

Dear Editors,

We sincerely thank you and the reviewers for the constructive and valuable comments on our manuscript. These suggestions have greatly helped us improve the clarity and overall quality of the paper.

We have carefully addressed all comments and revised the manuscript accordingly. The main improvements are summarized as follows:

(1) The novelty and scientific contribution of the proposed Hapke-HSR + MARMIT-2 framework have been clarified and strengthened in the Introduction. (2) The model coupling strategy and parameter estimation procedures have been clearly described, including the roles of key parameters and the integration workflow. (3) Additional analyses and discussions have been included to improve the interpretation of model performance, including the introduction of bias analysis to better compare different models. (4) The presentation of the manuscript has been improved, including clearer parameter descriptions, consistent terminology, and revised figures using color vision deficiency–friendly schemes.

A detailed, point-by-point response to all reviewers' comments is provided in the accompanying response document. All revisions have been clearly indicated in the manuscript.

We hope that the revised manuscript meets the requirements for publication and sincerely look forward to your consideration.

Thank you for your time and effort.

Sincerely,

Anxin Ding & co-authors

## Reviewer: 1

The manuscript proposes a coupled soil radiative transfer model by integrating the improved Hapke-HSR model with the MARMIT-2 model to simulate soil spectral reflectance under varying moisture conditions. The topic is relevant, and the study demonstrates promising results across multiple datasets. The manuscript shows clear novelty, with a well-organized structure and generally good writing quality. I recommend publication after major revisions. I encourage the authors to address the following comments.

We sincerely thank the reviewer for the positive and encouraging assessment of our manuscript. We appreciate the recognition of the relevance, novelty, and overall quality of the work, as well as the constructive suggestions for improvement.

In response, we have thoroughly revised the manuscript to address all comments. The main revisions include clarifying the novelty and model contribution, improving the description of the coupling strategy and parameter estimation, strengthening the discussion of model applicability and limitations, and expanding comparisons with existing approaches.

We believe these revisions have improved the clarity of the manuscript. All comments are addressed in detail below, and we hope the revised version meets the reviewer's expectations.

1. Although the proposed coupled model shows promising performance, the novelty of the study could be further clarified. It is recommended to explicitly highlight the key innovations of the proposed approach in comparison with existing models (e.g., Hapke or MARMIT-2) in the Introduction, to strengthen the overall contribution of the manuscript.

Response 1: We sincerely thank the reviewer for this insightful and constructive suggestion. We fully agree that the novelty of the proposed framework should be more explicitly highlighted, particularly in comparison with existing soil radiative transfer models.

In the revised manuscript, we have substantially strengthened the Introduction to clearly articulate the key innovations and scientific contributions of this study. Specifically, the novelty of the proposed Hapke-HSR + MARMIT-2 model framework can be summarized as follows:

**(1) A physically consistent unified modeling framework:**

Unlike existing models that separately describe either dry soil scattering (e.g., Hapke model) or moisture-induced absorption (e.g., MARMIT-2 model), the proposed approach establishes a unified formulation that explicitly integrates particle scattering and moisture-dependent absorption/refraction processes within a single radiative transfer framework.

**(2) Elimination of the dependency on externally prescribed dry soil reflectance:**

The MARMIT-2 model requires prior knowledge of dry soil reflectance, which is often unavailable in practical applications. In this study, the improved Hapke-HSR model dynamically generates dry reflectance under varying observation geometries, thereby removing this limitation and improving model applicability.

**(3) Joint spectral–directional simulation capability under varying moisture conditions:**

By inheriting the angular formulation of the Hapke model and incorporating moisture effects via MARMIT-2 model, the proposed model enables consistent simulation of soil reflectance across both spectral and directional domains, which is not achievable with either model alone.

**(4) Improved performance under high soil moisture conditions:**

The coupled framework significantly enhances model accuracy and stability at high soil moisture levels ( $SMC \geq 30\%$ ), where existing models typically exhibit systematic biases, particularly in strong water absorption regions.

We believe that these revisions have clarified the novelty and strengthened the overall contribution of the manuscript.

Please see p. 3, lines 87-94, in the revised manuscript.

2. Several models are listed in Table 1; however, it is not entirely clear why the Hapke-HSR and MARMIT-2 models were selected for coupling instead of other

possible combinations (e.g., BSM with Hapke-HSR). A brief justification for this choice would improve the clarity of the study design.

Response 2: We thank the reviewer for this important comment. We agree that the rationale for selecting the Hapke-HSR and MARMIT-2 models should be clarified.

In the revised manuscript, we have added a concise justification based on the complementary physical characteristics of the two models. The Hapke-HSR model provides a physically based description of directional scattering and enables simulation of dry soil reflectance, whereas the MARMIT-2 model explicitly represents moisture-related absorption and refraction processes but does not account for angular effects and requires prescribed dry reflectance.

