BRDF model parameters, which describe the anisotropic scattering behavior of the land surface (Schaaf et al., 2002). These parameters were used as auxiliary inputs for the radiative transfer simulations to account for surface bidirectional reflectance effects in the aerosol layer height retrieval.

l287

"Under typical aerosol loading conditions (AOD = 0.3–1)"

It is unclear what aerosol type is used in Fig 1 and what the AOD is, since it wavelength-dependent. This should be specified properly and consistently throughout the text. Also see next item.

Fig 1.

The AOD is a wavelength-dependent parameter. In Fig.1 the AOD ranges from 0-5 for all wavelengths, representing markedly different ranges, since the AOD at 441 is 2-3 times higher that at 873 for smoke. It is not clear what is used and the figures should be scaled to the same ranges to be compared as they are in the text now.

Response:

We thank the reviewer for the constructive comments. In this analysis, the aerosol type considered in Fig. 1 is smoke aerosol. Regarding AOD, the values at different wavelengths were converted to the equivalent AOD at 441 nm using the AE to ensure a consistent reference across spectral bands (i.e., AOD_{441} = 0.3–1). The corresponding AOD values at other wavelengths were calculated using the Ångström exponent and used in the radiative transfer simulations. Therefore, the comparisons among different spectral bands are based on consistent aerosol loading conditions. The descriptions in the manuscript and the caption of Fig. 1 have been revised accordingly to explicitly specify the smoke aerosol assumption and the use of AOD_{441}。

Revision:

Lines 403-406: To ensure a physically consistent comparison among different spectral bands, AOD was specified at 441 nm as the reference aerosol loading condition. The corresponding AOD values at other wavelengths were calculated using the AE and used in the radiative transfer simulations (Eck et al., 1999).

Lines 420-425: Fig. 1. Characteristics of ALH-AOD_{441}-DFS distributions derived from radiance under different spectral bands and surface reflectance conditions. The simulations were conducted for smoke aerosols, with AOD_{441} ranging from 0 to 5 as the common reference aerosol loading parameter. For all subplots, the x-axis represents AOD_{441}. Rows correspond to wavelengths of 441 nm, 549 nm, 669 nm, and 873 nm, respectively, while columns represent different surface reflectance conditions ($A_s = 0.00, 0.05, 0.20, 0.50$). Colors indicate the DFS values of ALH, ranging from 0 to 1.

l336

Same as before: aerosol optical depth (AOD = 0.3) is ambiguous. Please, describe exactly how Fig. 2-6 were produced.

Fig. 2-6. The captions should describe what's in the figures: I guess relative azimuth from 0-180 degrees? The axis from 0-10 is unexplained.

Response:

We thank the reviewer for the thoughtful comments. Regarding the ambiguity of aerosol optical depth (AOD), we have addressed this issue in our response to the previous comment and have revised the corresponding descriptions in the manuscript.

Regarding the insufficient descriptions of Fig. 2–6 in the original manuscript. In the revised version, we have clarified the meaning of the coordinates in these figures. Specifically, the radial coordinate (0–10 km) represents aerosol layer height (ALH), while the angular coordinate (0–180°) represents relative azimuth angle (RAA). The figure captions have also been revised accordingly to explicitly describe the coordinate definitions.

Revision:

Lines 484-490: Fig. 2. Comparison of ALH-DFS distributions under different combinations of observational information (*I* vs. *I*+*P*) and surface reflectance conditions. The DFS values were calculated by varying ALH (radial axis: 0–10 km) and RAA (angular axis: 0–180°), while other parameters were fixed at VZA = 0°, SZA = 40°, and AOD = 0.3. The top row shows DFS calculated using radiance (*I*) only, while the bottom row shows DFS obtained by combining radiance and polarization information (*I*+*P*). Columns correspond to different surface reflectance values ($A_s =$ $0.00,\ 0.05,\ 0.20,\ 0.50$).

l370

"DFS exhibits a pronounced enhancement with viewing geometry. "

I do not understand this. Viewing geometry in this sense is ambiguous. Please rephrase.

Response:

We thank the reviewer for the valuable comments. In this analysis, the DFS variation mainly corresponds to changes in VZA. Therefore, we have revised the statement by explicitly referring to VZA and clarified the physical mechanism associated with the increased atmospheric path length.

l372-274:

Under large viewing angles, the intensity-only observations are not so bad. I think the conclusion is that for small viewing angles the additional information form polarisation measurements is largest.

Response:

We thank the reviewer for this insightful comment. Following the reviewer's suggestion, we have revised the discussion to clarify that large viewing angles enhance the overall ALH information content, while polarization measurements provide complementary information and achieve the largest additional contribution under small viewing angles.

Revision:

Lines 511-521: In addition, DFS exhibits a strong dependence on VZA. As VZA increases from 0° to 80°, the longer atmospheric scattering path enhances the sensitivity of radiance measurements to aerosol vertical distribution, resulting in increased ALH information content. Under large viewing angles, intensity-only observations already provide considerable information due to the enhanced atmospheric path length. However, the additional contribution from polarization measurements is more pronounced at small viewing angles, where intensity-only observations provide relatively limited constraints on ALH. By incorporating polarization information, the DFS is substantially enhanced under these geometries, demonstrating that polarization measurements provide complementary information beyond radiance observations. Furthermore, under high surface reflectance conditions ($A_s = 0.50$), polarization measurements help maintain higher DFS values by reducing the impact of surface background effects on ALH retrieval.

l428-430: Repetition, move theoretical treatments to section 3.

Response:

Following the reviewer's suggestion, we have moved the relevant theoretical treatments to Section 3, where the theoretical framework and retrieval methodology are introduced. The corresponding discussion in the original section has been removed or revised to avoid redundancy and improve the overall organization of the manuscript.

l475.

The reference Fromm et al from 2010 is a strange reference to describe the transport of a 2025 smoke plume. Please, rephrase. I guess the idea is that this happens more often or happened before.

Response:

We thank the reviewer for the comments.Our intention was to indicate that long-range transport of Canadian wildfire smoke under the influence of prevailing westerly winds and large-scale atmospheric circulation has occurred previously. Therefore, we have revised the sentence to clarify that Fromm et al. (2010) provides an example of similar transport processes rather than evidence for the 2025 event.

Revision:

Lines 731-736: In late May 2025, multiple wildfires occurred in western and central Canada. Under the influence of prevailing westerly winds and large-scale atmospheric circulation, a substantial amount of smoke aerosols was transported to the central and eastern regions of North America, forming a pronounced long-range transport plume. Similar long-range transport of Canadian wildfire smoke driven by large-scale atmospheric circulation has also been reported in previous studies (Fromm et al., 2010).