

A unified Hapke-HSR + MARMIT-2 soil radiative transfer model for reflectance simulation under varying moisture conditions

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Abstract. Soil radiative transfer models (RTMs) provide a physical basis for interpreting surface reflectance and retrieving land-surface parameters. However, most existing soil RTMs represent either the spectral-directional scattering behavior of dry soils or the moisture-induced absorption effects of wet soils, and a physically consistent formulation capable of jointly describing both processes remains limited. In this study, we develop a unified soil RTM by refining the Hapke-based hyperspectral reflectance model (Hapke-HSR) using dry soil reflectance and dynamically coupling it with the improved multilayer RTM of soil reflectance (MARMIT-2). The proposed Hapke-HSR + MARMIT-2 model explicitly represents the interaction between particle scattering and moisture-dependent absorption and refraction processes, enabling joint spectral-directional simulation of soil reflectance under varying soil moisture conditions. The model is systematically evaluated using eight independent soil spectral databases spanning a wide range of textures and moisture levels. Results show that the Hapke-HSR + MARMIT-2 model consistently improves simulation accuracy and stability relative to the individual Hapke-HSR and MARMIT-2 models, with particularly pronounced gains at high soil moisture regimes ($SMC \geq 30\%$). Across all datasets, the proposed model achieves higher performance ($R^2 = 0.993$, $RMSE = 0.007$) than MARMIT-2 ($R^2 = 0.983$, $RMSE = 0.012$) and Hapke-HSR ($R^2 = 0.909$, $RMSE = 0.028$). The proposed framework provides a physically interpretable and extensible basis for soil reflectance modeling and offers a robust foundation for future developments in multi-angular hyperspectral remote sensing and land-surface parameter inversion.

Keywords: Soil radiative transfer model, soil reflectance, Hapke-HSR model, MARMIT-2 model, soil moisture content

1 Introduction

Soils are fundamental components of the Earth's surface system and play a critical role in agricultural productivity, ecosystem functioning, and hydrological processes (Fan et al., 2025; Rizzo et al., 2023; Shoshany et al., 2022). Soil reflectance is governed by a range of physical properties, including soil moisture content (SMC), particle size, and surface roughness, which jointly regulate the scattering and absorption of solar radiation (Gholami et al., 2018; Labarre et al., 2019;

45 Nolin and Liang, 2008; Sheng et al., 2024). Among these factors, SMC is one of the most influential and dynamic variables,
 46 exerting a dominant control on soil spectral behavior, particularly in the shortwave infrared region where water absorption
 47 features are pronounced (Babliet et al., 2019; Jiang and Fang, 2019; Xu et al., 2025). Because soil reflectance constitutes a
 48 fundamental component of optical remote sensing signals, a physically consistent soil radiative transfer model is essential for
 49 reliably linking observed reflectance to soil and vegetation parameters and for supporting the inversion of land-surface
 50 biophysical variables (Gao et al., 2024; Lei et al., 2025; Yang et al., 2025).

51 Soil radiative transfer models (RTMs) describe the absorption, scattering, and transmission of solar radiation within soil
 52 media and provide a physically based framework for linking surface reflectance to soil properties (Bach and Mauser, 1994;
 53 Jacquemoud, 1992; Liang and Townshend, 1996a, b; Sadeghi et al., 2007). Owing to their explicit physical formulation,
 54 RTMs have become a fundamental tool for the inversion of soil-related parameters from optical remote sensing observations.
 55 Moreover, soil reflectance constitutes a key background component of vegetation canopies and directly affects the accuracy
 56 of vegetation radiative transfer models and the retrieval of biophysical vegetation variables (Ni and Li, 2000; Ma et al.,
 57 2017a, b; Yang, 2022; Zeng et al., 2021). Despite substantial progress, important limitations remain in current soil RTMs.
 58 Many widely used models, including the multilayer radiative transfer model of soil reflectance (MARMIT) (Babliet et al.,
 59 2019) and the brightness-shape-moisture (BSM) model (Verhoef et al., 2018), rely on simplified assumptions of soil
 60 reflectance behavior and do not fully capture the combined spectral variability and moisture-dependent effects of natural
 61 soils (Jiang and Fang, 2019; Yang, 2022). Consequently, the development of physically consistent soil RTMs that can jointly
 62 represent spectral behavior and moisture-driven processes remains a critical requirement for reliable surface radiative
 63 transfer modeling and parameter inversion (Cheng et al., 2022; Jiang et al., 2023; Li et al., 2021; Verhoef et al., 2007). To
 64 address this requirement, appropriate model selection is essential for constructing a physically consistent framework. Among
 65 the soil models listed in Table 1, the Hapke-HSR and MARMIT-2 models were selected due to their complementary
 66 physical characteristics. The Hapke-HSR model explicitly represents directional scattering and provides a physically based
 67 description of dry soil reflectance, whereas the MARMIT-2 model focuses on moisture-related absorption processes but does
 68 not account for angular effects and depends on externally prescribed dry soil reflectance. Their combination therefore
 69 enables a physically consistent integration of directional and moisture-dependent processes, which is not achievable with
 70 semi-empirical models such as the BSM model.

71 **Table 1.** Summary of the strengths and limitations of existing soil radiative transfer models.

Models/References	Modelling dry soil	Absorption of water film	Irregular water film thickness	BRDF signatures
Ångström (Ångström, 1925)	×	√	×	×
Lekner & Dorf (1988)	×	√	×	×
Bach & Mauser (1994)	×	√	×	×
SPLITS (Kimmel & Baranoski, 2007)	√	√	√	√
Hapke (2012)	√	√	×	√
Sadeghi et al.(2017)	×	√	×	×
BSM (Verhoef et al., 2018)	√	√	√	×
MARMIT (Babliet et al., 2018)	×	√	√	×
Hapke-HSR (Ding et al., 2022)	√	√	×	√
MARMIT-2 (Dupiau et al., 2022)	×	√	√	×

72
 73 The Hapke model has been widely applied in remote sensing for the retrieval of soil physical and biochemical
 74 properties (Hapke, 2012; Zhao et al., 2023). Building on this framework, we previously developed a hyperspectral Hapke
 75 model (i.e, the Hapke-HSR model) by establishing a wavelength-dependent representation of the single scattering albedo
 76 (SSA) (Ding et al., 2022), which enables improved simulation of dry soil spectral reflectance (SSR). Nevertheless, important
 77 limitations remain in the representation of moisture-dependent soil reflectance. First, the Hapke-HSR model is primarily

78 parameterized for dry soil conditions, and its extension to wet soils relies on simplified assumptions. Second, the influence
 79 of soil moisture is represented through an idealized surface water layer, which restricts the model's ability to accurately
 80 characterize reflectance variations across a broad range of soil moisture content (SMC) and leads to systematic biases in
 81 major water absorption regions. Recently, the MARMIT-2 model has demonstrated strong performance in simulating SSR
 82 under varying moisture conditions by incorporating reflectance properties from diverse soil types (Dupiau et al., 2022).
 83 However, the MARMIT-2 model does not explicitly represent angular effects and requires prior knowledge of dry soil
 84 reflectance, which is often difficult to obtain from field or satellite observations. These limitations indicate that neither
 85 Hapke-HSR nor MARMIT-2 model alone provides a fully consistent framework for modeling soil reflectance under varying
 86 moisture conditions, thereby motivating the development of a unified soil modeling framework.

87 To address these limitations, this study develops a unified soil radiative transfer framework by coupling the improved
 88 Hapke-HSR model with the MARMIT-2 model. The main contributions can be summarized as follows. First, the proposed
 89 framework integrates directional scattering and moisture-related absorption processes within a physically consistent
 90 formulation, which is not achieved by existing models. Second, the improved Hapke-HSR model dynamically provides dry
 91 soil reflectance, thereby removing the dependency of MARMIT-2 model on externally prescribed dry reflectance. Third, the
 92 coupled model enhances simulation robustness under high soil moisture conditions, particularly in strong water absorption
 93 regions. Overall, this study presents a unified and physically interpretable framework for simulating soil reflectance under
 94 varying moisture conditions, improving both consistency and applicability for remote sensing inversion.

95 2 Soil radiative transfer models

96 2.1 Hapke-HSR model

97 In the Hapke-HSR model, dry soil is treated as a semi-infinite horizontal surface containing irregularly and randomly
 98 distributed absorbing particles (Ding et al., 2022), and the formulas of this model are defined as follows:

$$R_d(\theta_s, \theta_v, \varphi, \lambda) = \frac{\omega}{4} \frac{1}{\cos \theta_s + \cos \theta_v} \{ [P(g, g')(1 + B(g))] + H(\cos \theta_s)H(\cos \theta_v) - 1 \} \quad (1)$$

$$P(g, g') = 1 + b \cos g + \frac{c(3 \cos^2 g - 1)}{2} + b' \cos g' + \frac{c'(3 \cos^2 g' - 1)}{2} \quad (2)$$

$$\cos g = \cos \theta_s \cos \theta_v + \sin \theta_s \sin \theta_v \cos \varphi \quad (3)$$

$$\cos g' = \cos \theta_s \cos \theta_v - \sin \theta_s \sin \theta_v \cos \varphi \quad (4)$$

$$B(g) = \frac{B_0}{1 + \frac{1}{h} \tan\left(\frac{g}{2}\right)} \quad (5)$$

$$H(x) = \frac{1 + 2x}{1 + 2x\sqrt{1 - \omega}} \quad (6)$$

99 where $R_d(\theta_s, \theta_v, \varphi, \lambda)$ represents dry soil reflectance, and ω denotes the average SSA. $B(g)$ is the hotspot function, with $B_0 =$
 100 0.4 and $h = 0.1$ adopted as optimal values. The scattering phase function is represented by $P(g, g')$, for which the parameters
 101 are set as $b' = 0.4$, $c = 0.4$, and $c' = 0$. These parameter values are adopted from previous studies (e.g., Hapke, 2012; Ding et
 102 al., 2022) and represent commonly used or empirically optimized values for soil surfaces, providing stable and physically
 103 reasonable simulations across a wide range of conditions. The $P(g, g')$ function is employed in this study because it provides
 104 an improved representation of soil anisotropic scattering characteristics. Here, θ_s , θ_v , and φ denote the solar zenith angle
 105 (SZA), view zenith angle (VZA), and relative azimuth angle (RAA), respectively.

106 The relationship between the soil SSA and wavelength was established via radiative transfer theory. The association
 107 formulas of the SSA and wavelength are as follows:

$$\omega = 1 - \frac{4\pi M \chi_{soil}}{\lambda} \quad (7)$$

108 where M represents the soil particle size and shape-dependent parameter and χ_{soil} is the soil refractive index, which is
 109 important for simulating soil optical properties.

110 In the Hapke-HSR model, the relationship between SSA (ω) and wavelength is further simplified as follows:

$$\omega_1 = 1 - \frac{1}{A_0 \lambda + A_1} \quad (8)$$

$$\omega_2 = 1 - \frac{1}{A_2 \lambda + A_3} + \Delta d \quad (9)$$

111 where Δd refers to the offset of two soil spectra and where A_0, A_1, A_2 and A_3 represent the soil spectral parameters.

112 Considering the influence of SMC, we assume that the equivalent water thickness represents the transition from dry soil
 113 to wet soil (Yang et al., 2011). The formulas are defined as follows:

$$R_w(\theta_s, \theta_v, \varphi, \lambda) = R_d(\theta_s, \theta_v, \varphi, \lambda) \times e^{-\alpha_{water} f} \quad (10)$$

$$\alpha_{water} = \frac{4\pi \chi_{water}}{\lambda} \quad (11)$$

114 where $R_w(\theta_s, \theta_v, \varphi, \lambda)$ represents wet soil reflectance, χ_{water} is the water refractive index for water, and f is the equivalent
 115 water thickness.

