



Mach cone refraction in layered snow slabs during supershear fracture

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Abstract. When a crack accelerates beyond the material shear-wave speed, shear-waves pile up into a shock front, a striking Doppler-induced signature of supershear fractures. While supershear fracture is best known from dynamic rupture in strike-slip earthquakes, it has recently been observed and modeled in snow slab avalanches triggered by the failure of a weak layer beneath cohesive slabs. These slabs are typically stratified, with layers of distinct mechanical properties that strongly influence wave propagation. Here, we combine numerical simulations with a long snow-fracture experiment to identify Mach cones associated with supershear crack propagation in layered snow slabs. We show that slab layering alters both the crack propagation speed and Mach cone geometry, and that curvature of the cone can be explained by wave refraction at layer interfaces. These findings advance our understanding of avalanche release and supershear fracture dynamics and provide a framework for detecting and interpreting Mach cones in snow-fracture experiments.

1 Introduction

Dry snow slab avalanches are among the most common types of snow avalanches in mountain terrains and are responsible for more than 90% of avalanche fatalities (Ava). They are caused by the failure of a weak snow layer buried beneath a cohesive slab of snow, which leads to the apparent instantaneous release and sliding of a large snow slab (Schweizer et al., 2003). Due to its sparse and highly porous microstructure, snow exhibits bulk densities ranging from approximately 50 to 550 kg.m⁻³ (Khuller et al., 2021). This unique structure enables significant structural rearrangement and volumetric compaction following mechanical failure. Snow can therefore fail under compressive loading and collapse in a mode I-like manner, with closure rather than opening of the crack faces, often interpreted as mode -I and referred to as an ‘anticrack’. Both compressive and shear loading play a role in the failure of the weak layer in the slope parallel directions as well as on a flat terrain (Reiweger et al., 2015; Bobillier et al., 2020, 2021). The associated fracture mechanism is therefore commonly described as mixed-mode anticrack propagation: mode -I/II for up- or down-slope crack propagation, and mode -I/III for cross-slope crack propagation. This theory was introduced in 1967 by Bradley and Bowles for snow slab avalanches and later generalized by Heierli et al. in 2008, as an alternative to the shear-failure theory of McClung (1979), which could not account for avalanches triggered from



low-angle terrain. In mixed-mode anticrack propagation, the crack speed is bounded by the Rayleigh-wave speed (Siron et al., 2023), a surface-wave speed slightly lower than the shear-wave speed.

25 In 2018, Gaume et al. introduced a Cohesive Modified Cam Clay model (Roscoe and Burland, 1968; McClung, 1979) in
a Material Point Method (MPM) (Sulsky et al., 1995; Nguyen et al., 2023; Stomakhin et al., 2013) scheme to successfully
simulate, for the first time, the complete avalanche release process at the slope scale, encompassing failure initiation, crack
propagation, avalanche release, and subsequent flow dynamics. Building on this new model, and following real-scale observa-
tions of avalanche triggering showing propagation speeds greatly exceeding the snow shear wave speed (Hamre et al., 2014),
30 Trottet et al. (2022) and Bobillier et al. (2024a) unveiled, through numerical simulations, the existence of a regime of crack
propagation called “supershear” or “intersonic” (the two terms are used interchangeably in this paper), where the crack prop-
agates at speeds greater than the slab’s shear (S)-wave speed and lower than its pressure (P)-wave speed. More precisely, a
transition occurs from mixed-mode anticrack to a pure shear intersonic failure type after reaching a so-called “super critical
crack length” typically between 3 and 6 meters (Trottet et al., 2022). This transition follows the Burridge-Andrews mechanism
35 (Burridge, 1973; Andrews, 1976), with the appearance of a “daughter” shear crack ahead of the initial anticrack, propagating
up and down-slope at an increasing speed until it reaches a stable regime exceeding the shear wave speed in the material. This
phenomenon, well documented in shallow strike-slip earthquakes (Bouchon et al., 2010; Bouchon and Vallée, 2003; Rosakis
et al., 2023, 2025; Bao et al., 2022; Huang et al., 2016; Zeng et al., 2025), further underscores the parallels between slab
avalanche release and earthquake rupture mechanics.

40 When a crack tip propagates along an interface, it continuously emits compressional and shear waves that radiate in all
directions. While the speeds of these waves are governed solely by the material properties, their wavelength is modified by
the crack-tip speed through a Doppler-like effect. If the crack tip travels faster than one of these wave speeds, it overtakes
the waves it emits. In any plane parallel to the direction of crack propagation, the corresponding wavefronts coalesce along a
common tangent passing through the crack tip (see Methods). Consequently, the shear strain energy transported by the waves
45 accumulates along this tangent, generating a sharp and intense perturbation as it sweeps through the medium. Similar effects
occur in the wake of a moving boat or in the Mach cone produced by an aircraft flying faster than sound. In the latter case, the
sonic boom heard after a fighter jet has passed corresponds to the arrival of this tangent wave at the observer.