Compared with semi-empirical models such as BSM, which lack explicit directional representation, the Hapke-HSR model offers a stronger physical basis for angular reflectance, while MARMIT-2 provides a more physically interpretable treatment of moisture effects. Their coupling therefore enables a physically consistent integration of directional and moisture-related processes.

Please see p. 2-3, lines 63-86, in the revised manuscript.

3. The manuscript should provide clearer references or methodological explanations regarding how the optimal values of parameters such as  $B_0$ ,  $h$ ,  $b'$ ,  $c$ , and  $c'$  are determined. In addition, since both  $b$  and  $c$  influence the BRDF shape in the Hapke model, it would be helpful to clarify why the discussion primarily focuses on parameter  $b$ .

Response 3: We thank the reviewer for this insightful comment. We agree that the determination of model parameters and their physical interpretation should be clarified.

In the revised manuscript, we have explicitly stated that the parameters  $B_0$ ,  $h$ ,  $b'$ ,  $c$ , and  $c'$  are adopted from previous studies and represent commonly used or empirically optimized values for soil surfaces (e.g., Hapke, 2012; Ding et al., 2022). These parameter values have been shown to provide stable and physically reasonable representations of soil reflectance across a wide range of conditions. To improve clarity, we have added appropriate references and briefly described their physical

meaning and typical value ranges. Regarding the roles of parameters  $b$  and  $c$ , we have added further clarification in Section 4.1. Although both parameters influence the BRDF shape through the phase function, their sensitivities and physical effects differ. The parameter  $b$  primarily controls the width and anisotropy of the scattering lobe, and exhibits a stronger and more stable influence on reflectance across viewing geometries. In contrast, the parameter  $c$  mainly affects the asymmetry between forward and backward scattering and generally shows lower sensitivity under the observational configurations considered in this study.

Therefore, this study focuses primarily on parameter  $b$  in the discussion, as it plays a dominant role in shaping the directional reflectance characteristics under the given conditions. We have added this explanation to the manuscript to improve the transparency and physical interpretation of the model parameters.

Please see p. 3, lines 99-105, in the revised manuscript.

#### References

Ding, A., Ma, H., Liang, S., and He, T.: Extension of the Hapke model to the spectral domain to characterize soil physical properties, *Remote Sensing of Environment*, 269, 112843, 2022.

Hapke, B.: Bidirectional reflectance spectroscopy 7: the single particle phase function hockey stick relation, *Icarus*, 221, 1079–1083, 2012.

4. In the current manuscript, the distinction between input parameters and retrieved parameters is not always clear, particularly in some tables. It is recommended to clearly differentiate these parameter types and ensure consistent terminology throughout the manuscript.

Response 4: We thank the reviewer for this helpful suggestion. In the revised manuscript, we have clarified the distinction between input and retrieved parameters throughout the text. Specifically, input parameters refer to variables prescribed in the forward simulations, whereas retrieved parameters are estimated through inversion.

To improve clarity, we have revised the relevant tables and descriptions to explicitly distinguish these parameter types. In addition, Table 1 provides a summary of the input parameters for each soil model, ensuring a consistent and transparent

presentation. We have also ensured consistent terminology across the manuscript. These revisions improve the clarity and reproducibility of the modeling framework.

Table 1. Input parameters of each soil model.

Models	Input parameters
Hapke-HSR (Ding et al., 2022)	SZA, VZA, RAA, $b$ , $A_0$ , $A_1$ , $A_2$ , $A_3$ , and $f$
Improved Hapke-HSR (dry soil)	SZA, VZA, RAA, $b$ , $M$ , and $\chi_{\text{soil}}$
MARMIT-2 (Dupiau et al., 2022)	$\delta$ , $L$ and $\varepsilon$
Hapke-HSR model and MARMIT-2	SZA, VZA, RAA, $b$ , $M$ , $\chi_{\text{soil}}$ , $\delta$ , $L$ and $\varepsilon$

The meanings of the parameters

- (1) Solar zenith angle (SZA)
- (2) View zenith angle (VZA)
- (3) Relative azimuth angle (RAA)
- (4) Coefficient of the scattering phase function ( $b$ )
- (5) Soil spectral fitting parameters ( $A_0$ ,  $A_1$ ,  $A_2$  and  $A_3$ )
- (6) Equivalent water thickness ( $f$ )
- (7) Soil particle size and shape-dependent parameter ( $M$ )
- (8) The imaginary component of the soil complex refractive index ( $\chi_{\text{soil}}$ )
- (9) Volume fraction of the soil particles ( $\delta$ )
- (10) Thickness of the water layer ( $L$ )
- (11) Surface coverage fraction of water ( $\varepsilon$ )

Please see p. 8, lines 201-208, in the revised manuscript.