116 2.2 Improvement of the Hapke-HSR model

117 The primary challenge in addressing the statistical relationship between the SSA and wavelength of the Hapke-HSR model is
 118 to provide a stable parameter χ_{soil} . In this study, we assume that the shape of the dry SSR is consistent with the variation in
 119 the SSA with wavelength (Ding et al., 2022). A method is proposed to calculate the parameter χ_{soil} via the dry SSR and
 120 improve the Hapke-HSR model. The SSA typically has a significant influence on the SSR, with multiple scattering events
 121 often disregarded in the theoretical derivation (Yang, 2022). The relationships between dry SSR and SSA are as follows:

$$R_d(\theta_s, \theta_v, \varphi, \lambda) = C_1 \times \omega \quad (12)$$

$$C_1 = \frac{1}{4(\cos \theta_s + \cos \theta_v)} \{ [P(g, g') (1 + B(g))] - 1 \} \quad (13)$$

$$\omega = 1 - \frac{C_2 \times \chi_{soil}}{\lambda} \quad (14)$$

$$C_2 = 4\pi M \quad (15)$$

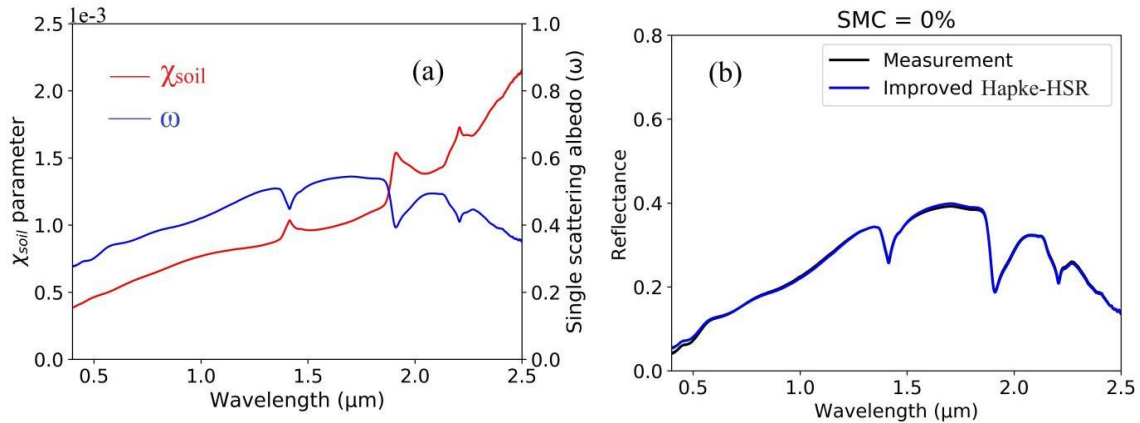
$$\chi_{soil} = \frac{\lambda}{C_2} \times \left(1 - \frac{R_d(\theta_s, \theta_v, \varphi, \lambda)}{C_1} \right) \quad (16)$$

122 where C_1 and C_2 are the shape adjustment parameters of the dry SSR. We use $C_1 = 1$ and $C_2 = 1$ as the initial values to
 123 simplify the calculation, and we then further calculate the shape adjustment parameters.

124 To incorporate the impact of multiple scattering within the soil, we first calculate the χ_{soil} parameter via Eq. (16). This
 125 parameter is then utilized in the improved Hapke-HSR model to determine the dry SSR. We adjust the dry SSR estimated
 126 with the improved Hapke-HSR model via the measured SSR and then calculate the adjusted dry SSR. The relationship
 127 between the dry SSR simulated via the improved Hapke-HSR model and the measured dry SSR can be expressed via the
 128 following formula:

$$R'_d(\theta_s, \theta_v, \varphi, \lambda) = C_3 \times R_d(\theta_s, \theta_v, \varphi, \lambda) + C_4 \quad (17)$$

129 where C_3 and C_4 are spectral shape adjustment parameters for dry soil reflectance, obtained through linear regression
 130 between simulated and measured reflectance, and used as empirical correction terms to compensate for discrepancies arising
 131 from the simplified treatment of multiple scattering in the Hapke-HSR model. Here, $R_d(\theta_s, \theta_v, \varphi, \lambda)$ denotes the reflectance
 132 simulated by the improved Hapke-HSR model, and $R'_d(\theta_s, \theta_v, \varphi, \lambda)$ represents the measured dry soil reflectance.



133
 134 **Figure 1: The variation in the soil refractive index (parameter χ_{soil}) and SSA of dry soil with wavelength (a). The**
 135 **measured dry soil reflectance (i.e., dup20_009) and soil reflectance calculated with the improved Hapke-HSR model**
 136 **(b).**

137 Figure 1(a) shows the variation in the parameter χ_{soil} and soil SSA calculated with the dry SSR (i.e., dup20_009)
 138 considering the wavelength. The parameter χ_{soil} increases with wavelength, the slope clearly increases at wavelengths of 2.0–
 139 2.5 μm , and there are obvious peak values in the absorption band of water (centred at 1.47, 1.90 and 2.21 μm). The soil SSA
 140 is highly similar to the dry SSR. With increasing wavelength, the SSA increases significantly in the spectral range of 0.4–
 141 1.36 μm . In the spectral range of 1.36–2.5 μm , obvious valleys occur in the absorption band of water, which is similar to that
 142 of dry SSR. In general, the SSA and dry SSR curves display very high similarity in terms of shape, but the SSA curve is
 143 flatter. Figure 1(b) shows the measured dry SSR and the SSR estimated with the improved Hapke-HSR model. Clearly, the
 144 SSR modelled by the improved Hapke-HSR model matches well with the measured dry SSR and can characterize the dry
 145 SSR characteristics well. The accuracy of this improved model in fitting these typical data is shown in Table 4 ($R^2 = 1.00$,
 146 RMSE = 0.001). These analyses suggest that calculating the parameter χ_{soil} via dry SSR data is feasible and can solve the
 147 problems associated with the statistical relationship between the SSA and wavelength of the Hapke-HSR model.

148 2.3 Coupling strategy between the improved Hapke-HSR and MARMIT-2 models

149 (1) Physical coupling mechanism between the improved Hapke-HSR model and MARMIT-2 model

150 The improved Hapke-HSR model provides an effective description of multi-angular dry soil spectral reflectance, whereas the
 151 MARMIT-2 model explicitly accounts for the influence of SMC on soil reflectance but assumes that dry reflectance is
 152 known and does not incorporate directional information. By coupling these two models, their complementary strengths can
 153 be integrated to achieve a more physically consistent representation of both the spectral and moisture-dependent behavior of
 154 soils. In the proposed framework, dry soil reflectance under different viewing geometries is first simulated using the
 155 improved Hapke-HSR model and subsequently used as input to the MARMIT-2 model. As a result, the MARMIT-2 model
 156 no longer requires externally prescribed dry reflectance. Moreover, because the simulations of the Hapke-HSR model retain
 157 angular information, the coupled Hapke-HSR + MARMIT-2 model is able to account for the influence of observation
 158 geometry on soil reflectance. Through this coupling strategy, the reflectance properties of both dry and wet soils can be
 159 simulated under varying angular and moisture conditions within a unified framework (Ding, 2025).

160 (2) Radiative transfer formulation of the coupled Hapke-HSR + MARMIT-2 model

161 In the MARMIT-2 model, wet soil is described as dry soil overlaid with a thin layer of water (Dupiau et al., 2022). Therefore,
 162 the wet SSR is described as follows:

$$R_{mod}(\theta_s, \theta_v, \varphi, \lambda) = \varepsilon R_w(\theta_s, \theta_v, \varphi, \lambda) + (1 - \varepsilon) R_d(\theta_s, \theta_v, \varphi, \lambda) \quad (18)$$

163 where $R_m(\theta_s, \theta_v, \varphi, \lambda)$ is the wet SSR, $R_d(\theta_s, \theta_v, \varphi, \lambda)$ is the dry SSR calculated via the improved Hapke-HSR model, $R_w(\theta_s,$
 164 $\theta_v, \varphi, \lambda)$ refers to the fully wet SSR, ε is the wet soil fraction, and $R_w(\theta_s, \theta_v, \varphi, \lambda)$ is described as:

$$R_w(\theta_s, \theta_v, \varphi, \lambda) = \frac{t_{12}t_{21}R_d(\theta_s, \theta_v, \varphi, \lambda)T_w^2}{1 - r_{21}R_d(\theta_s, \theta_v, \varphi, \lambda)T_w^2} \quad (19)$$

165 where t_{12} and t_{21} are the interface transmittance of light passing into and out of the water layer, respectively.

166 To address the presence of multiple scattering events within the water layer, transmittance (T_w) is considered.

$$T_w = (1 - \alpha_{water}L)e^{-\alpha_{water}L} + \alpha_{water}L \int_{\alpha_{water}L}^{\infty} \frac{e^{-x}}{x} dx \quad (20)$$

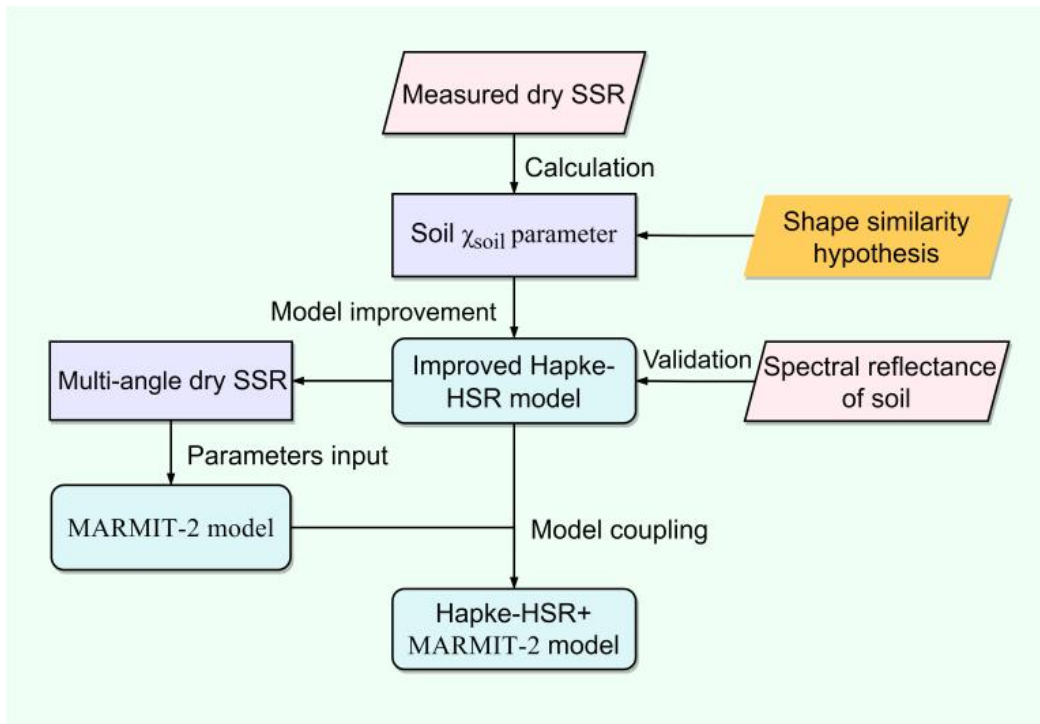
167 where L refers to the water layer thickness.

$$n_{mix} = \delta n_{soil} + (1 - \delta)n_{water} \quad (21)$$

168 where n_{mix} represents the result of the linear weighting of the complex refractive index of water and soil, n_{soil} and n_{water}
 169 represents the complex refractive index of water and soil particles respectively, and δ represents the volume fraction of soil
 170 particles.

171 (3) Computational procedure and evaluation scheme

172 Figure 2 illustrates the workflow of the improved Hapke-HSR model and its coupling with the MARMIT-2 model. First, the
 173 soil parameter χ_{soil} is estimated from measured dry SSR by assuming similarity between the spectral shapes of the single-
 174 scattering albedo and dry SSR. The derived χ_{soil} is then incorporated into the Hapke-HSR model, which alleviates the
 175 limitations associated with piecewise fitting of the single scattering albedo and improves the numerical stability of the
 176 formulation. The refined Hapke-HSR model is subsequently used to simulate dry SSR under different viewing geometries,
 177 and these simulations are provided as input to the MARMIT-2 model, thereby enabling the dynamic coupling of the two
 178 formulations. For parameter estimation, the coupled model allows flexible inversion strategies. In this study, the parameters
 179 can be estimated either simultaneously by jointly optimizing all model parameters, or sequentially by first determining the
 180 dry-soil scattering parameters (e.g., b and M) using the Hapke-HSR model, followed by the estimation of moisture-related
 181 parameters (e.g., ε , L and δ) using the MARMIT-2 model. Finally, the performance of the coupled Hapke-HSR + MARMIT-
 182 2 model is evaluated using eight independent soil spectral databases spanning a range of soil moisture conditions. Model
 183 performance is assessed using multiple statistical metrics, including the coefficient of determination (R^2), root mean square
 184 error (RMSE), normalized RMSE (NRMSE), mean relative error (MRE), and bias.



185

186 **Figure 2: The workflow of the improved Hapke-HSR model and the coupled MARMIT-2 model.**

187 **3 Databases and methods**

188 **3.1 Databases of soil spectral reflectance**

189 In this section, eight different databases provided by Dupiau et al. (2022) are used to verify the Hapke-HSR + MARMIT-2
 190 model (Table 2). These eight datasets were acquired primarily via the Analytical Spectra Devices (ASD) FieldSpec
 191 spectroradiometer. The soil types in the datasets are diverse, consisting primarily of clay, silt, and sand as the main
 192 components. The Bab16, Dup20, and Liu02 databases provide the soil components of each sample, such as organic matter,
 193 iron, nitrogen and organic carbon. These eight databases provide dry and wet SSR data for 219 soil samples, spanning 1984
 194 spectra in the 0.4-2.5 μm range. The data quality of these databases is high, but there are some uncertainties in the 2.4-2.5
 195 μm range. Therefore, these databases offer crucial data for validating the ability of the Hapke-HSR + MARMIT-2 model to
 196 describe SSR features.

197 **Table 2.** The main information on the eight soil databases.