This piled-up tangent wave, commonly referred to as a “shock wave” in aerospace engineering and as a “shock front” or
“Mach cone” in seismology, is purely a wave-propagation phenomenon. The two latter terms will be used interchangeably in
50 the rest of this paper. Therefore, if a crack in a discontinuity propagates faster than the shear wave speed in the surrounding
medium, a shear shock front should appear in the medium. More precisely, a shear shock front appears if and only if the
discontinuity propagates at intersonic speed, making it a direct signature of supershear crack propagation. Such Mach cones
have already been observed in scaled laboratory experiments on polymers (Gori et al., 2018; Needleman, 1999; Liu and
Lapusta, 2008) and remote earthquake measurements (Rosakis et al., 2023, 2025; Bao et al., 2022; Zeng et al., 2025) where
55 this shock front leads to more destructive action on the ground surface (Dunham and Bhat, 2008).

The existence of a supershear crack propagation regime in snow slab avalanche release is now supported by numerical
simulations (Trottet et al., 2022; Bobillier et al., 2024b, a), indirect avalanche observations (Trottet et al., 2022) and crack



speed measurements from long snow fracture experiments called Propagation Saw Tests (PST) (Bergfeld et al., 2025). However, Mach cones have not yet been identified and characterized in snow supershear fracture, and the influence of slab layering on their geometry remains unknown. Here, we address these two questions using slope-scale two-dimensional MPM simulations of long PSTs. We first identify Mach cones in homogeneous slabs, exhibiting classical geometries consistent with theoretical expectations. We then investigate how snow layering affects crack propagation and Mach cone shape. Finally, we relate these findings to recent in situ observations of supershear failure by Bergfeld et al. (2025) and discuss implications for future snow-fracture experiments.

2 Results

2.1 Numerical simulations

We study dynamic crack propagation in snow through two-dimensional MPM simulations of PST experiments. The PST is a standard field test introduced by Gauthier and Jamieson (2008), used to study crack propagation in buried weak snow layers. In this test, a snow column 30 cm wide (cross-slope) and at least 100 cm long (downslope) is isolated from the surrounding snow cover. An artificial crack is then initiated in a pre-identified weak layer by pushing the back of a snow saw, generally in the upslope direction, until the crack propagates spontaneously (see Figure 1). The critical cut length before spontaneous crack propagation is measured as an indicator of snow-cover stability.

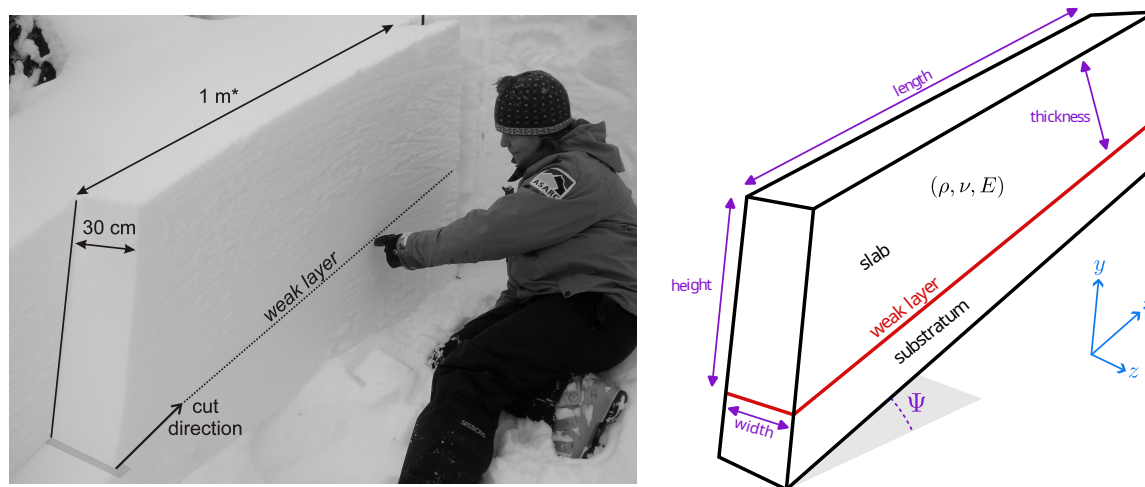


Figure 1. Propagation Saw Test (PST) configuration, with an *in situ* example (left) and schematic representation (right). Credit for left picture : ASARC photo (Gauthier and Jamieson, 2007)