#### References

Ding, A., Ma, H., Liang, S., and He, T.: Extension of the Hapke model to the spectral domain to characterize soil physical properties, *Remote Sensing of Environment*, 269, 112843, 2022.

Hapke, B.: Bidirectional reflectance spectroscopy 7: the single particle phase function hockey stick relation, *Icarus*, 221, 1079–1083, 2012.

5. The study is mainly conducted under a fixed observation geometry. It would be beneficial to further discuss how this assumption may influence the model results, as

well as the applicability of the model under different viewing and illumination conditions.

Response 5: We thank the reviewer for this valuable comment. We agree that the assumption of a fixed observation geometry should be further discussed in terms of its influence on the results and the applicability of the proposed model.

In the revised manuscript, we have added a clarification regarding this aspect. In this study, a fixed observation geometry is adopted primarily because the available datasets do not provide multi-angular measurements. This simplification allows us to focus on evaluating the spectral performance of the model under controlled conditions. We note that the proposed Hapke-HSR + MARMIT-2 framework is not restricted to a fixed geometry. The Hapke-HSR component explicitly accounts for viewing and illumination angles, and therefore the coupled model can be directly extended to simulate soil reflectance under varying geometries. However, the use of fixed geometry in this study may limit the characterization of angular effects and the full assessment of BRDF behavior.

We have added a discussion to clarify that future work will incorporate multi-angular datasets to further evaluate and validate the model under varying observation conditions. This extension will allow a more comprehensive assessment of the model's applicability in real remote sensing scenarios. We appreciate the reviewer's suggestion, which has helped improve the discussion of model applicability and limitations.

Please see p. 8, lines 201-208, in the revised manuscript.

6. Although the validation results are generally convincing, the study could be further strengthened by including comparisons with additional models (e.g., GSV or similar approaches), or by providing more discussion on the differences between the proposed model and existing methods.

Response 6: We thank the reviewer for this constructive suggestion. We agree that additional comparisons and discussion with existing models would further strengthen the manuscript.

In the revised version, we have expanded the discussion to provide a clearer comparison between the proposed Hapke-HSR + MARMIT-2 model framework and other representative soil reflectance models. Specifically, we discuss the differences in model structure, physical assumptions, and applicability, with particular emphasis on models such as GSV and BSM models. We note that models such as GSV and BSM are primarily semi-empirical or hybrid approaches, which are effective in describing spectral reflectance variability but do not explicitly represent directional scattering processes. In contrast, the proposed framework is based on a physically consistent coupling of directional scattering (Hapke-HSR model) and moisture-related absorption processes (MARMIT-2 model), enabling joint simulation across both spectral and angular domains. In addition, we clarify that the primary objective of this study is to develop and validate a unified physically based framework. Therefore, the comparison focuses on models that explicitly represent moisture processes (e.g., MARMIT-2 model), which provides a more direct and meaningful benchmark for evaluating model improvements.

We have incorporated this discussion into the manuscript to improve the positioning of the proposed model relative to existing approaches. We appreciate the reviewer's suggestion, which has helped enhance the completeness and clarity of the study.

Please see p. 20, lines 489-492, in the revised manuscript.

7. The coupling strategy between the improved Hapke-HSR model and the MARMIT-2 model is not entirely clear. It would be useful to clarify whether the parameters ( $b$ ,  $M$ ,  $\delta$ ,  $L$ ,  $\epsilon$ ) are retrieved sequentially or simultaneously, and to provide a clearer description of the integration workflow.

Response 7: We thank the reviewer for this insightful comment. We agree that the coupling strategy and parameter estimation procedure should be described more clearly.

The coupling between the improved Hapke-HSR and MARMIT-2 models is implemented in a sequential manner at the model level. Specifically, the improved Hapke-HSR model is first used to simulate dry soil reflectance under different observation geometries, based on scattering-related parameters (e.g.,  $b$ ,  $M$ ). The

resulting dry soil reflectance is then used as input to the MARMIT-2 model, which simulates wet soil reflectance by accounting for moisture-related parameters (e.g.,  $\delta$ ,  $L$ ,  $\epsilon$ ). This structure follows the physical process in which scattering from dry soil is subsequently modified by moisture-related absorption and refraction effects. Regarding parameter estimation, two strategies can be applied depending on the application scenario. In this study, the parameters can be retrieved simultaneously using the coupled Hapke-HSR + MARMIT-2 model, allowing all five parameters ( $b$ ,  $M$ ,  $\delta$ ,  $L$ ,  $\epsilon$ ) to be jointly optimized. Alternatively, a sequential inversion strategy can also be adopted, in which the dry soil parameters ( $b$ ,  $M$ ) are first estimated using the Hapke-HSR model, followed by the inversion of moisture-related parameters ( $\delta$ ,  $L$ ,  $\epsilon$ ) using the MARMIT-2 model.