Databases	Locations	Number of soil spectrum	Spectral range (μm)	Spectral resolution (μm)	SMCg (%)
Bab16	ONERA, Toulouse (France)	106	0.4-2.4	0.001	0-79.2
Dup20	ONERA, Toulouse (France)	72	0.4-2.5	0.001	0-68.91
Hum15	ONERA, Toulouse (France)	455	0.4-2.298	0.001	0-67
Les08	ONERA, Toulouse (France)	190	0.4-2.4	0.001	0-87
Liu02	INRAE, Avignon (France)	367	0.4-2.4	0.001	0-81.1
Lob02	Standford University, Standford (CA, USA)	41	0.4-2.49	0.001	0-109.4
Mar12	CEA, Bruyères le Chatel (France)	258	0.4-2.4	0.001	0-52.9
Phil14	Cornell University, Ithaca (NY, USA)	405	0.4-2.5	0.001	0-50.71

198 Note: SMCg is the soil moisture content (SMC) as a weight percent.

200 4.1 Parameters analysis of the Hapke-HSR + MARMIT-2 model

201 In this section, we first analyse the effects of the main input parameters of the Hapke-HSR + MARMIT-2 (HM) model on
 202 the soil reflectance properties. Table 3 shows the main input variables of the HM model. The Hapke-HSR model has many
 203 input variables, and the variables of this model were optimized in previous studies (Ding et al., 2022; Dupiau et al., 2022;
 204 Hapke, 2012; Verhoef et al., 2018); five main variables were used to describe the soil spectral and angular features. The
 205 MARMIT-2 model includes three main parameters that describe the influence of the SMC on SSR. We analyse the effects of
 206 different parameters in the HM model on the simulated SSR.

207 **Table 3.** The main input variables of the Hapke-HSR + MARMIT-2 (HM) model.

Databases	Parameters	Range of values	Units
Hapke-HSR	Solar zenith angle (SZA)	0-90	degrees (°)
	View zenith angle (VZA)	0-90	degrees (°)
	Relative azimuth angle (RAA)	0-180	degrees (°)
	The coefficient of the scattering phase function (b)	0-6	--
	Soil particle size and shape-dependent parameter (M)	0-1	mm
MARMIT-2	Volume fraction of soil particles (δ)	0-0.25	--
	Thickness of water layer (L)	0-0.15	cm
	Surface coverage fraction of water (ϵ)	0-1	--

208 Figure 3 illustrates the impact of the main variables of the Hapke-HSR + MARMIT-2 (HM) model on the SSR. For the
 209 influence of angle variation, the SSR calculated via the HM model increases with increasing SZA parameters. The increase
 210 in SSR is obvious in the range of 0.4–1.36 μm ; however, the rate of increase in SSR subsequently decreases. The impact of
 211 VZA parameters on SSR is consistent with the influence of the SZA parameters since the Hapke-HSR model is reciprocal.
 212 Furthermore, the influence of the RAA parameters on SSR is basically the same as that of the SZA parameters with
 213 increasing RAA parameters; however, the changes in soil reflectance are slightly different. With increasing parameter b , the
 214 SSR simulated by the HM model decreases, with a more pronounced reduction in the range of 0.4–1.36 μm than in 1.36–2.5
 215 μm . Both parameters b and c influence the BRDF shape through the phase function. However, b primarily controls the
 216 overall anisotropy of scattering, whereas c governs the forward–backward asymmetry. Under the observational
 217 configurations considered in this study, b shows a stronger and more stable influence on reflectance, while the sensitivity of
 218 c is relatively weaker. Therefore, the discussion focuses on parameter b (Ding et al., 2022, 2026). The SSR simulated with
 219 the HM model also decreases with increasing M ; this finding corroborates the experimental observations regarding spectral
 220 variations due to soil particle size reported by Sun et al.(2023). However, the variation in the parameter M of the SSR is
 221 basically the same at different wavelengths. A possible reason is that the influence of the parameter M in the range of 0.4-2.5
 222 μm on the SSR is consistent, which may be related to the structure of Eq. (7). For the variables related to the SMC, with
 223 increasing parameter δ , the SSR decreases, whereas in the strong absorption band of water, this effect is smaller. A possible
 224 reason for this result is that the absorption of water weakens the impact of the parameter δ on the SSR. As the parameter L
 225 increases, the SSR decreases in the range of 1.0–2.5 μm , whereas the variation in the parameter L has no effect on the soil
 226 reflectance in the range of 0.4–1.0 μm . With increasing parameter ϵ , the SSR decreases. In the strong absorption band of
 227 water, the simulated SSR quickly decreases. In summary, the main parameters of the Hapke-HSR model are related to the
 228 influence of dry SSR and angular variation characteristics, and the variables of the MARMIT-2 model mainly account for
 229 the influence of SMC on the SSR. Therefore, the HM model can characterize the spectral and angular reflectance attributes
 230 of dry and wet soils by coupling the Hapke-HSR and MARMIT-2 models.

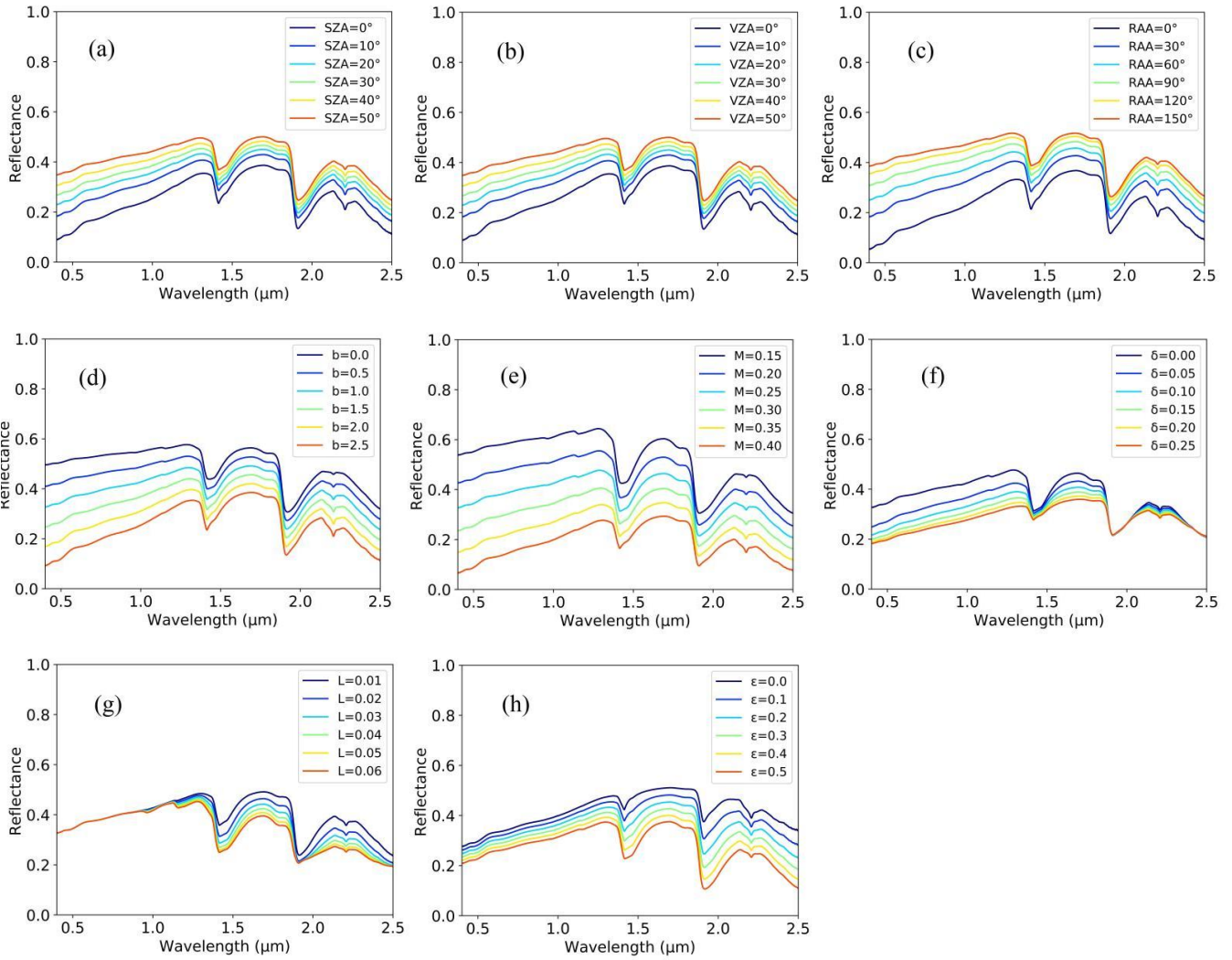


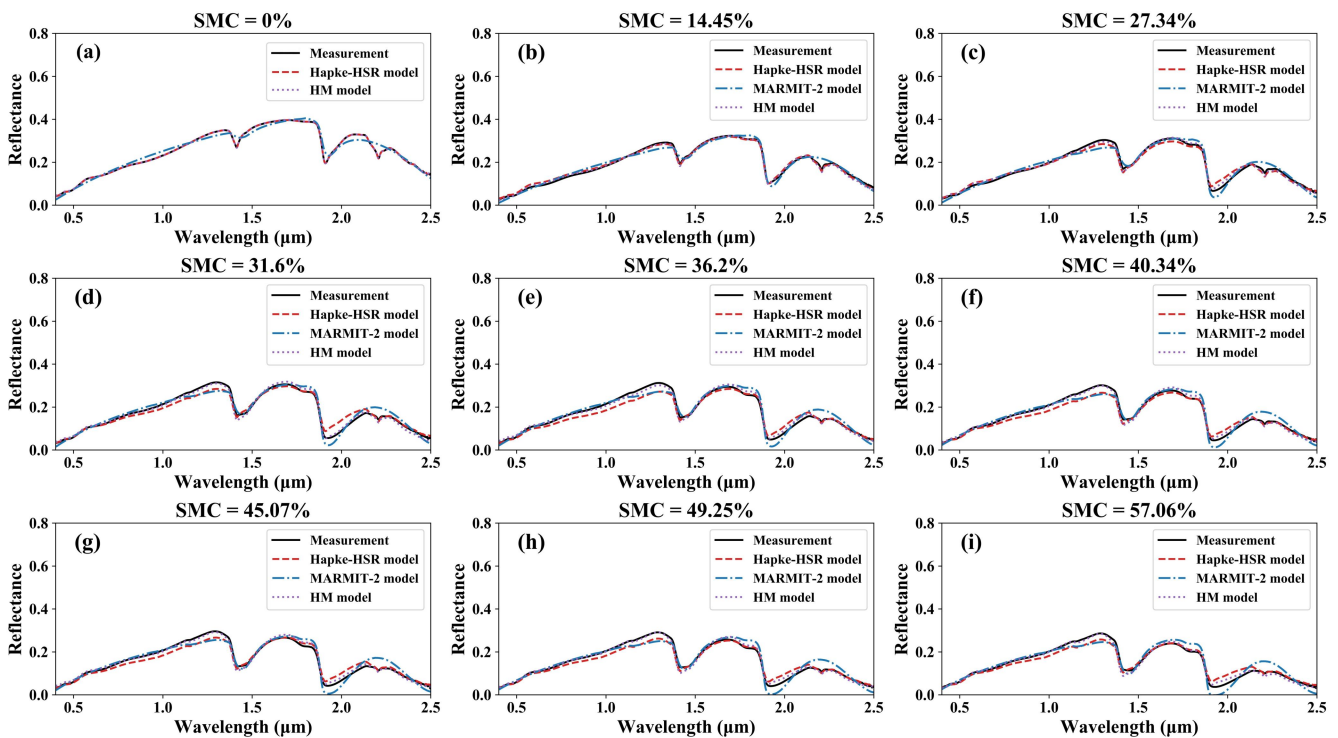
Figure 3: Influence of the SZA (a), VZA (b), RAA (c), b (d), M (e), δ (f), and ϵ (h) in the Hapke-HSR + MARMIT-2 (HM) model on soil reflectance.

