This study introduces a soil radiative transfer framework designed to reproduce soil reflectance behavior under varying SMC conditions. The subject is of clear importance to the remote sensing field. The manuscript is logically organized, and the evaluation using diverse datasets indicates that the proposed approach has strong potential. That said, several aspects require further clarification before the work can be fully appreciated. With these revisions, the manuscript is expected to reach the standard required for publication.

We sincerely thank the reviewer for the positive and constructive assessment of our manuscript. We appreciate the recognition of the importance of the topic, the logical organization, and the strong potential of the proposed framework.

We also thank the reviewer for the valuable suggestions regarding aspects that require further clarification. In response, we have carefully revised the manuscript to address all comments and to improve the clarity of the methodology, parameter description, and model applicability. We believe that these revisions have strengthened the manuscript and improved its overall quality. All comments are addressed in detail below, and we hope the revised version meets the reviewer's expectations.

We appreciate the reviewer's suggestion, which has helped improve the discussion of model applicability and future development.

Comments to the Authors

1. Lines, 90-94, and 211-216. The manuscript emphasizes parameter b within the Hapke-HSR framework, yet the procedure used to determine its optimal value is not sufficiently described. Moreover, although parameter c also contributes to shaping the phase function, its role is not adequately addressed. A clearer explanation of how these parameters are treated would improve the transparency of the modeling approach.

Response 1: We thank the reviewer for this helpful comment. We agree that the treatment of the phase function parameters should be clarified.

In the revised manuscript, we have explicitly described the determination of parameter b. Specifically, b is estimated through model fitting by minimizing the difference between simulated and observed reflectance under given observation geometries. This parameter plays a dominant role in controlling the anisotropy of the scattering pattern and shows high sensitivity in the inversion. Regarding parameter *c*, we have added clarification that, although it also influences the BRDF shape through the phase function, its sensitivity is comparatively weaker under the observational configurations considered in this study. Therefore, *c* is assigned a fixed value following previous studies (e.g., Hapke, 2012; Ding et al., 2022), while the analysis focuses primarily on parameter *b*. These explanations have been incorporated into the manuscript to improve the transparency and physical interpretation of the modeling approach. We appreciate the reviewer's suggestion, which has helped clarify the treatment of model parameters.

Please see p. 8, lines 213-218, in the revised manuscript.

References

Ding, A., Ma, H., Liang, S., and He, T.: Extension of the Hapke model to the spectral domain to characterize soil physical properties, Remote Sensing of Environment, 269, 112843, 2022.

Hapke, B.: Bidirectional reflectance spectroscopy 7: the single particle phase function hockey stick relation, Icarus, 221, 1079-1083, 2012.

2. Lines, 117-118. The parameters C3 and C4 introduced in Eq. (17) require further clarification. Providing additional details on their physical interpretation and derivation process would help readers better understand their function within the model.

Response 2: Thank you very much for this valuable comment. In the revised manuscript, we have clarified the definition and derivation of the parameters (C_3) and (C_4), as well as their physical role in the model.

In Eqs.(1)–(5), the relationship between dry soil spectral reflectance (SSR) and single-scattering albedo (SSA) is established under a simplified formulation, where the effects of multiple scattering are not explicitly considered. In this context, the parameters (C_1) and (C_2) are introduced as shape adjustment parameters of the dry SSR curve and are set to ($C_1 = 1$) and ($C_2 = 1$) to simplify the calculations.

However, neglecting multiple scattering leads to discrepancies between the simulated dry soil reflectance and the measured reflectance. To account for this effect, we introduce the parameters (C_3) and (C_4), which are determined through a linear regression between the dry soil reflectance simulated by the Hapke-HSR model and the corresponding measured dry soil reflectance, as described in Eq. (6). These parameters serve as spectral shape adjustment coefficients that compensate for the bias caused by the simplified treatment of multiple scattering.