To improve clarity, we have revised the manuscript to explicitly describe this integration workflow and to clarify the relationship between the sequential coupling of the models and the flexible parameter inversion strategies (Figure 1). These revisions make the modeling framework more transparent and easier to interpret. We appreciate the reviewer’s suggestion, which has helped improve the clarity of the methodological description.

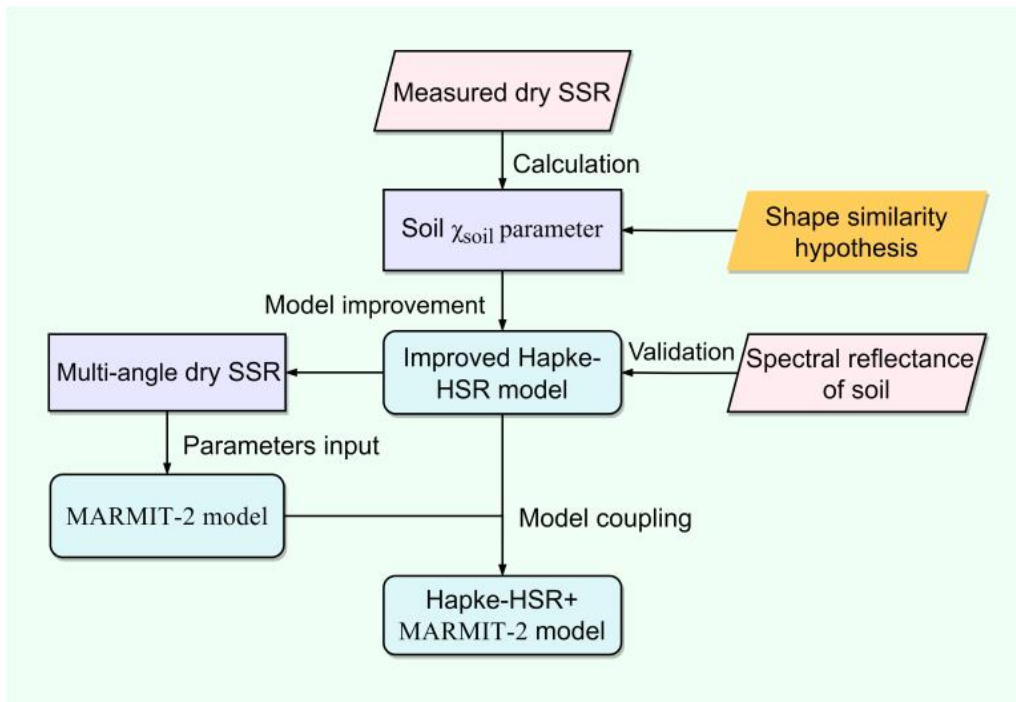


Figure 1: The workflow of the improved Hapke-HSR model and the coupled MARMIT-2 model.

Please see p. 6, lines 178-181, in the revised manuscript.

8. In the Discussion, the manuscript attributes the decrease in model accuracy (when using averaged  $\chi_{\text{soil}}$ ) mainly to differences in soil types. However, soil properties (e.g., organic carbon content, texture) can vary significantly even within the same soil type and may strongly affect spectral absorption. It is suggested to consider incorporating soil property variability or grouping spectrally similar soils when defining  $\chi_{\text{soil}}$ , which may further improve model performance.

Response 8: We thank the reviewer for this valuable suggestion. We agree that soil properties such as organic carbon content and texture can vary significantly within the same soil type and may influence spectral absorption characteristics.

In the current study, the use of an averaged  $\chi_{\text{soil}}$  is intended as a simplified representation to evaluate the overall performance of the coupled model under varying moisture conditions. While this approach captures the general behavior across different soils, it does not explicitly account for within-type variability, which may contribute to the observed decrease in accuracy. We agree that incorporating soil property variability or grouping spectrally similar soils represents a promising direction for improving model performance. In the revised manuscript (Section 5.X), we have added a discussion to clarify this limitation and to highlight potential improvements. In particular, defining  $\chi_{\text{soil}}$  based on soil-specific properties or spectral similarity classes could provide a more accurate representation of absorption behavior. However, implementing such a strategy requires detailed soil property information or clustering analyses, which are beyond the scope of the present study. Our primary objective is to establish a unified physical framework, and the use of averaged  $\chi_{\text{soil}}$  serves as a first-order approximation. We have revised the manuscript to explicitly acknowledge this limitation and to outline future work aimed at incorporating soil-specific or spectrally grouped parameterization strategies.

We appreciate the reviewer's suggestion, which has helped improve the discussion of model applicability and future development.

Please see p. 17, lines 408-416, in the revised manuscript.