4.2 Validating the Hapke-HSR + MARMIT-2 model to describe soil reflectance properties

In previous studies, we assessed the Hapke-HSR model to describe soil BRDF features (Ding et al., 2022). Therefore, we focus mainly on evaluating the soil spectral characteristics in this paper. In addition, these eight soil databases do not provide angle-related information. Therefore, we use $SZA = 45^\circ$, $VZA = 0^\circ$ and $RAA = 0^\circ$ as fixed values because SSR is usually measured in the nadir direction. Figure 4 shows that the Hapke-HSR, MARMIT-2 and Hapke-HSR + MARMIT-2 (HM) models effectively fit the influence of the typical fitted SSR (dup20_009-001) at SMCs = 0%, 14.45%, 27.34%, 31.6%, 36.2%, 40.34%, 45.07%, 49.25%, and 57.06%, respectively. The SSR decreases significantly with increasing SMC, and the main absorption bands (centred at 1.47 and 1.90 μm) of water become wider. The outcomes simulated using the Hapke-HSR, MARMIT-2 and HM models can be used to determine the change in SSR with increasing SMC and are highly in line with the measured SSR values. On the basis of a comparison of the results, the HM model fits the measured SSR better than the Hapke-HSR and MARMIT-2 models do, especially at $SMC \geq 30\%$. Compared with the MARMIT-2 model, the HM model yields slightly better results. The main reason is that the MARMIT-2 model yields very high accuracy in characterizing SSR characteristics. The ability of the Hapke-HSR model to accurately fit the measured SSR decreases with increasing SMC. The correlation analysis results indicate that the fitting ability of these two models meets the relevant requirements. However, at $SMC \geq 30\%$, the fitting capability of the MARMIT-2 model is relatively low due to slight underestimations in the range of 0.4-1.36 μm , and there is an overestimation in the strong water absorption band; moreover, the Hapke-HSR model has

252 difficulty capturing the SSR characteristics in the absorption band of water, which leads to significant underestimation of the
 253 fitted SSR at a wavelength of approximately 1.90 μm . However, the HM model effectively considers the influence of these
 254 factors, resulting in high accuracy for characterizing SSR attributes. To show the differences between the SSR values
 255 simulated with these three models and the measured spectral reflectance values, we calculate the bias between them, as
 256 shown in Appendix Figure A1. In addition, the Hapke-HSR and HM models are applied to simulate the dry SSR, and the
 257 SSR simulated via the HM model is more consistent with the measured results than the SSR obtained with the Hapke-HSR
 258 model is.

259 Table 4 shows that the Hapke-HSR, MARMIT-2 and HM models fit the parameters and statistical results of the SSR.
 260 With increasing SMC, the parameters L and ε in the MARMIT-2 model increase significantly, whereas the parameter δ
 261 shows little variation. The parameter f increases significantly. Moreover, the SSR fitting accuracy of the Hapke-HSR and
 262 MARMIT-2 models decreases with increasing SMC, especially at $\text{SMC} \geq 30\%$. This finding may be because the MARMIT-
 263 2 model ignores the variations in soil scattering characteristics, particle size and shape with increasing SMC. In the Hapke-
 264 HSR model, a dry soil surface is overlaid with a water layer to reflect the influence of the SMC on the SSR. This simple
 265 assumption limits the ability of the Hapke-HSR model to fit the variable characteristics of the SMC. The overall R^2 values
 266 for the Hapke-HSR model in SSR fitting vary from 0.952 to 0.971, with RMSE values varying from 0.016 to 0.019, and the
 267 MARMIT-2 model achieves R^2 values between 0.957 and 0.995 in SSR fitting, with RMSE values ranging from 0.007 to
 268 0.021 and negligible bias. These results indicate that these two models can effectively characterize the variation in SSR with
 269 SMC and yield high fitting accuracy. However, the HM model is accurate (RMSE = 0.008), presenting a high R^2 ($R^2 = 0.991$)
 270 and a small bias in relation to the measured SSR. This is because the HM model considers the variations in the soil scattering
 271 characteristics, particle size and particle shape with increasing SMC. Therefore, the HM model can effectively characterize
 272 the attributes of SSR and exhibits greater accuracy than the Hapke-HSR and MARMIT-2 models do, especially at $\text{SMC} \geq$
 273 30%.



274
 275 **Figure 4: The Hapke-HSR (red), MARMIT-2 (blue) and Hapke-HSR + MARMIT-2 (HM) (lime) models fit the**
 276 **measured soil reflectance (black) at SMC = 0% (a), 14.45% (b), 27.34% (c), 31.6% (d), 36.2% (e), 40.34% (f), 45.07%**
 277 **(g), 49.25% (h), and 57.06% (i), respectively.**

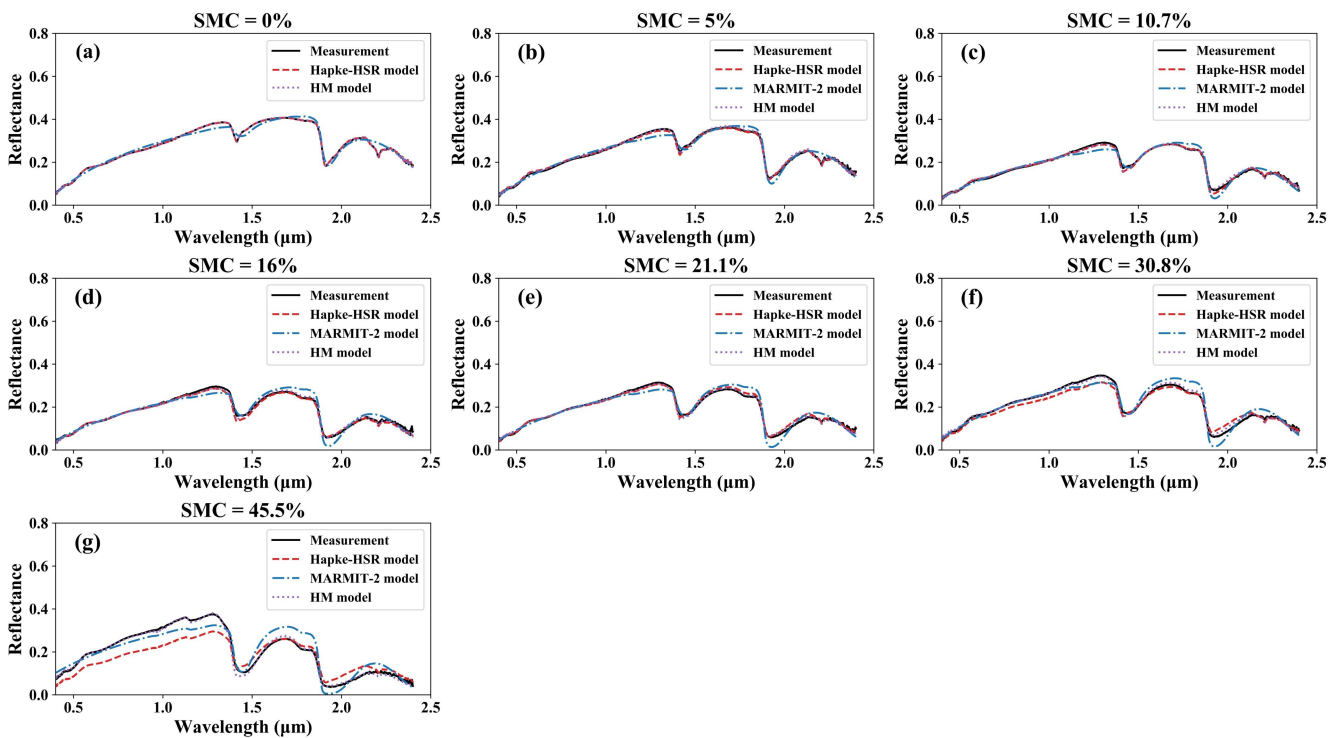
Table 4. The Hapke-HSR, MARMIT-2 and Hapke-HSR + MARMIT-2 (HM) models fit the soil reflectance variables and statistical outcomes.

Models	SMC (%)	b	f/M	A_0/δ	A_1/L	A_2/ε	A_3	R^2	RMSE	bias
Hapke-HSR	0.0	4.4	0.4	0.782	0.723	-1.141	4.178	0.972	0.016	0.001
	14.45	4.8	1.0	0.545	0.796	-0.708	3.054	0.976	0.013	0.001
	27.34	3.0	1.6	0.861	0.677	-1.151	4.336	0.968	0.016	0.000
	31.6	1.4	2.0	1.586	0.405	-1.908	6.738	0.962	0.019	0.001
	36.2	1.6	2.2	1.431	0.479	-1.795	6.326	0.954	0.020	0.002
	40.34	1.0	2.4	1.699	0.392	-1.924	6.948	0.948	0.020	0.002
	45.07	0.0	2.8	2.772	-0.009	-2.420	9.426	0.945	0.021	0.001
	49.25	0.8	2.8	1.747	0.384	-1.757	6.801	0.938	0.022	0.001
	57.06	0.2	3.4	2.316	0.188	-1.390	7.026	0.936	0.023	0.001
	All	--	--	--	--	--	--	--	0.952	0.019
MARMIT-2	0	--	--	--	--	--	--	--	--	--
	14.45	--	--	0.030	0.01	0.3	--	0.995	0.007	0.001
	27.34	--	--	0.000	0.03	0.3	--	0.984	0.013	-0.002
	31.6	--	--	0.000	0.03	0.3	--	0.957	0.019	-0.002
	36.2	--	--	0.000	0.02	0.4	--	0.955	0.021	-0.008
	40.34	--	--	0.000	0.03	0.4	--	0.970	0.018	-0.008
	45.07	--	--	0.000	0.03	0.4	--	0.956	0.018	-0.003
	49.25	--	--	0.000	0.04	0.4	--	0.969	0.017	-0.005
	57.06	--	--	0.000	0.05	0.4	--	0.957	0.018	-0.002
	All	--	--	--	--	--	--	0.971	0.016	-0.003
Hapke-HSR + MARMIT- 2	0	2.0	0.30	0.000	0.00	0.0	--	1.000	0.001	0.000
	14.45	2.2	0.29	0.018	0.01	0.3	--	0.994	0.006	-0.001
	27.34	3.1	0.24	0.011	0.02	0.4	--	0.991	0.008	0.000
	31.6	4.0	0.21	0.006	0.02	0.5	--	0.989	0.009	-0.001
	36.2	3.3	0.23	0.000	0.02	0.5	--	0.986	0.010	0.000
	40.34	4.0	0.21	0.006	0.03	0.5	--	0.987	0.009	0.000
	45.07	3.3	0.23	0.001	0.03	0.5	--	0.987	0.009	0.002
	49.25	4.0	0.21	0.005	0.04	0.5	--	0.986	0.009	0.001
	57.06	3.7	0.22	0.000	0.05	0.5	--	0.984	0.009	-0.001
	All	--	--	--	--	--	--	0.991	0.008	0.000

280 Figure 5 shows that the Hapke-HSR, MARMIT-2 and HM models fit the typical measured SSR (bab16_014-008) at
281 SMC = 0%, 5%, 10.7%, 16%, 21.1%, 30.8%, and 45.5%, respectively. This set of typical data is suspected to have a specular
282 reflection effect when SMC = 30.8% and 45.5%. Therefore, we further validate the capacity of the HM model for describing
283 the relevant SSR attributes. The outcomes of the Hapke-HSR, MARMIT-2 and HM models match the typical measured SSR
284 when the SMC < 30%. The Hapke-HSR and MARMIT-2 models cannot effectively consider the specular reflectance
285 characteristics at high SMCs (SMC \geq 30%), and the results of the HM model display greater consistency with the measured

286 SSR values. When the SMC is high and there is a specular reflectance effect, the fitting capability of the Hapke-HSR model
 287 is significantly underestimated in the range of 0.5-1.2 μm and two strong water absorption bands; moreover, there is a slight
 288 overestimation in the range of 1.5-1.9 μm . The outcomes of the MARMIT-2 model are marginally underestimated across the
 289 0.4–1.36 μm interval, and there is a slight overestimation across the spectral interval of 1.36–2.5 μm , particularly in the
 290 strong water absorption region. The HM model can match the measured SSR well, especially at SMC = 30.8% and 45.5%,
 291 possibly because this model accounts for the specular scattering characteristics of high SMCs ($\text{SMC} \geq 30\%$) on the basis of
 292 the coefficient (b) and soil particle shape-dependent parameter (M). To better show the differences between the SSR values
 293 simulated with these three models and the measured SSR values, we calculate the bias between them, as shown in Appendix
 294 Figure A2. The results indicate that the HM model can describe SSR features effectively at $\text{SMC} \geq 30\%$, and the simulated
 295 values exhibit very high consistency with the measured SSR values.

296 Table 5 shows that the Hapke-HSR, MARMIT-2 and HM models fit to the SSR parameters and statistical results. The
 297 overall precision of the Hapke-HSR and MARMIT-2 models in terms of fitting measured SSR was high ($R^2 = 0.943\text{-}0.946$,
 298 $\text{RMSE} = 0.006\text{-}0.022$), especially at $\text{SMC} < 30\%$; however, these two models were not suitable at $\text{SMC} \geq 30\%$, which needs
 299 to be improved by accounting for specular reflectance. The HM model shows greater accuracy in fitting the variation in the
 300 SMC than the Hapke-HSR and MARMIT-2 models; the overall R^2 is 0.995, the RMSE is 0.009, and the bias is negligible.
 301 When $\text{SMC} = 30.8\%$ and 45.5% , the measured SSR are suspected to have a specular reflection effect, and the HM model
 302 maintains a higher fitting precision ($R^2 = 0.990\text{-}0.993$, $\text{RMSE} = 0.009$) than the Hapke-HSR and MARMIT-2 models ($R^2 =$
 303 $0.846\text{-}0.973$, $\text{RMSE} = 0.019\text{-}0.053$). These results indicate that these two models can be combined by coupling the Hapke-
 304 HSR and MARMIT-2 models (i.e., HM model), which can effectively determine the variation in SSR with increasing SMC,
 305 particularly in the presence of specular reflectance.