The PST is a simple and widely used field procedure for avalanche research and snow-stability evaluation. However, its relatively small length usually does not allow crack propagation to reach a steady state (van Herwijnen et al., 2010; Bergfeld

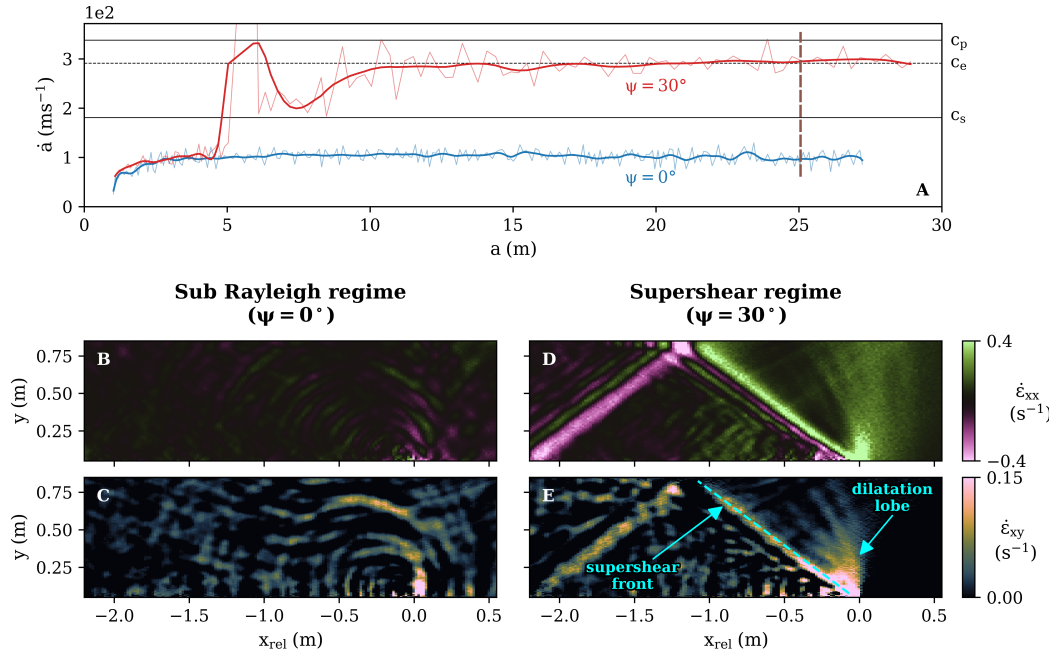


Figure 2. Identical PST simulations at slope angles of 0° and 30° . (A) Crack propagation speed ($\dot{a}$) versus crack tip position (a) along the x -axis. Low and high opacity lines correspond to the raw and smoothed crack speeds, respectively. The crack tip position is computed by taking the average x position of the five furthest plasticized particles. (B) to (E) Close-up views of the slope-parallel (B-D) and shear (C-E) strain rates in the slab for the low (B-C) and high (D-E) angle cases. Close-ups (D) and (E) show a supershear Mach cone and a dilatation (pressure waves) lobe.

et al., 2022; Trottet et al., 2022), especially in the intersonic regime. To overcome this limitation, we perform numerical PST simulations at a much larger scale, with PST lengths between 20–60 m. In addition, the numerical results are compared with experimental data from an approximately 10 m long PST.

This study focuses on the evolution of the crack in the weak layer along the slope-parallel direction and the deformation behaviour along the depth of the slab. Given that the PST is essentially a 2D problem, MPM simulations were conducted in two dimensions with a plane strain approximation for computational efficiency. This assumption enabled high-resolution simulations of long PST experiments. The MPM model is based on a finite strain elastoplastic framework. In the present study, the slab is modeled as purely elastic using Hencky elasticity (Hencky, 1928, 1929) and therefore cannot fail in tension. Regarding the weak layer, the Cohesive Cam Clay elastoplastic model is applied in addition to Hencky's elasticity, allowing the weak layer to fail and collapse under multi-axial loading (see Methods section for more information on the mechanical model) (Gaume et al., 2018; Trottet et al., 2022). The cut in the weak layer is created by a virtual saw, which sets the Young's modulus of the particles in its vicinity to zero. For faster computations, the saw cuts the first meter of the weak layer all at once, starting