Therefore, while (C_1) and (C_2) are fixed constants, (C_3) and (C_4) are empirically estimated parameters that improve the agreement between model simulations and observations. This procedure has been clarified in the revised manuscript to improve the transparency and reproducibility of the model.

$$R_d(\theta_s,\theta_v,\varphi,\lambda) = C_1 \times \omega \tag{1}$$

$$C_1 = \frac{1}{4(\cos\theta_s + \cos\theta_v)}\{[P(g,g^{'})(1+B(g))]-1\} \tag{2}$$

$$\omega = 1 - \frac{C_2 \times \chi_{soil}}{\lambda} \tag{3}$$

$$C_2 = 4\pi M \tag{4}$$

$$\chi_{soil} = \frac{\lambda}{C_2} \times (1 - \frac{R_d(\theta_s,\theta_v,\varphi,\lambda)}{C_1}) \tag{5}$$

where C_1 and C_2 are the shape adjustment parameters of the dry SSR curve. We use $C_1 = 1$ and $C_2 = 1$ as the initial values to simplify the calculation, after which we further calculate the shape adjustment parameters.

The relationship between the dry SSR simulated with the Hapke-HSR model and the measured dry SSR can be expressed via the following formula:

$$R_d^{'}(\theta_s,\theta_v,\varphi,\lambda) = C_3 \times R_d(\theta_s,\theta_v,\varphi,\lambda) + C_4 \tag{6}$$

where C_3 and C_4 represent the spectral shape adjustment parameters of the dry SSR. Note that $R_d(\theta_s, \theta_v, \varphi, \lambda)$ is calculated via the Hapke-HSR model on the basis of the shape similarity assumption.

Please see p. 5, lines 129-132, in the revised manuscript.

3. In the Table 3. To enhance clarity, it would be beneficial to summarize the parameter settings of the Hapke-HSR model, the MARMIT-2 model, and their coupled formulation. In particular, explicitly categorizing parameters into fixed, prescribed inputs, and those subject to retrieval would improve consistency and readability throughout the manuscript.

Response 3: We thank the reviewer for this constructive suggestion. In the revised manuscript, we have improved the presentation of model parameters to enhance clarity and consistency.

Specifically, we have added a summary table (Table 1) that provides a unified overview of the input parameters for the Hapke-HSR model, the improved Hapke-HSR model (dry soil), the MARMIT-2 model, and the coupled Hapke-HSR + MARMIT-2 framework. In addition, we have explicitly clarified the roles of different parameters in the text, distinguishing between prescribed input parameters and those subject to retrieval.

These revisions improve the consistency and readability of the parameter settings across different model components. We appreciate the reviewer's suggestion, which has helped improve the overall presentation of the manuscript.

Table 1. Input parameters of each soil model.

Models	Input parameters
Hapke-HSR (Ding et al., 2022)	SZA, VZA, RAA, b, A_0, A_1, A_2, A_3, and f
Improved Hapke-HSR (dry soil)	SZA, VZA, RAA, b, M, and χ_{soil}
MARMIT-2 (Dupiau et al., 2022)	δ, L and ε
Hapke-HSR model and MARMIT-2	SZA, VZA, RAA, b, M, χ_{soil}, δ, L and ε

Please see p. 11-12, lines 280-282, in the revised manuscript.

4. In the Figure 4, it is not easy to see the difference among different models. Could you explain the difference? The authors only showed the results for these two samples in Table 4.

Response 4: We thank the reviewer for this helpful comment. We agree that the differences among the models are not sufficiently clear in Figure 4 in the revised manuscript.

To address this issue, we have added a figure in the revised manuscript (Appendix Figure A) showing the bias (i.e., simulated reflectance minus measured reflectance) for the Hapke-HSR, MARMIT-2, and coupled Hapke-HSR + MARMIT-2 (HM) models across a wide range of soil moisture conditions. This figure provides a more direct and quantitative comparison of model performance. The results show that the HM model consistently reduces the bias across most wavelengths and soil moisture conditions compared to the individual models. In particular, the improvement is more evident under moderate to high soil moisture levels.

We have added a corresponding description in the revised manuscript (Appendix A) to clarify these differences and improve the interpretation of model performance. We appreciate the reviewer's suggestion, which has helped improve the clarity of the comparison.