306
 307 **Figure 5: The Hapke-HSR (red), MARMIT-2 (blue) and Hapke-HSR + MARMIT-2 (HM) (lime) models fit the**
 308 **measured soil reflectance (black) at SMC = 0% (a), 5% (b), 10.7% (c), 16% (d), 21.1% (e), 30.8% (f), and 45.5% (g),**
 309 **respectively.**

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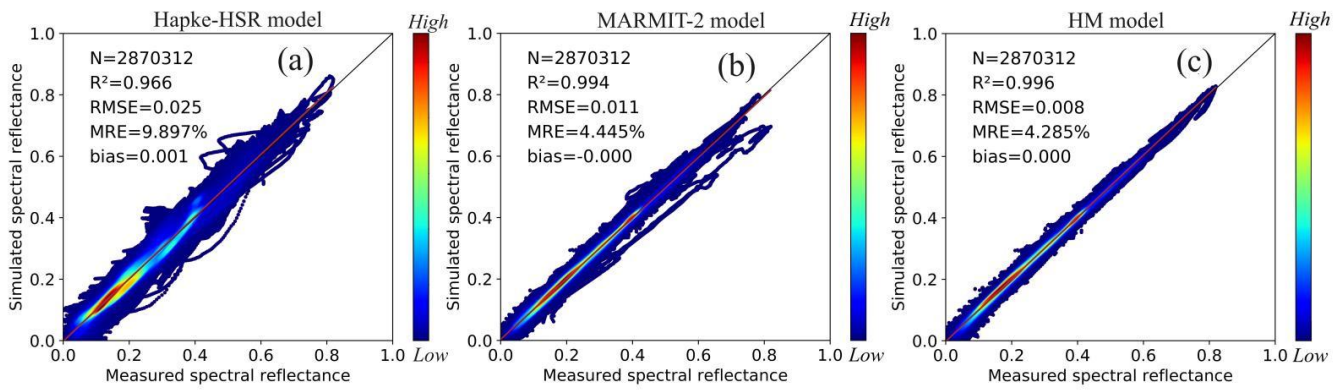
Table 5. The Hapke-HSR, MARMIT-2 and Hapke-HSR + MARMIT-2 (HM) models fit to the soil reflectance parameters and statistical results.

Models	SMC (%)	b	f/M	A_0/δ	A_1/L	A_2/ε	A_3	R^2	RMSE	bias
Hapke- HSR	0.0	1.4	0.6	2.447	0.180	-4.150	12.010	0.974	0.014	0.000
	5.0	1.4	1.0	2.016	0.322	-3.477	10.222	0.971	0.014	-0.001
	10.7	0.0	1.6	2.685	0.091	-5.119	14.339	0.960	0.015	-0.002
	16.0	0.2	2.0	2.506	0.189	-4.601	13.129	0.944	0.018	0.001
	21.1	0.0	2.2	3.144	-0.023	-5.618	15.939	0.941	0.020	0.000
	30.8	0.2	2.4	3.544	-0.173	-6.051	17.081	0.936	0.023	0.003
	45.5	1.0	4.0	2.377	0.439	-4.292	12.350	0.884	0.037	0.004
	All	--	--	--	--	--	--	--	0.943	0.022
MARMIT- 2	0.0	--	--	--	--	--	--	--	--	--
	5.0	--	--	0.000	0.01	0.2	--	0.996	0.006	-0.001
	10.7	--	--	0.020	0.01	0.5	--	0.993	0.006	-0.001
	16.0	--	--	0.010	0.03	0.4	--	0.991	0.009	-0.005
	21.1	--	--	0.000	0.02	0.4	--	0.988	0.009	0.002
	30.8	--	--	0.000	0.04	0.3	--	0.973	0.019	-0.009
	45.5	--	--	0.000	0.04	0.4	--	0.846	0.053	-0.024
	All	--	--	--	--	--	--	--	0.946	0.022
Hapke- HSR + MARMIT- 2	0.0	2.0	0.30	0.000	0.00	0.0	--	1.000	0.001	0.000
	5.0	2.3	0.28	0.020	0.01	0.2	--	0.997	0.005	0.002
	10.7	1.3	0.38	0.001	0.02	0.3	--	0.994	0.006	0.000
	16.0	1.7	0.33	0.000	0.02	0.4	--	0.991	0.007	-0.001
	21.1	2.4	0.27	0.009	0.03	0.4	--	0.992	0.007	0.000
	30.8	3.0	0.24	0.002	0.03	0.4	--	0.990	0.009	0.002
	45.5	5.8	0.17	0.002	0.05	0.6	--	0.993	0.009	-0.002
	All	--	--	--	--	--	--	--	0.995	0.007

316

317 Figure 6 presents a comparison of the SSR results obtained from the Hapke-HSR, MARMIT-2 and HM models and the
 318 measured SSR values from eight databases. These three models are generally highly accurate in terms of capturing SSR
 319 features. However, the HM and MARMIT-2 models ($R^2 = \sim 0.993$) fit the measured SSR data with slightly greater
 320 correlation accuracy than did the Hapke-HSR model ($R^2 = 0.957$), and the RMSE values of the HM (RMSE = 0.010) and
 321 MARMIT-2 (RMSE = 0.012) models were significantly lower than the RMSE of the Hapke-HSR (RMSE = 0.027) model.
 322 Additionally, the MRE values of the HM and MARMIT-2 models are approximately 5.74% and 6.43% lower than that of the
 323 Hapke-HSR model, respectively. These findings indicate that the HM model yields the highest level of accuracy in fitting the
 324 measured SSR, followed by the MARMIT-2 model, whereas the Hapke-HSR model has the worst fitting effect on the basis
 325 of the measured SSR. The main reason is that the Hapke-HSR model includes a simple assumption regarding the effect of
 326 the SMC on SSR. In addition, the SSR simulated by the Hapke-HSR and MARMIT-2 models is considerably uncertain at
 327 high SMCs (e.g., Figure 5(f)-(g)), whereas the HM model results display greater consistency with the fitted SSR value. In
 328 general, the HM and MARMIT-2 models excellently characterize the SSR attributes of soil and yield greater accuracy than

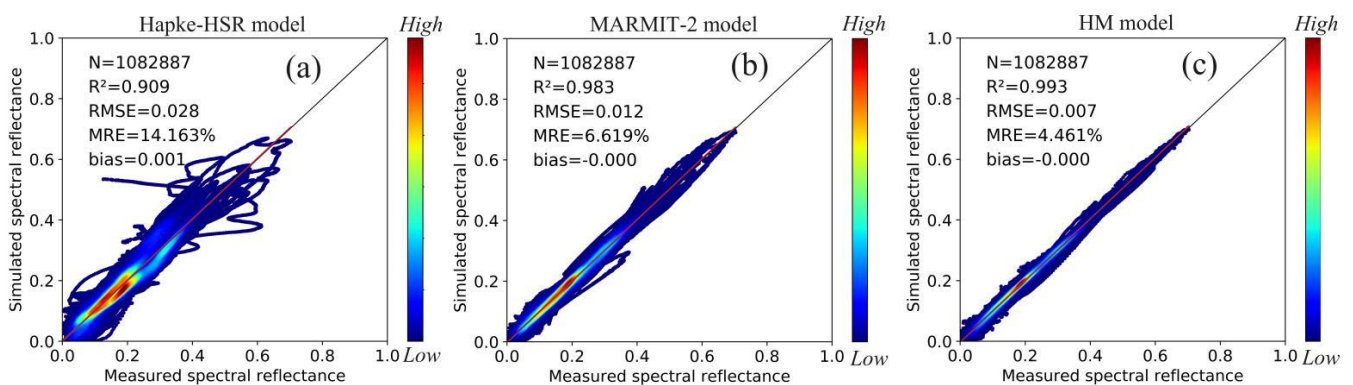
329 the Hapke-HSR model for the eight soil databases does, and the SSR estimates produced by the HM model are marginally
 330 more accurate than those of the MARMIT-2 model are.



331
 332 **Figure 6: Comparison results of the all soil reflectance simulated by the Hapke-HSR (a), MARMIT-2 (b) and Hapke-**
 333 **HSR + MARMIT-2 (HM) (c) models with all measured soil reflectance.**

334 **4.3 Validating the Hapke-HSR + MARMIT-2 model for high SMC**

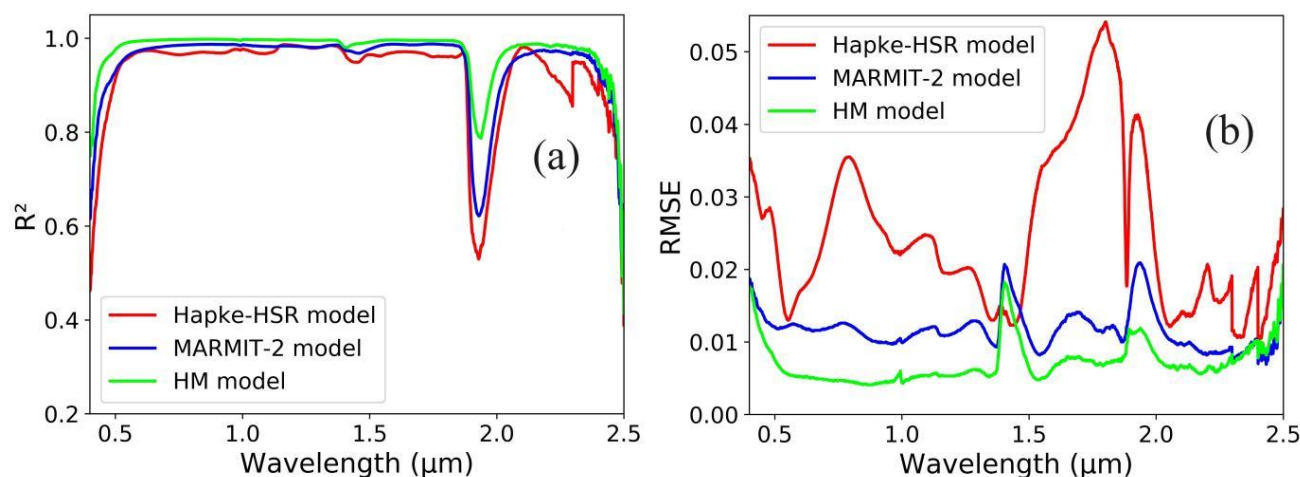
335 The MARMIT-2 and HM models achieve excellent fitting accuracy at SMC levels ranging from 0-30%, whereas the Hapke-
 336 HSR and MARMIT-2 models exhibit moderate fitting capability at $SMC \geq 30\%$ (e.g., Figures 3 and 4). Therefore, focus is
 337 placed on comparing the fitting results of the above three models under the condition of an $SMC \geq 30\%$. Figure 7 shows the
 338 comprehensive results obtained with the Hapke-HSR, MARMIT-2 and HM models at $SMC \geq 30\%$, and these three models
 339 exhibit strong agreement with the measured SSR ($R^2 > 0.90$), with RMSE values ranging from 0.0070.028. However, the
 340 accuracy of the HM model ($R^2 = 0.993$, $RMSE = 0.007$) for fitting SSR data is slightly better than that of the MARMIT-2
 341 model ($R^2 = 0.983$, $RMSE = 0.012$) and significantly greater than that of the Hapke-HSR model ($R^2 = 0.909$, $RMSE = 0.028$).
 342 Compared with those of the MARMIT-2 and Hapke-HSR models, the RMSE values of the HM model are 41.7% and 66.7%
 343 lower, and the MRE values are 2.158% and 9.702% lower, respectively. Moreover, the HM model has the ability to
 344 improved the inadequate fitting outcomes of the Hapke-HSR model. These findings show that the HM model can describe
 345 SSR attributes more effectively than the other models can, especially at $SMC \geq 30\%$. The key factor is that the HM model
 346 combines the strengths of both the Hapke-HSR and MARMIT-2 models to better describe the changes in SSR with
 347 increasing SMC. The MARMIT-2 model also exhibits higher accuracy at $SMC \geq 30\%$ since it fully considers the effect of
 348 the SMC on SSR.