## **Reviewer: 2**

This study introduces a soil radiative transfer framework designed to reproduce soil reflectance behavior under varying SMC conditions. The subject is of clear importance to the remote sensing field. The manuscript is logically organized, and the evaluation using diverse datasets indicates that the proposed approach has strong potential. That said, several aspects require further clarification before the work can be fully appreciated. With these revisions, the manuscript is expected to reach the standard required for publication.

We sincerely thank the reviewer for the positive and constructive assessment of our manuscript. We appreciate the recognition of the importance of the topic, the logical organization, and the strong potential of the proposed framework.

We also thank the reviewer for the valuable suggestions regarding aspects that require further clarification. In response, we have carefully revised the manuscript to address all comments and to improve the clarity of the methodology, parameter description, and model applicability. We believe that these revisions have strengthened the manuscript and improved its overall quality. All comments are addressed in detail below, and we hope the revised version meets the reviewer's expectations.

We appreciate the reviewer's suggestion, which has helped improve the discussion of model applicability and future development.

Comments to the Authors

1. Lines, 90-94, and 211-216. The manuscript emphasizes parameter  $b$  within the Hapke-HSR framework, yet the procedure used to determine its optimal value is not sufficiently described. Moreover, although parameter  $c$  also contributes to shaping the phase function, its role is not adequately addressed. A clearer explanation of how these parameters are treated would improve the transparency of the modeling approach.

Response 1: We thank the reviewer for this helpful comment. We agree that the treatment of the phase function parameters should be clarified.

In the revised manuscript, we have explicitly described the determination of parameter  $b$ . Specifically,  $b$  is estimated through model fitting by minimizing the difference between simulated and observed reflectance under given observation geometries. This parameter plays a dominant role in controlling the anisotropy of the scattering pattern and shows high sensitivity in the inversion. Regarding parameter  $c$ , we have added clarification that, although it also influences the BRDF shape through the phase function, its sensitivity is comparatively weaker under the observational configurations considered in this study. Therefore,  $c$  is assigned a fixed value following previous studies (e.g., Hapke, 2012; Ding et al., 2022), while the analysis focuses primarily on parameter  $b$ . These explanations have been incorporated into the manuscript to improve the transparency and physical interpretation of the modeling approach. We appreciate the reviewer's suggestion, which has helped clarify the treatment of model parameters.

Please see p. 8, lines 213-218, in the revised manuscript.

#### References

Ding, A., Ma, H., Liang, S., and He, T.: Extension of the Hapke model to the spectral domain to characterize soil physical properties, *Remote Sensing of Environment*, 269, 112843, 2022.

Hapke, B.: Bidirectional reflectance spectroscopy 7: the single particle phase function hockey stick relation, *Icarus*, 221, 1079-1083, 2012.

2. Lines, 117-118. The parameters  $C_3$  and  $C_4$  introduced in Eq. (17) require further clarification. Providing additional details on their physical interpretation and derivation process would help readers better understand their function within the model.

Response 2: Thank you very much for this valuable comment. In the revised manuscript, we have clarified the definition and derivation of the parameters ( $C_3$ ) and ( $C_4$ ), as well as their physical role in the model.

In Eqs.(1)–(5), the relationship between dry soil spectral reflectance (SSR) and single-scattering albedo (SSA) is established under a simplified formulation, where the effects of multiple scattering are not explicitly considered. In this context, the parameters ( $C_1$ ) and ( $C_2$ ) are introduced as shape adjustment parameters of the dry SSR curve and are set to ( $C_1 = 1$ ) and ( $C_2 = 1$ ) to simplify the calculations.

However, neglecting multiple scattering leads to discrepancies between the simulated dry soil reflectance and the measured reflectance. To account for this effect, we introduce the parameters ( $C_3$ ) and ( $C_4$ ), which are determined through a linear regression between the dry soil reflectance simulated by the Hapke-HSR model and the corresponding measured dry soil reflectance, as described in Eq. (6). These parameters serve as spectral shape adjustment coefficients that compensate for the bias caused by the simplified treatment of multiple scattering.

Therefore, while ( $C_1$ ) and ( $C_2$ ) are fixed constants, ( $C_3$ ) and ( $C_4$ ) are empirically estimated parameters that improve the agreement between model simulations and observations. This procedure has been clarified in the revised manuscript to improve the transparency and reproducibility of the model.

$$R_d(\theta_s, \theta_v, \varphi, \lambda) = C_1 \times \omega \quad (1)$$

$$C_1 = \frac{1}{4(\cos \theta_s + \cos \theta_v)} \{[P(g, g')(1 + B(g))] - 1\} \quad (2)$$

$$\omega = 1 - \frac{C_2 \times \chi_{soil}}{\lambda} \quad (3)$$

$$C_2 = 4\pi M \quad (4)$$

$$\chi_{soil} = \frac{\lambda}{C_2} \times \left(1 - \frac{R_d(\theta_s, \theta_v, \varphi, \lambda)}{C_1}\right) \quad (5)$$

where  $C_1$  and  $C_2$  are the shape adjustment parameters of the dry SSR curve. We use  $C_1 = 1$  and  $C_2 = 1$  as the initial values to simplify the calculation, after which we further calculate the shape adjustment parameters.