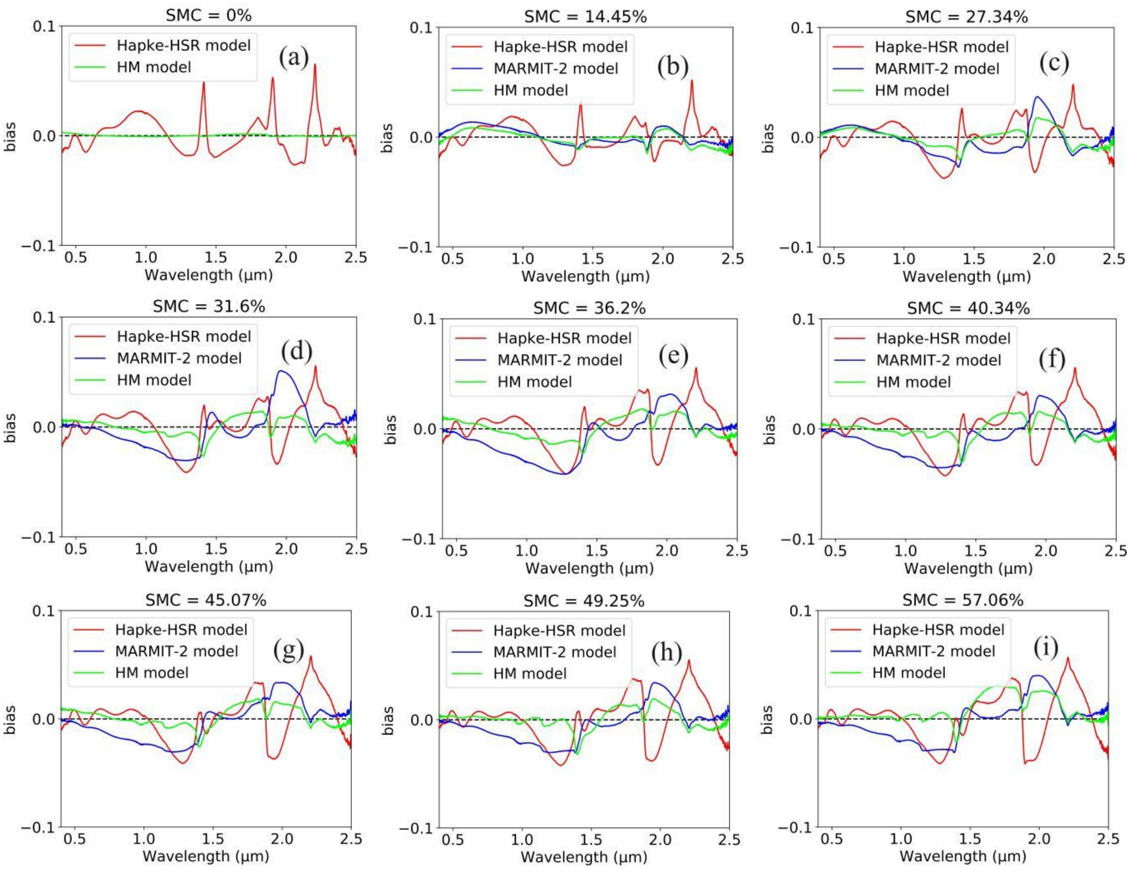

Figure A1: The bias (i.e., simulated reflectance of these models - measured reflectance) between the simulated spectral reflectance of the Hapke-HSR (red), MARMIT-2 (blue) and Hapke-HSR + MARMIT-2 (HM) (lime) models and the fitted soil reflectance at SMC = 0% (a), 14.45% (b), 27.34% (c), 31.6% (d), 36.2% (e), 40.34% (f), 45.07% (g), 49.25% (h), and 57.06% (i).