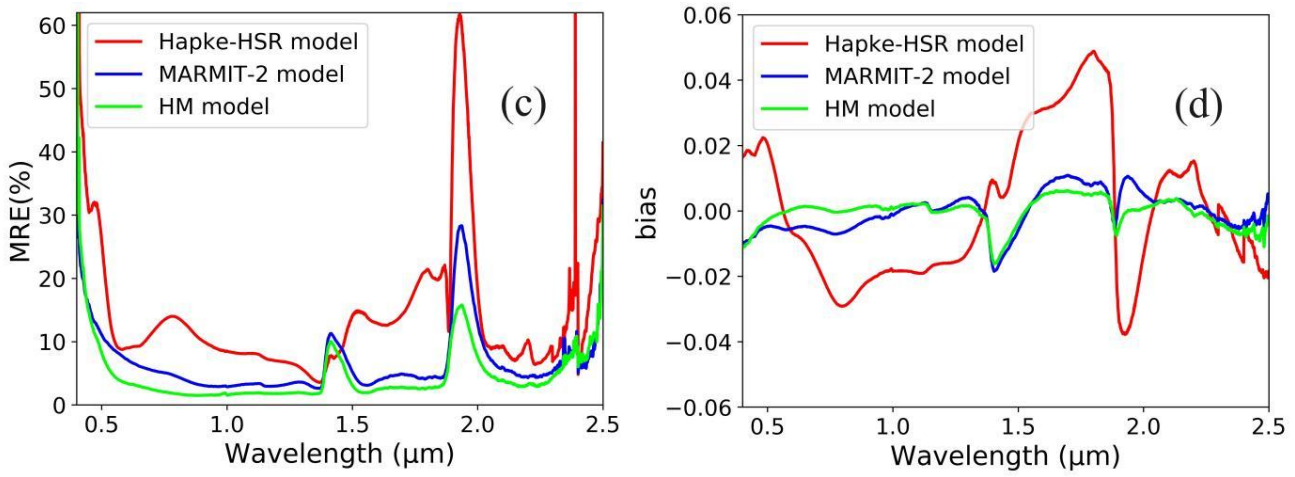
349
 350 **Figure 7: Comparison results of the soil reflectance values simulated by the Hapke-HSR (a), MARMIT-2 (b) and**
 351 **Hapke-HSR + MARMIT-2 (HM) (c) models with the measured soil reflectance at $SMC \geq 30\%$.**

352 Next, we further evaluate the ability of the Hapke-HSR, MARMIT-2 and HM models to fit the measured SSR for
 353 different bands (0.4-2.5 μm) at $SMC \geq 30\%$. Figure 8 shows the comparison results between the simulated SSR values of the
 354 three models and the measured SSR values. The results of these three models are highly consistent with the measured values.

355 The R^2 values of these three models are generally very high, in the range of 0.4–2.5 μm . The R^2 value of the HM model is
 356 the largest, followed by those of the MARMIT-2 model and, finally, the Hapke-HSR model. However, the consistency
 357 between the outcomes of these three models and the measured values in the strong absorption band of water (centred at 1.90
 358 μm) is significantly lower than that for the other bands. The RMSE results from the HM model were the smallest, followed
 359 by those of the MARMIT-2 model. The maximum RMSE value of the Hapke-HSR model was approximately 0.05 because
 360 the Hapke-HSR model uses a simplistic assumption to reflect the effect of the SMC. However, the HM and MARMIT-2
 361 models significantly outperform the Hapke-HSR model, specifically at two major absorption bands of water (centred at 1.47
 362 and 1.90 μm), since the HM and MARMIT-2 models fully consider the changes in SSR characteristics with variations in the
 363 SMC. The MRE trends of the three models are basically similar to the RMSE trends, and the HM model demonstrates the
 364 highest level of accuracy, with the MARMIT-2 model following closely behind. The MRE value of the Hapke-HSR model is
 365 approximately 60% at the major absorption band of water (centred at 1.90 μm), which is significantly greater than that of the
 366 MARMIT-2 model (28%) and the HM model (15%). The bias values of the HM model approach 0.4–2.5 μm , whereas those
 367 of the Hapke-HSR model exhibit a large range of variation. These studies indicate that the HM model results are more in line
 368 with the fitted SSR, whereas the Hapke-HSR model results are more different from the measured SSR values. In general, the
 369 variations between the outputs of these three models and the measured values are in the wavelength ranges of 0.4–0.6 μm
 370 and 2.4–2.5 μm and represent the two major absorption bands of water. The soil reflectance is low over the spectral region
 371 from 0.40.6 μm , and the soil reflectance variation remains insignificant in this wavelength range as the SMC increases. The
 372 soil measurements have great uncertainty in the range of 2.4–2.5 μm , resulting in poor correlations between the fitting results
 373 of these three models and the measured values. The SSR changes rapidly in the strong absorption band of water, which leads
 374 to great uncertainty in the fitting results of these models. Compared with the observed SSR model, the Hapke-HSR model
 375 has the lowest accuracy, followed by the MARMIT-2 model. The HM and MARMIT-2 models are better than the Hapke-
 376 HSR model at $\text{SMC} \geq 30\%$ because these two models fully consider the variation in SSR characteristics with the variation in
 377 SMC.



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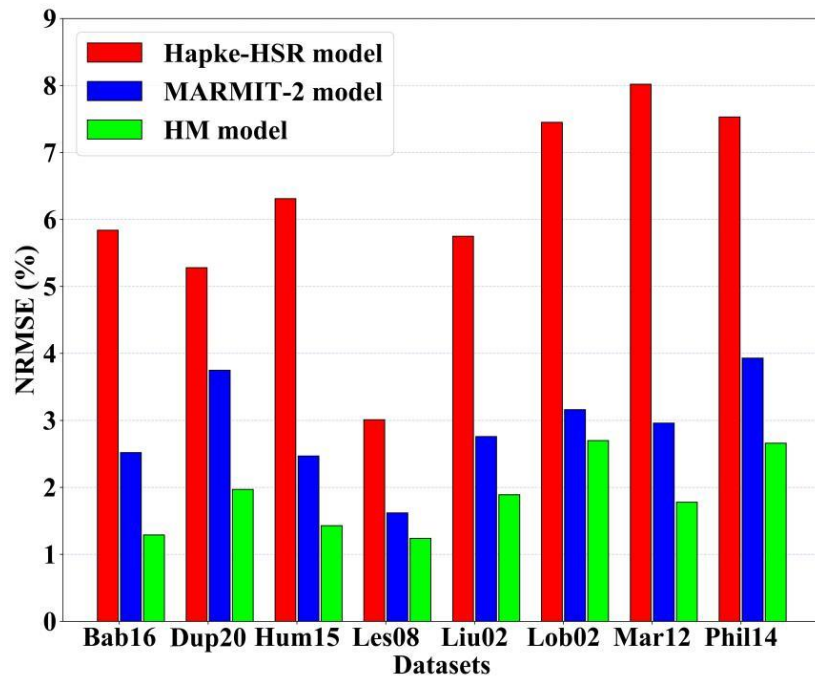


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380 **Figure 8. Evaluation of the Hapke-HSR (red), MARMIT-2 (blue) and Hapke-HSR + MARMIT-2 (HM) (lime) models**
 381 **in fitting measured soil reflectance at $SMC \geq 30\%$. The assessment indices are the R^2 (a), RMSE (b), MRE (c), and**
 382 **bias (d) values.**

383

384 Finally, we analysed the fitting performance of the Hapke-HSR, MARMIT-2, and HM models across eight soil spectral
 385 databases, as shown in Figure 9. All three models achieve high accuracy (NRMSE < 9%), with the HM model generally
 386 outperforming the individual models. The performance varies across datasets, reflecting differences in soil properties,
 387 reflectance levels, and sensitivity to soil moisture. The HM model shows clear improvements for most datasets (e.g., Bab16,
 388 Dup20, Hum15, Liu02, Mar12, and Phil14), indicating the effectiveness of coupling directional scattering and moisture-
 389 related processes. However, the improvement is less pronounced for certain datasets. For example, the MARMIT-2 model
 390 already achieves high accuracy for the Les08 dataset, resulting in limited additional improvement. For the Lob02 dataset, the
 391 relatively low reflectance leads to larger NRMSE values, which reduces the apparent gain. Overall, these results demonstrate
 392 the robustness of the proposed framework across diverse datasets, while also highlighting its dependence on dataset
 characteristics.

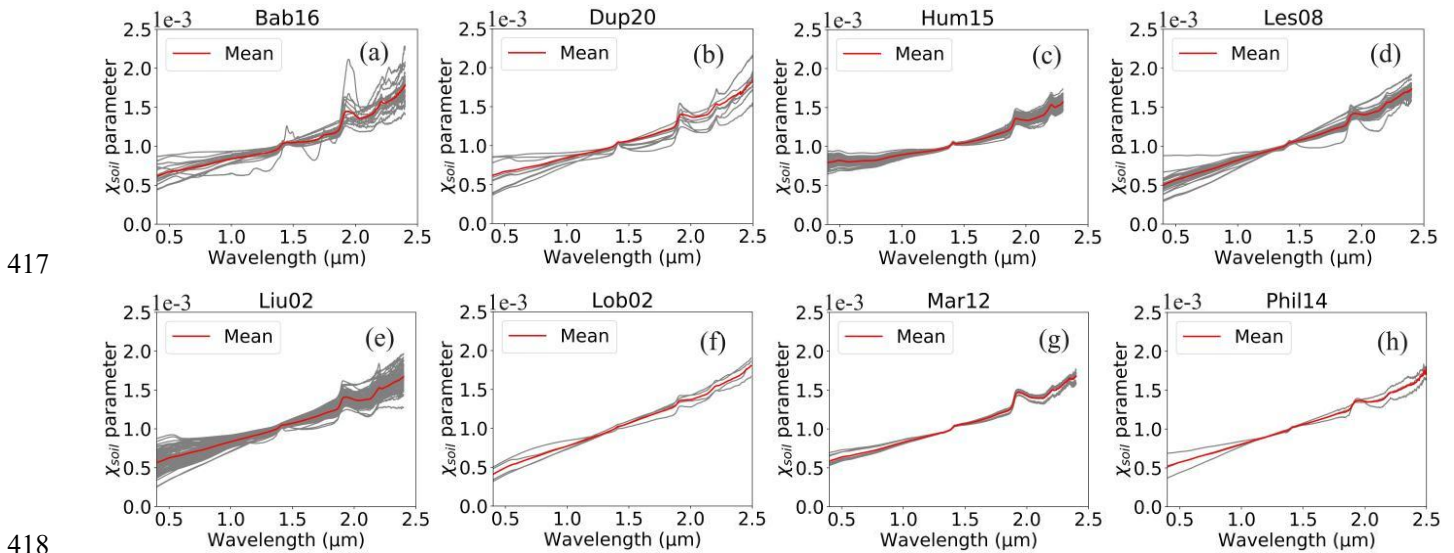


393

394 **Figure 9: The NRMSE values of the measured soil reflectance fit with the Hapke-HSR (red), MARMIT-2 (blue) and**
 395 **Hapke-HSR + MARMIT-2 (HM) (lime) models for eight different databases.**

397 5.1 Analysis of the variation in the parameter χ_{soil}

398 In this section, we analyse the imaginary component of the soil index (χ_{soil} parameter) calculated from the dry soil reflectance
 399 for eight different databases with the wavelengths shown in Figure 10. For different soil databases, the change trend of the
 400 χ_{soil} parameter is basically the same. The χ_{soil} parameter increases with wavelength, and there is an obvious peak at the two
 401 strong absorption bands of water (i.e., centred at 1.47 and 1.90 μm). However, there are still some differences between
 402 different soil databases. The values of the parameter χ_{soil} are greater in the strong absorption band of water for the Bab16 and
 403 Liu02 databases, whereas the parameter values χ_{soil} are lower for the Hum15, Lob02 and Phil14 databases in the strong
 404 absorption band of water; moreover, the influence of the SMC on these three databases is small. For the same soil database,
 405 the change in the parameter χ_{soil} is small, and only the difference between the Bab16 database results is large. In addition, the
 406 parameter χ_{soil} is obviously greater in the two water absorption bands, and the overall change range also decreases. This
 407 decrease may further affect the accuracy of fitting the measured SSR. Therefore, the method of determining the parameter
 408 χ_{soil} in each soil database should be theoretically feasible. The parameter χ_{soil} is derived based on the assumption that the
 409 spectral shape of dry soil reflectance is similar to that of the single scattering albedo. Therefore, this approach depends on the
 410 availability of dry soil reflectance, which is often difficult to obtain from field measurements or satellite observations.
 411 Providing a representative dry soil reflectance remains an important challenge for future work. In addition, the use of an
 412 averaged χ_{soil} does not explicitly account for variability in soil properties within the same soil type, such as differences in
 413 organic carbon content or texture, which may influence spectral absorption. This simplification may contribute to the
 414 observed decrease in model accuracy. In future work, the definition of χ_{soil} could be refined by incorporating soil-specific
 415 properties or by grouping spectrally similar soils, which is expected to provide a more accurate representation of absorption
 416 characteristics and further improve model performance.