The relationship between the dry SSR simulated with the Hapke-HSR model and the measured dry SSR can be expressed via the following formula:

$$R'_d(\theta_s, \theta_v, \varphi, \lambda) = C_3 \times R_d(\theta_s, \theta_v, \varphi, \lambda) + C_4 \quad (6)$$

where  $C_3$  and  $C_4$  represent the spectral shape adjustment parameters of the dry SSR. Note that  $R_d(\theta_s, \theta_v, \varphi, \lambda)$  is calculated via the Hapke-HSR model on the basis of the shape similarity assumption.

Please see p. 5, lines 129-132, in the revised manuscript.

3. In the Table 3. To enhance clarity, it would be beneficial to summarize the parameter settings of the Hapke-HSR model, the MARMIT-2 model, and their coupled formulation. In particular, explicitly categorizing parameters into fixed, prescribed inputs, and those subject to retrieval would improve consistency and readability throughout the manuscript.

Response 3: We thank the reviewer for this constructive suggestion. In the revised manuscript, we have improved the presentation of model parameters to enhance clarity and consistency.

Specifically, we have added a summary table (Table 1) that provides a unified overview of the input parameters for the Hapke-HSR model, the improved Hapke-HSR model (dry soil), the MARMIT-2 model, and the coupled Hapke-HSR + MARMIT-2 framework. In addition, we have explicitly clarified the roles of different parameters in the text, distinguishing between prescribed input parameters and those subject to retrieval.

These revisions improve the consistency and readability of the parameter settings across different model components. We appreciate the reviewer's suggestion, which has helped improve the overall presentation of the manuscript.

Table 1. Input parameters of each soil model.

Models	Input parameters
Hapke-HSR (Ding et al., 2022)	SZA, VZA, RAA, $b$ , $A_0$ , $A_1$ , $A_2$ , $A_3$ , and $f$
Improved Hapke-HSR (dry soil)	SZA, VZA, RAA, $b$ , $M$ , and $\chi_{\text{soil}}$
MARMIT-2 (Dupiau et al., 2022)	$\delta$ , $L$ and $\varepsilon$
Hapke-HSR model and MARMIT-2	SZA, VZA, RAA, $b$ , $M$ , $\chi_{\text{soil}}$ , $\delta$ , $L$ and $\varepsilon$

Please see p. 11-12, lines 280-282, in the revised manuscript.

4. In the Figure 4, it is not easy to see the difference among different models. Could you explain the difference? The authors only showed the results for these two samples in Table 4.

Response 4: We thank the reviewer for this helpful comment. We agree that the differences among the models are not sufficiently clear in Figure 4 in the revised manuscript.

To address this issue, we have added a figure in the revised manuscript (Appendix Figure A) showing the bias (i.e., simulated reflectance minus measured reflectance) for the Hapke-HSR, MARMIT-2, and coupled Hapke-HSR + MARMIT-2 (HM) models across a wide range of soil moisture conditions. This figure provides a more direct and quantitative comparison of model performance. The results show that the HM model consistently reduces the bias across most wavelengths and soil moisture conditions compared to the individual models. In particular, the improvement is more evident under moderate to high soil moisture levels.

We have added a corresponding description in the revised manuscript (Appendix A) to clarify these differences and improve the interpretation of model performance. We appreciate the reviewer's suggestion, which has helped improve the clarity of the comparison.