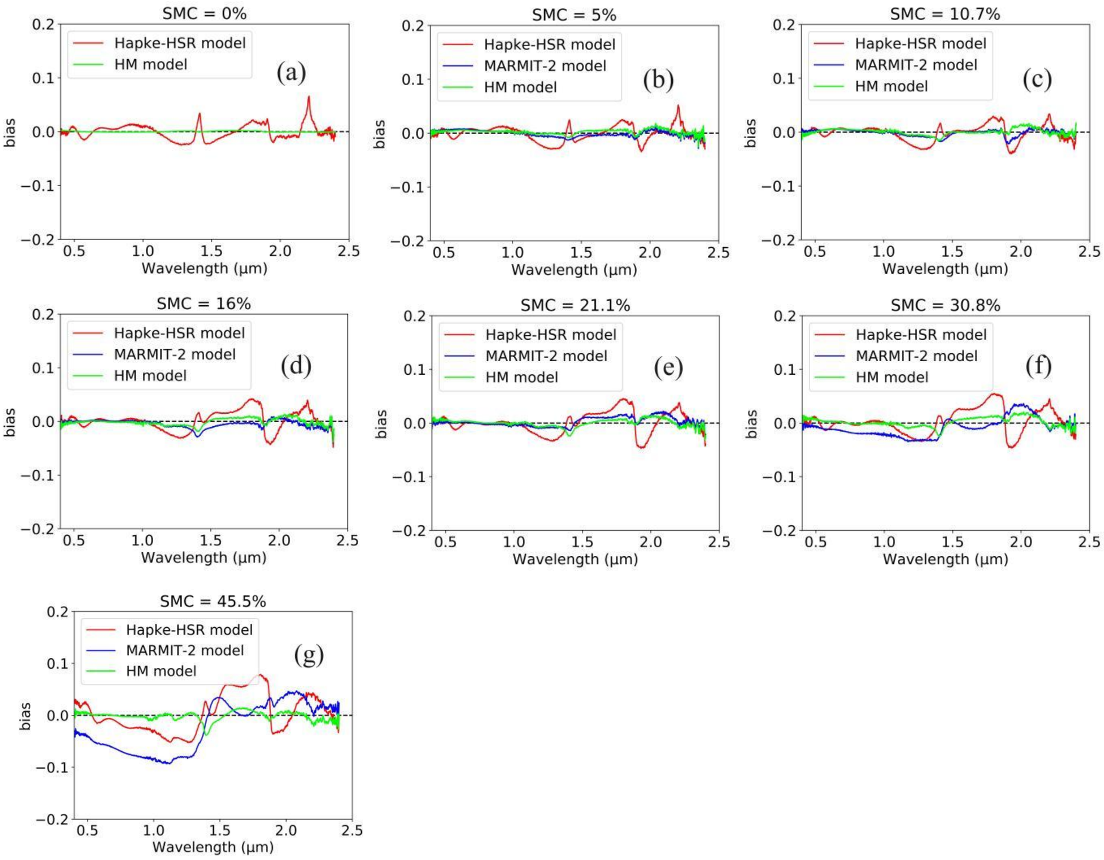

Figure A2. The bias (i.e., simulated reflectance of these models - measured reflectance) between the simulated reflectance of the Hapke-HSR (red), MARMIT-2 (blue) and Hapke-HSR + MARMIT-2 (HM) (lime) models and the fitted soil reflectance at SMC = 0% (a), 5% (b), 10.7% (c), 16% (d), 21.1% (e), 30.8% (f), and 45.5% (g).

Please see p. 22-23, lines 542-555, in the revised manuscript.

5. In the Figure 9. The model is evaluated using multiple datasets, which demonstrates its potential. However, additional commentary on the variability of model performance across these datasets would provide deeper insight into its robustness and limitations.

Response 5: We thank the reviewer for this constructive suggestion. We agree that further discussion on the variability of model performance across different datasets can provide additional insight into the robustness and limitations of the proposed framework.

In the revised manuscript, we have expanded the discussion associated with Figure 9. The results show that the performance of the three models varies across datasets, mainly due to differences in soil properties, spectral reflectance levels, and moisture sensitivity. The proposed HM model generally achieves improved accuracy across most datasets, particularly where moisture-related absorption effects are more pronounced. However, the improvement is less evident for certain datasets. For example, for the Les08 dataset, the MARMIT-2 model already provides high accuracy, resulting in limited additional improvement from the coupled model. For the Lob02 dataset, the relatively low reflectance leads to larger NRMSE values, which reduces the apparent improvement.

These observations have been incorporated into the revised manuscript to better explain the variability of model performance and to clarify the robustness and limitations of the proposed approach. We appreciate the reviewer's suggestion, which has helped improve the interpretation of the results.

Please see p. 16, lines 383-392, in the revised manuscript.

6. The current manuscript mainly presents results from forward simulations. Given that the Introduction emphasize the role of soil reflectance in canopy reflectance and vegetation parameter retrieval, it may be helpful to include a brief discussion or additional comparison to further illustrate this connection.