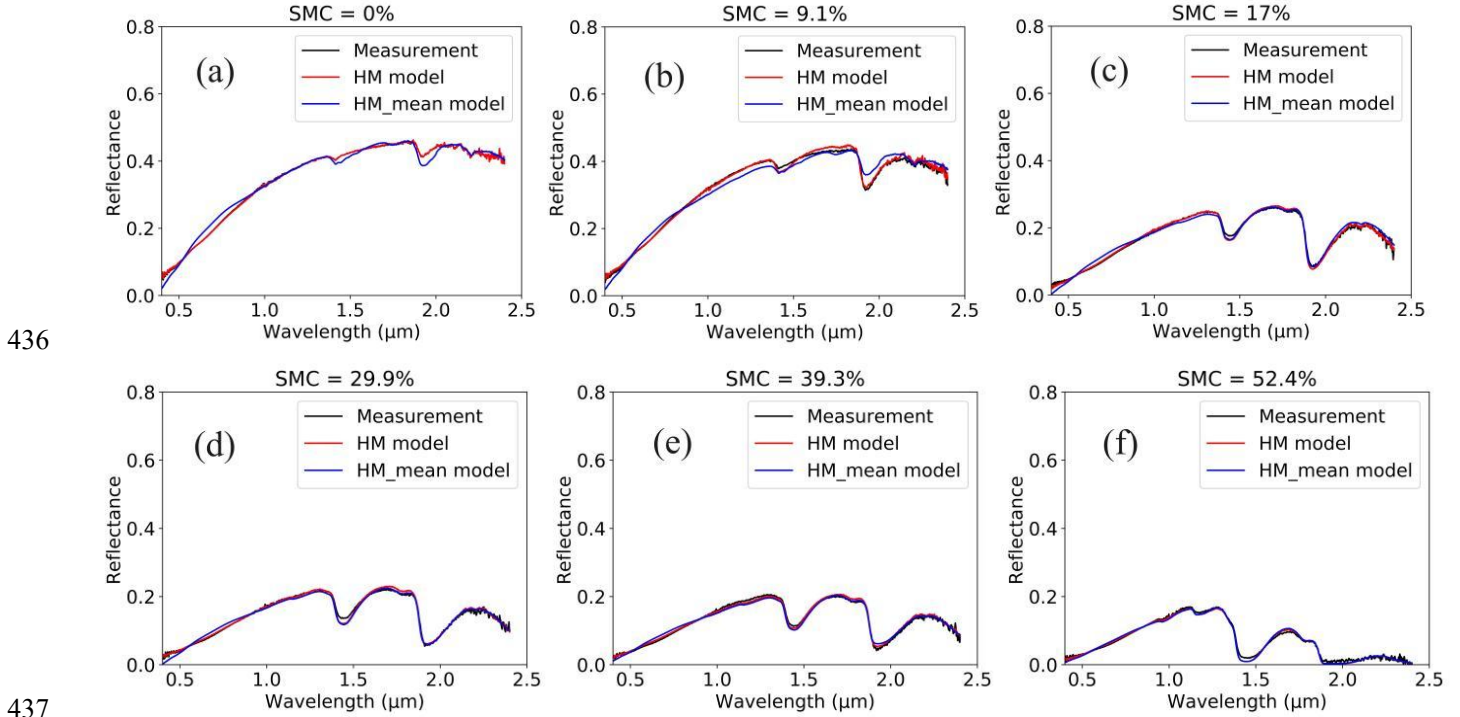


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 419 **Figure 10: Analysis of the imaginary component of the soil index (parameter χ_{soil}) for eight different databases (i.e.,**
 420 **Bab16 (a), Dup20 (b), Hum15 (c), Les08 (d), Liu02 (e), Lob02 (f), Mar12 (g), and Phil14 (h)) with wavelengths.**

421 5.2 Validating the Hapke-HSR + MARMIT-2 model using the average parameter χ_{soil}

422 We used the average parameter χ_{soil} (i.e., Figure 10(a)) to validate the HM model to characterize the SSR attributes; this
 423 model is called the HM_mean model in the following section. Figure 11 shows that the HM and HM_mean models fit the
 424 influence of the typical measured SSR value (i.e., bab16_056-051) at SMC = 0%, 9.1%, 17%, 29.9%, 39.3%, and 52.4%,

425 respectively. This set of typical data is thought to have a specular reflection effect when SMC = 52.4%. The HM and
 426 HM_mean models match well with the typical measured SSR values. However, the HM model shows greater consistency
 427 with the fitted SSR value than does the HM_mean model, especially at SMC < 10%. The HM_mean model results in
 428 significant underestimation and overestimation at SMC = 0% and 9.1%, respectively, because the average parameter χ_{soil} is
 429 obviously greater in the water absorption band (Figure 10(a)), which further affects the accuracy of the HM_mean model.
 430 The HM and HM_mean model fitting results can capture the change in SSR with increasing SMC and are highly in line with
 431 the measured SSR values at high SMCs, which may be caused by the obvious SSR broadening in the strong absorption band
 432 of water with increasing SMC. In general, the HM model, which uses the average parameter χ_{soil} , can still effectively
 433 describe the SSR characteristics, especially at high SMCs. However, the average parameter χ_{soil} leads to a significant
 434 broadening of the strong absorption band of water at low SMCs, which further leads to obvious overestimation or
 435 underestimation of the SSR fitted by the HM_mean model in the strong absorption band of water.



438 **Figure 11: The HM (red) and HM_mean (blue) models fit the measured soil reflectance (black) at SMC = 0% (a),**
 439 **9.1% (b), 17% (c), 29.9% (d), 39.3% (e), and 52.4% (f).**

440 **Table 6.** The HM and HM_mean models fit the soil reflectance variables and statistical outputs.

Models	SMC	b	M	δ	L	ϵ	R ²	RMSE	bias
HM model	0.0	2.0	0.30	0.00	0.00	0.0	1.000	0.001	0.000
	9.1	3.6	0.22	0.14	0.01	0.2	0.996	0.007	0.003
	17.0	1.8	0.32	0.05	0.01	0.6	0.993	0.006	0.000
	29.9	2.2	0.28	0.07	0.02	0.6	0.990	0.006	0.001
	39.3	1.2	0.38	0.06	0.02	0.6	0.990	0.006	0.001
	52.4	1.8	0.32	0.01	0.05	0.9	0.991	0.005	-0.002
All	--	--	--	--	--	--	0.998	0.005	0.001
HM_mean model	0.0	2.2	0.18	0.04	0.00	0.9	0.989	0.012	0.001
	9.1	2.2	0.18	0.14	0.00	1.0	0.977	0.016	-0.001
	17.0	0.6	0.34	0.13	0.01	0.6	0.980	0.009	0.002

29.9	0.6	0.34	0.09	0.02	0.6	0.981	0.008	-0.001
39.3	1.0	0.26	0.10	0.03	0.6	0.981	0.008	0.000
52.4	2.6	0.16	0.03	0.06	0.9	0.988	0.006	-0.002
All	--	--	--	--	--	0.993	0.011	0.000

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Table 6 shows that the HM and HM_mean models fit the SSR parameters and statistical outputs. The overall accuracy of the HM and HM_mean models in terms of fitting the measured SSR is high ($R^2 = 0.991\text{--}0.993$, $RMSE = 0.005\text{--}0.011$). According to the simulation results, the HM model is more accurate with respect to the measured SSR than the HM_mean model is, especially at $SMC < 10\%$, because the average parameter χ_{soil} is obviously greater in the water absorption band. The HM and HM_mean models can effectively describe SSR features, especially high SMCs ($R^2 > 0.98$ and $RMSE < 0.01$). When $SMC = 52.4\%$, the measured soil spectral data are suspected to have a specular reflection effect, and the HM model maintains a higher fitting accuracy ($R^2 = 0.991$, $RMSE = 0.005$) than does the Hapke_mean model ($R^2 = 0.988$, $RMSE = 0.006$). These results indicate that the method of assuming an average parameter χ_{soil} in each soil database should be theoretically feasible. However, all the soil types may have large differences in the parameter χ_{soil} . How to select the parameter χ_{soil} of each soil type will be particularly important in our subsequent study.

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Finally, we calculate the overall average parameter χ_{soil} to determine the dependence of the HM model on the dry SSR. First, we normalize all χ_{soil} parameters to the same order of magnitude and then average these indices in each band. Finally, we use all the SSR data to verify the accuracy of this method. Table 7 shows that the HM_mean model fit the statistical results for all the SSR data. Compared with the measured SSR, the HM_mean model has high fitting accuracy. The R^2 value of the HM_mean model is 0.988, and the RMSE is 0.014, indicating negligible bias. However, the fitting accuracy of the overall SSR data of the HM_mean model is lower than that of the MARMIT-2 and HM models (i.e., Figure 6(b)-(c)). The main reason is that there are notable discrepancies in the parameter χ_{soil} among various soil types (Figure 10). The model operation can be simplified by averaging all the χ_{soil} parameters of the soil, but this approach also results in an accuracy decline for the HM_mean model. For SMC values $\geq 30\%$, the HM_mean model results in a lower RMSE than that for SMC values $< 30\%$, which is consistent with the findings illustrated in Figure 11 and Table 6. However, the NRMSE and MRE values of the HM_mean model at $SMC \geq 30\%$ were lower than those at $SMC < 30\%$, possibly because the dry SSR is greater than that of wet soil. In conclusion, the HM_mean model demonstrates proficiency in describing SSR attributes, and the overall average parameter χ_{soil} of the soil can be used to determine the dependence of the HM model on the SSR. In this study, we based our analysis solely on the soil databases, which has limitations. In future research, we will consider soil properties or spectrally similar soils and account for more factors affecting the χ_{soil} parameter to improve the accuracy of the Hapke-HSR + MARMIT-2 model.

Table 7. The HM_mean model fit the statistical results for all the soil reflectance data.

Data	Number	R^2	RMSE	NRMSE (%)	MRE (%)	bias
SMC < 30%	2257585	0.989	0.015	1.811	5.117	0.000
SMC \geq 30%	1082887	0.980	0.013	1.858	6.988	0.000
All	3340472	0.988	0.014	1.743	5.723	0.000

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5.3 Analysing the Parameter Influence of the Hapke-HSR + MARMIT-2 model on the soil BRDF shape

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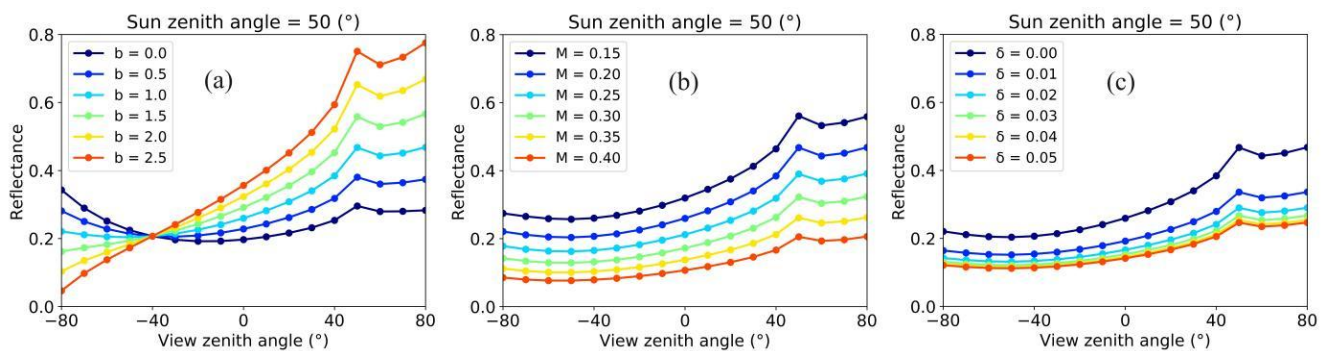
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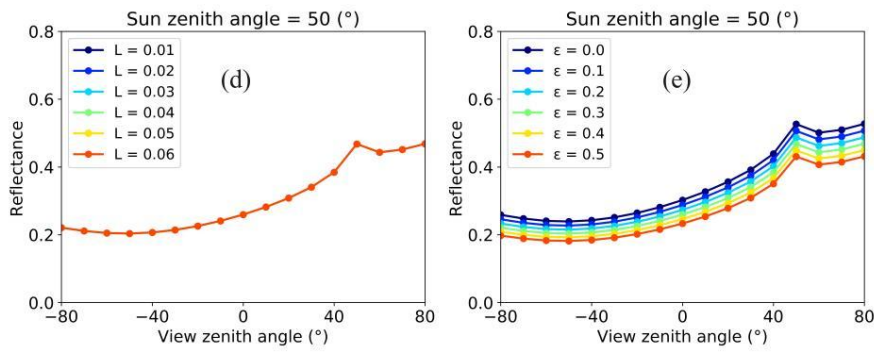
Finally, we analyse the impact of the soil parameters on the BRDF shape obtained with the Hapke-HSR + MARMIT-2 model. Considering that we previously used various BRDF data sources to examine the role of the Hapke-HSR model in modelling soil BRDF features, we analyse how the model parameters affect the shape of the BRDF curve (Ding et al., 2022). Figure 12 shows the effects of parameters b , M , δ , L and ϵ in the Hapke-HSR + MARMIT-2 (HM) model on the soil BRDF

474 shape in the principal plane (PP). With increasing parameter b , the soil reflectance gradually decreases in the forwards
 475 direction but increases in the backwards reflection direction. The parameter b has a relatively large contribution to the
 476 anisotropy characteristics of the soil. When the parameter M increases, the soil reflectance continuously decreases. The soil
 477 anisotropy is strongest when the parameter M is relatively small. With the increase in the parameter δ , the impact of the
 478 parameter δ on the soil reflectance is relatively large in the range of 0-0.01, and the anisotropy of the soil is strong; however,
 479 the influence of the parameter δ on the SSR is relatively low, and the soil anisotropy is significantly weak. As the parameter
 480 L increases, L does not impact the soil reflectance, which corresponds with the results in Figure 3(g). With increasing value
 481 of the parameter ε , the equal interval of the soil reflectance decreases since the influence of the surface coverage fraction of
 482 water is proportionally related to this factor. In summary, the variation in the parameters b and M in the Hapke-HSR model
 483 has a notable effect on the soil BRDF shape, whereas the parameters of the MARMIT-2 model have a relatively minimal
 484 effect on the soil BRDF shape. Therefore, the ability of the HM model to describe the features of the soil BRDF is basically
 485 consistent with that of the Hapke-HSR model. This occurs because the MARMIT-2 model does not include additional
 486 BRDF-related information, whereas the Hapke-HSR model includes input parameters for angle-related information. In future
 487 studies, we will comprehensively assess the ability of the HM model to represent soil BRDF features, especially in the
 488 forwards direction for wet soil.