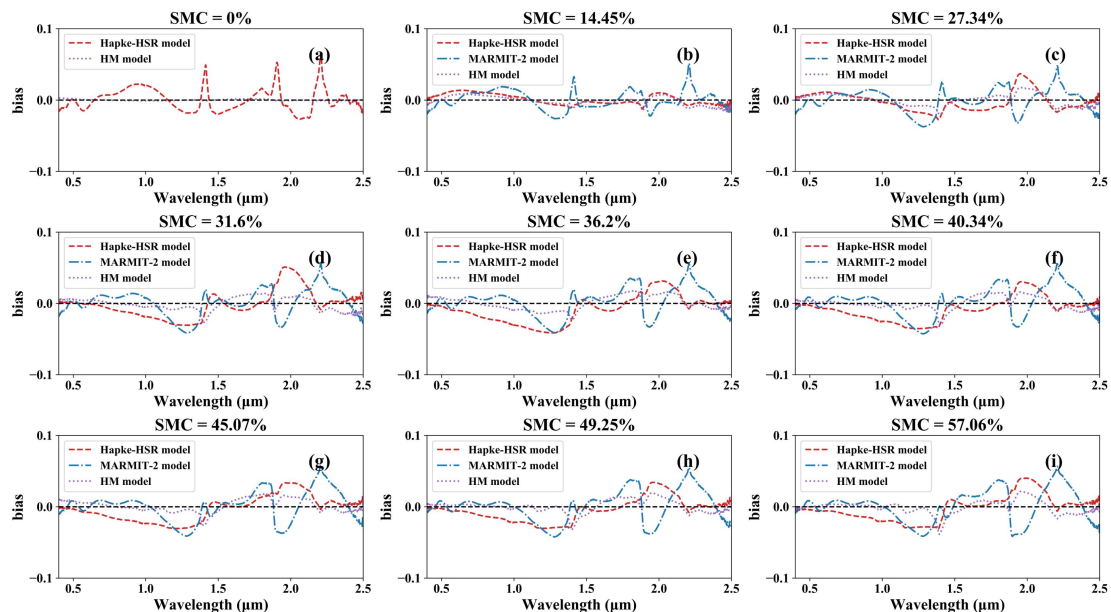


Figure A1: The bias (i.e., simulated reflectance of these models - measured reflectance) between the simulated spectral reflectance of the Hapke-HSR (red), MARMIT-2 (blue)

and Hapke-HSR + MARMIT-2 (HM) (lime) models and the fitted soil reflectance at SMC = 0% (a), 14.45% (b), 27.34% (c), 31.6% (d), 36.2% (e), 40.34% (f), 45.07% (g), 49.25% (h), and 57.06% (i).

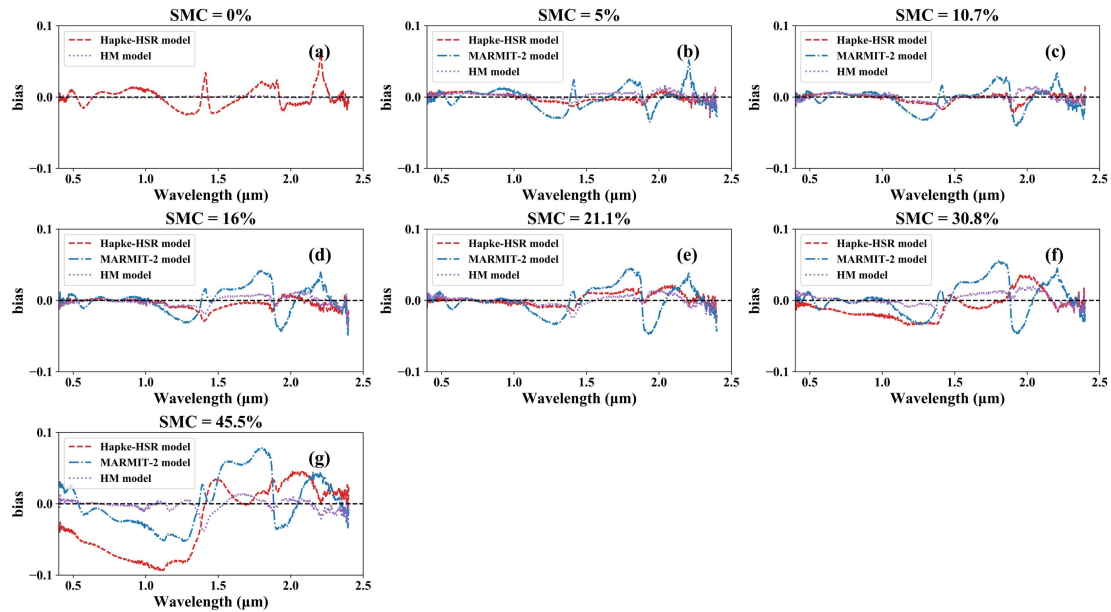


Figure A2. The bias (i.e., simulated reflectance of these models - measured reflectance) between the simulated reflectance of the Hapke-HSR (red), MARMIT-2 (blue) and Hapke-HSR + MARMIT-2 (HM) (lime) models and the fitted soil reflectance at SMC = 0% (a), 5% (b), 10.7% (c), 16% (d), 21.1% (e), 30.8% (f), and 45.5% (g).

Please see p. 22-23, lines 542-555, in the revised manuscript.

5. In the Figure 9. The model is evaluated using multiple datasets, which demonstrates its potential. However, additional commentary on the variability of model performance across these datasets would provide deeper insight into its robustness and limitations.

Response 5: We thank the reviewer for this constructive suggestion. We agree that further discussion on the variability of model performance across different datasets can provide additional insight into the robustness and limitations of the proposed framework.