We thank the reviewer for this constructive suggestion. We agree that clarifying the link between soil reflectance modeling and vegetation parameter retrieval would improve the overall relevance of the study.

In the revised manuscript, we have added a discussion to highlight this connection. Soil reflectance serves as a key background component in canopy radiative transfer models, directly influencing canopy reflectance and the retrieval accuracy of vegetation parameters. By improving the representation of soil spectral and directional reflectance under varying moisture conditions, the proposed Hapke-HSR +

MARMIT-2 model framework can provide more accurate soil background inputs for canopy modeling.

This improvement is expected to enhance the performance of coupled soil-vegetation radiative transfer models (e.g., PROSAIL) and to reduce uncertainties in vegetation parameter retrieval, particularly under sparse vegetation or dry/wet soil conditions.

We have incorporated this discussion into the revised manuscript (Section 5.3) to better illustrate the broader applicability of the proposed model. We appreciate the reviewer's suggestion, which has helped strengthen the interpretation and relevance of the study.

Please see p. 20-21, lines 492-496, in the revised manuscript.

Minor comments:

1. Line 268. How to define the range of high SMC?

We thank the reviewer for this helpful question. In the revised manuscript, we have clarified the definition of high soil moisture content (SMC). In this study, SMC values greater than approximately 30% are considered to represent relatively high moisture conditions. This threshold is chosen based on the datasets used and reflects the regime where moisture-related absorption becomes dominant in the spectral response, particularly in the shortwave infrared region.

Please see p.12, lines 287-293, in the revised manuscript.

2. Please keep consistency in the terminology used for soil models, soil reflectance models, and soil radiative transfer models.

We thank the reviewer for this helpful suggestion. In the revised manuscript, we have carefully reviewed and unified the terminology throughout the text. Specifically, the term "soil radiative transfer model (RTM)" is used consistently to describe the modeling framework, while alternative expressions such as "soil reflectance model" or "soil model" have been standardized to avoid ambiguity. These revisions improve the clarity and consistency of the manuscript.

Please see p.12, lines 287-293, in the revised manuscript.

3. Table 3, how to determine the ranges of model parameters? Please add some references.

We thank the reviewer for this helpful comment. In the revised manuscript, we have clarified the determination of the parameter ranges listed in Table 3 and added appropriate references. The parameter ranges are defined based on a combination of previous studies (e.g., Hapke, 2012; Verhoef et al., 2018; Dupiau et al., 2022; Ding et al., 2022) and physical considerations of soil properties. These ranges are selected to ensure physically realistic values while covering the variability observed across different soil types and moisture conditions. We have added corresponding references and brief explanations in the revised manuscript to improve transparency and reproducibility.

Please see p.8, lines 203-208, in the revised manuscript.

References

Ding, A., Ma, H., Liang, S., and He, T.: Extension of the Hapke model to the spectral domain to characterize soil physical properties, Remote Sensing of Environment, 269, 112843, 2022.

Dupiau, A., Jacquemoud, S., Briottet, X., Fabre, S., Viallefont-Robinet, F., Philpot, W., Di Biagio, C., Hébert, M., and Formenti, P.: MARMIT-2: an improved version of the MARMIT model to predict soil reflectance as a function of surface water content in the solar domain, Remote Sensing of Environment, 272, 112951, 2022.

Hapke, B.: Bidirectional reflectance spectroscopy 7: the single particle phase function hockey stick relation, Icarus, 221, 1079–1083, 2012.

Verhoef, W. and Bach, H.: Coupled soil–leaf–canopy and atmosphere radiative transfer modeling to simulate hyperspectral multi-angular surface reflectance and TOA radiance data, Remote Sensing of Environment, 109, 166–182, 2007.

4. The description of the Hapke-HSR model and its improved version is very confusing. Please explain it.

We thank the reviewer for this comment. In the revised manuscript, we have clarified that the improved Hapke-HSR model is not a separate model, but a refined formulation of the original Hapke-HSR model. The main difference lies in replacing the empirical spectral fitting with a physically based representation using the parameter χ_{soil}, which improves the physical consistency and simulation of dry soil reflectance. The corresponding description has been revised to enhance clarity.

Please see p.4, lines 117-128, in the revised manuscript.