489 Compared with existing models, the proposed Hapke-HSR + MARMIT-2 model provides a physically consistent
 490 integration of directional scattering and moisture-related processes. Semi-empirical models such as the BSM and general
 491 spectral vectors (GSV) capture spectral variability but lack explicit representation of angular effects (Ding et al., 2022; Jiang
 492 et al., 2019), which is critical for multi-angular observations and physically based parameter inversion. Moreover, soil
 493 reflectance acts as a key background component in canopy reflectance and influences vegetation parameter retrieval. By
 494 improving its spectral and directional representation under varying moisture conditions, the proposed framework can provide
 495 more reliable inputs for coupled soil-vegetation models (e.g., PROSAIL), thereby reducing uncertainties in vegetation
 496 parameter inversion. This study has certain limitations. To simplify the analysis, the effects of surface roughness and
 497 porosity are not explicitly considered (Ding et al., 2026). Despite this, the model achieves high accuracy, indicating its
 498 effectiveness in representing soil reflectance. The evaluation is conducted under a fixed observation geometry due to dataset
 499 limitations, which may constrain the characterization of angular effects. Nevertheless, the framework inherently accounts for
 500 viewing geometry through the Hapke formulation and can be extended to varying observation conditions. Future work will
 501 incorporate multi-angular datasets to further evaluate model performance.



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 504 **Figure 12: Influence of the coefficient of b (a), soil particle size and shape-dependent M (b), volume fraction δ (c),**
 505 **thickness L (d) and surface coverage fraction of water ϵ (e) parameters of the Hapke-HSR + MARMIT-2 (HM) model**
 506 **on the soil BRDF shape in the red band ($0.67 \mu\text{m}$).**

507 **6 Conclusion**

508 This study develops a unified soil radiative transfer framework by refining the improved Hapke-HSR model and dynamically
 509 coupling it with the MARMIT-2 model to improve the representation of soil reflectance under varying soil moisture
 510 conditions. The primary objective is to overcome the limitations of the Hapke-HSR model in wet soils and the dependence of
 511 MARMIT-2 model on externally prescribed dry reflectance, thereby extending the applicability of both models.

512 First, dry SSR is used to estimate the imaginary part of the soil refractive index (χ_{soil}), which alleviates the piecewise
 513 fitting limitation of the Hapke-HSR model by establishing a continuous statistical relationship between single scattering
 514 albedo and wavelength. The improved Hapke-HSR is then coupled with MARMIT-2 to integrate particle scattering and
 515 moisture-dependent absorption processes within a physically consistent framework. The proposed Hapke-HSR + MARMIT-
 516 2 (HM) model is evaluated using multiple independent soil spectral databases. The results show that all three models
 517 reproduce measured SSR with reasonable accuracy, whereas the coupled HM model achieves consistently higher
 518 performance ($R^2 = 0.993$, $\text{RMSE} = 0.007$) than MARMIT-2 ($R^2 = 0.983$, $\text{RMSE} = 0.012$) and Hapke-HSR ($R^2 = 0.909$,
 519 $\text{RMSE} = 0.028$), with particularly pronounced improvements at high soil moisture levels ($\text{SMC} \geq 30\%$). This study does not
 520 aim to replace the MARMIT-2 model, which already provides an effective description of moisture effects, but rather to
 521 improve the overall physical consistency of soil reflectance modeling through the integration of complementary mechanisms.
 522 The coupled framework provides a robust basis for future developments in soil parameter inversion, particularly for soil
 523 moisture, and for improved representation of soil background effects in land-surface radiative transfer modeling.

524 In summary, this work addresses two key modeling challenges: (1) improving the hyperspectral consistency of the
 525 Hapke-HSR using dry soil reflectance, and (2) establishing a unified coupling strategy that jointly represents spectral
 526 behavior and moisture-dependent effects. The proposed framework contributes to the theoretical and methodological
 527 foundation of soil radiative transfer modeling and supports future advances in optical remote sensing of land-surface
 528 parameters.

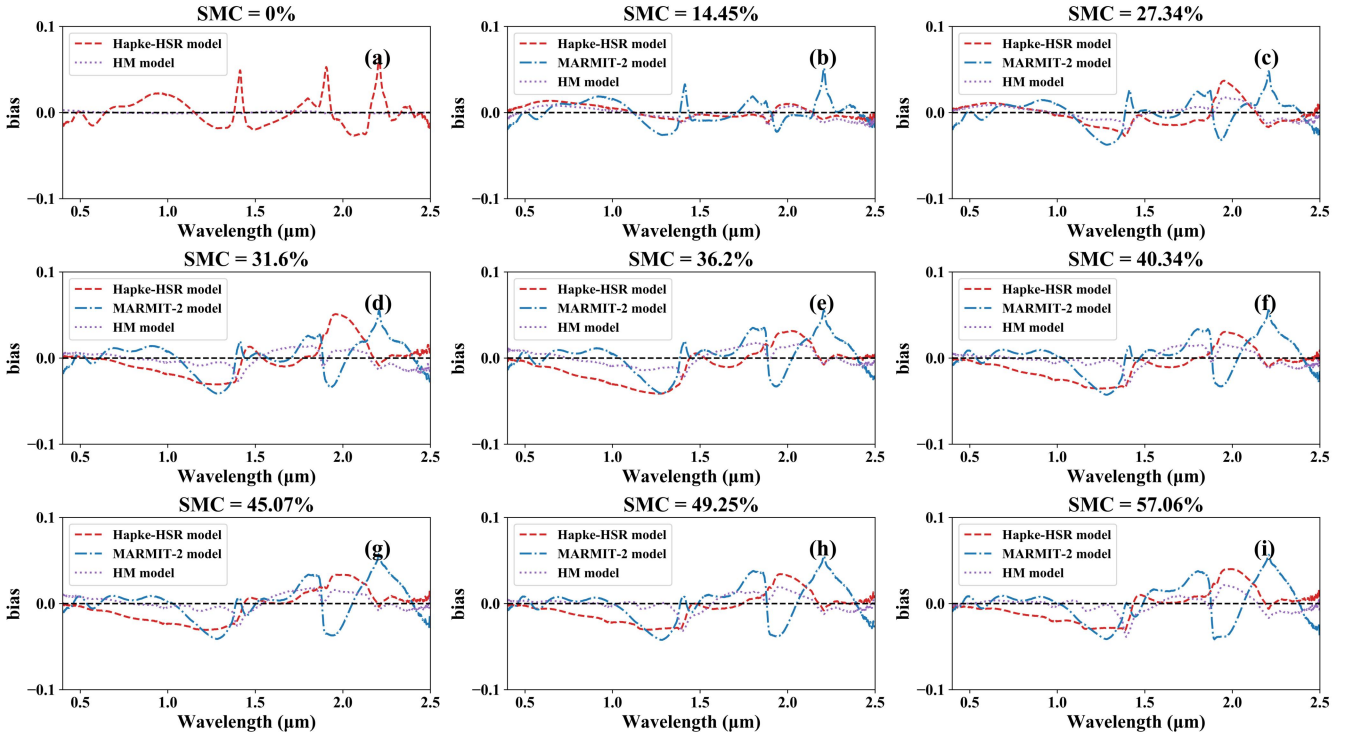
529 *Code and data availability.* The Hapke-HSR + MARMIT-2 model code and example datasets used in this study are archived
 530 on Zenodo (<https://doi.org/10.5281/zenodo.18366791>; Ding, 2025). The original soil databases are derived from the
 531 MARMIT framework (<https://pss-gitlab.math.univ-paris-diderot.fr/marmit/marmit>).

532 *Author contributions.* Conceptualization, Anxin Ding, Shunlin Liang; methodology, Anxin Ding, Han Ma, Shunlin Liang,
 533 Ziti Jiao and Alexander A. Kokhanovsky; formal analysis, Anxin Ding, Han Ma and Rui Xie; data curation, Han Ma, Ziti
 534 Jiao and Rui Xie; software, Han Ma; investigation, Ziti Jiao; theoretical support, Alexander A.Kokhanovsky; data processing
 535 and validation, Hanyu Shi; supervision, Shunlin Liang; writing—original draft, Anxin Ding; writing—review and editing,
 536 Han Ma, Shunlin Liang, Ziti Jiao, Hanyu Shi, Alexander A.Kokhanovsky and Rui Xie.

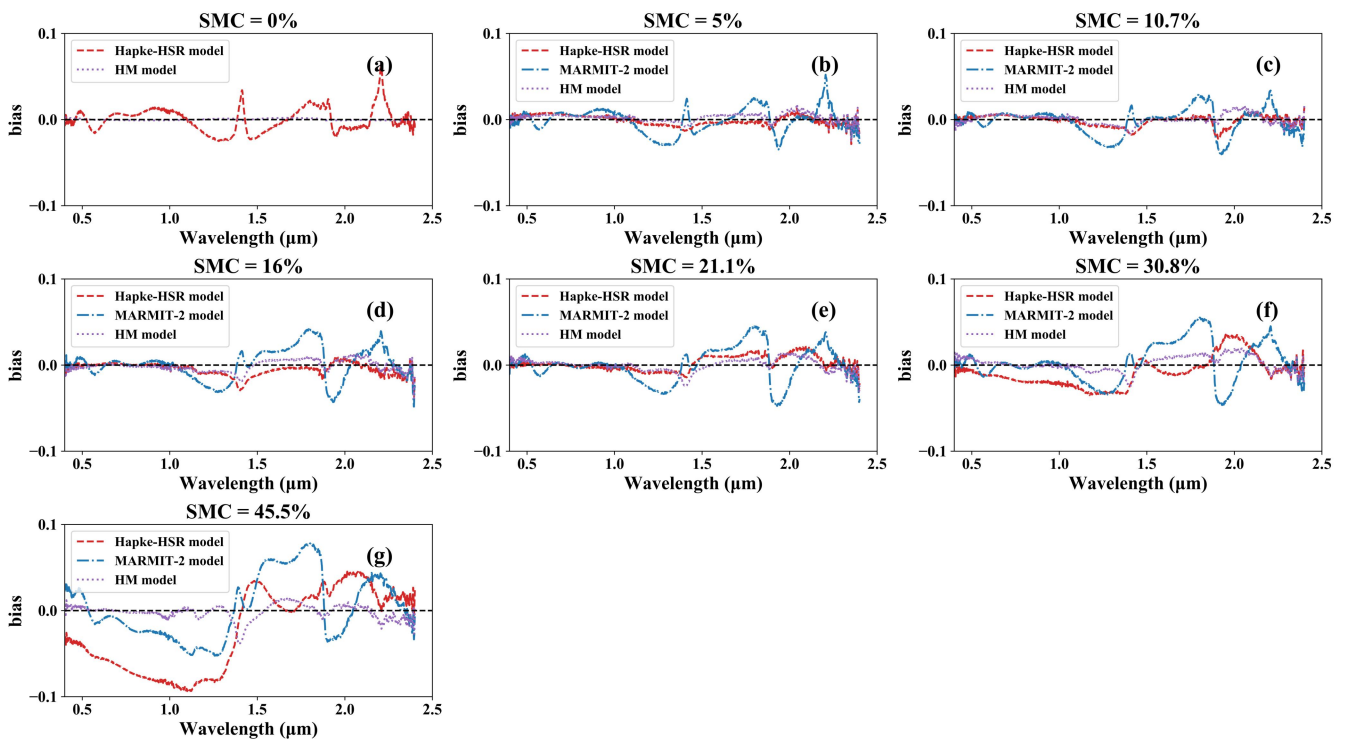
537 *Competing interests.* The authors have no conflict of interest.

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542 **Appendix**



543
544 **Figure A1: The bias (i.e., simulated reflectance of these models - measured reflectance) between the simulated**
545 **spectral reflectance of the Hapke-HSR (red), MARMIT-2 (blue) and Hapke-HSR + MARMIT-2 (HM) (lime) models**
546 **and the fitted soil reflectance at SMC = 0% (a), 14.45% (b), 27.34% (c), 31.6% (d), 36.2% (e), 40.34% (f), 45.07% (g),**
547 **49.25% (h), and 57.06% (i).**



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549 **Figure A2. The bias (i.e., simulated reflectance of these models - measured reflectance) between the simulated**
 550 **reflectance of the Hapke-HSR (red), MARMIT-2 (blue) and Hapke-HSR + MARMIT-2 (HM) (lime) models and the**
 551 **fitted soil reflectance at SMC = 0% (a), 5% (b), 10.7% (c), 16% (d), 21.1% (e), 30.8% (f), and 45.5% (g).**

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