In the revised manuscript, we have expanded the discussion associated with Figure 9. The results show that the performance of the three models varies across datasets, mainly due to differences in soil properties, spectral reflectance levels, and moisture sensitivity. The proposed HM model generally achieves improved accuracy across

most datasets, particularly where moisture-related absorption effects are more pronounced. However, the improvement is less evident for certain datasets. For example, for the Les08 dataset, the MARMIT-2 model already provides high accuracy, resulting in limited additional improvement from the coupled model. For the Lob02 dataset, the relatively low reflectance leads to larger NRMSE values, which reduces the apparent improvement.

These observations have been incorporated into the revised manuscript to better explain the variability of model performance and to clarify the robustness and limitations of the proposed approach. We appreciate the reviewer's suggestion, which has helped improve the interpretation of the results.

Please see p. 16, lines 383-392, in the revised manuscript.

6. The current manuscript mainly presents results from forward simulations. Given that the Introduction emphasize the role of soil reflectance in canopy reflectance and vegetation parameter retrieval, it may be helpful to include a brief discussion or additional comparison to further illustrate this connection.

We thank the reviewer for this constructive suggestion. We agree that clarifying the link between soil reflectance modeling and vegetation parameter retrieval would improve the overall relevance of the study.

In the revised manuscript, we have added a discussion to highlight this connection. Soil reflectance serves as a key background component in canopy radiative transfer models, directly influencing canopy reflectance and the retrieval accuracy of vegetation parameters. By improving the representation of soil spectral and directional reflectance under varying moisture conditions, the proposed Hapke-HSR + MARMIT-2 model framework can provide more accurate soil background inputs for canopy modeling.

This improvement is expected to enhance the performance of coupled soil-vegetation radiative transfer models (e.g., PROSAIL) and to reduce uncertainties in vegetation parameter retrieval, particularly under sparse vegetation or dry/wet soil conditions.

We have incorporated this discussion into the revised manuscript (Section 5.3) to better illustrate the broader applicability of the proposed model. We appreciate the

reviewer's suggestion, which has helped strengthen the interpretation and relevance of the study.

Please see p. 20-21, lines 492-496, in the revised manuscript.

Minor comments:

1. Line 268. How to define the range of high SMC?

We thank the reviewer for this helpful question. In the revised manuscript, we have clarified the definition of high soil moisture content (SMC). In this study, SMC values greater than approximately 30% are considered to represent relatively high moisture conditions. This threshold is chosen based on the datasets used and reflects the regime where moisture-related absorption becomes dominant in the spectral response, particularly in the shortwave infrared region.

Please see p.12, lines 287-293, in the revised manuscript.

2. Please keep consistency in the terminology used for soil models, soil reflectance models, and soil radiative transfer models.

We thank the reviewer for this helpful suggestion. In the revised manuscript, we have carefully reviewed and unified the terminology throughout the text. Specifically, the term "soil radiative transfer model (RTM)" is used consistently to describe the modeling framework, while alternative expressions such as "soil reflectance model" or "soil model" have been standardized to avoid ambiguity. These revisions improve the clarity and consistency of the manuscript.

Please see p.12, lines 287-293, in the revised manuscript.

3. Table 3, how to determine the ranges of model parameters? Please add some references.

We thank the reviewer for this helpful comment. In the revised manuscript, we have clarified the determination of the parameter ranges listed in Table 3 and added appropriate references. The parameter ranges are defined based on a combination of previous studies (e.g., Hapke, 2012; Verhoef et al., 2018; Dupiau et al., 2022; Ding et al., 2022) and physical considerations of soil properties. These ranges are selected to ensure physically realistic values while covering the variability observed across different soil types and moisture conditions. We have added corresponding references

and brief explanations in the revised manuscript to improve transparency and reproducibility.

Please see p.8, lines 203-208, in the revised manuscript.

#### References

Ding, A., Ma, H., Liang, S., and He, T.: Extension of the Hapke model to the spectral domain to characterize soil physical properties, *Remote Sensing of Environment*, 269, 112843, 2022.

Dupiau, A., Jacquemoud, S., Briottet, X., Fabre, S., Viallefont-Robinet, F., Philpot, W., Di Biagio, C., Hébert, M., and Formenti, P.: MARMIT-2: an improved version of the MARMIT model to predict soil reflectance as a function of surface water content in the solar domain, *Remote Sensing of Environment*, 272, 112951, 2022.

Hapke, B.: Bidirectional reflectance spectroscopy 7: the single particle phase function hockey stick relation, *Icarus*, 221, 1079–1083, 2012.

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4. The description of the Hapke-HSR model and its improved version is very confusing. Please explain it.

We thank the reviewer for this comment. In the revised manuscript, we have clarified that the improved Hapke-HSR model is not a separate model, but a refined formulation of the original Hapke-HSR model. The main difference lies in replacing the empirical spectral fitting with a physically based representation using the parameter  $\chi_{\text{soil}}$ , which improves the physical consistency and simulation of dry soil reflectance. The corresponding description has been revised to enhance clarity.

Please see p.4, lines 117-128, in the revised manuscript.