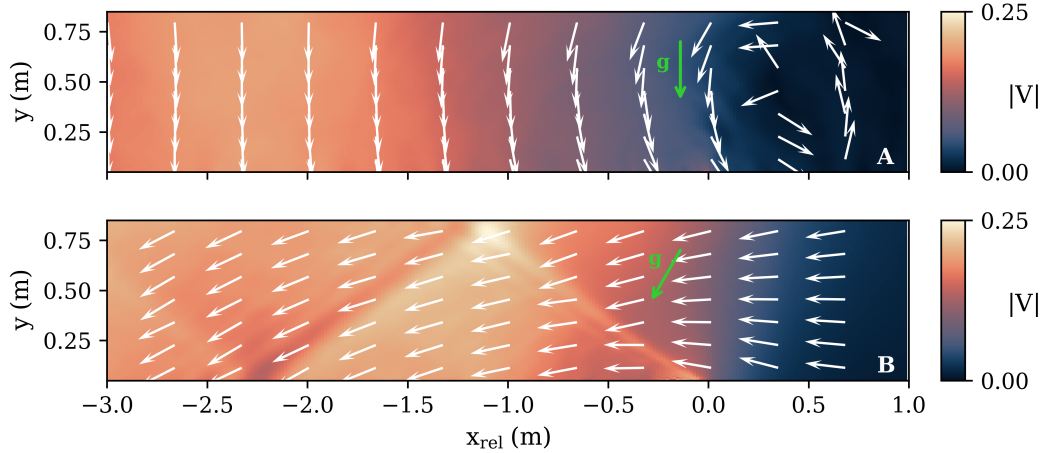


Figure 3. Speed amplitude and direction in the collapsing zone of the slab for sub-Rayleigh and supershear failures. The simulations and close-ups for sub-Rayleigh (A) and supershear (B) crack propagation are identical to those shown in Figure 2. The colormap shows the absolute amplitude of the speed, while the white arrows show the local direction of the displacement (at the base of the arrow) and are referred to as the "displacement field". The green arrow represents the direction of gravity.

immediately the spontaneous crack propagation. We verified that this simplification does not affect the supercritical crack length a_{sc} , i.e. the maximum crack length before the daughter crack appears, nor the transient phase or the steady-state crack-tip speed. Finally, we assume a rigid substratum for computational reasons as well. This approximation influences slightly the supercritical crack length a_{sc} but has no effect on the transient phase nor the steady state crack speed (see Figure B7).

Geometrical and material parameters of the simulation showcased in this study can be found in Table C1.

2.2 Intersonic Mach cone

To reveal the presence of a Mach cone in the supershear regime, we perform simulations of both a 0° slope angle sub-Rayleigh crack propagation and a 30° slope angle supershear crack propagation, with identical geometrical and mechanical properties.

In Figure 2, we observe the slope-parallel and shear strain rates, distinctly highlighting the Mach cone in the supershear regime.

In the case of a low slope angle PST (blue), the crack tip speed reaches an asymptotic speed of 126 m.s^{-1} , lower than the shear wave speed $c_s = \sqrt{E/(2\rho(1+\nu))} = 181 \text{ m.s}^{-1}$, where E is the Young's modulus, ρ the density and ν the Poisson's ratio. For a steep slope (red), a similar behaviour can be identified on the first 5 meters of the column. Thereafter, however, crack tip speed suddenly increases, briefly exceeding the pressure wave speed. This apparently nonphysical overshoot is an artefact of the crack-tip identification method. We define it as the mean x -position of the ten furthest plasticized particles in the weak layer. The apparition of a daughter crack ahead of the initial crack, following the Burridge-Andrews mechanism (Burridge, 1973; Andrews, 1976) causes an apparent instantaneous overshoot in the crack tip displacement, thus a nearly-infinite speed. This phenomenon is visualized in Figure B1, where we can also briefly see a trailing Rayleigh wave, another



feature of supershear earthquakes (Dunham and Archuleta, 2004; Rosakis et al., 2023, 2025). After this transition, the crack
105 reaches a new asymptotic speed equal to the elastic wave speed $c_e = \sqrt{E/\rho}$ (Trottet et al., 2022). This is the intersonic regime
: the crack propagates faster than the shear wave speed but slower than the pressure wave speed. In the case of $\psi = 30^\circ$, a
supershear Mach cone is observed. The cone angle measured from the horizontal (see appendix A4) is 36.6° , which deviates
by less than 3% from the theoretical prediction (37.6°) given by

$$\theta = \arcsin\left(\frac{c_s}{\dot{a}}\right) \quad (1)$$

110 where θ is the angle between the crack line and the cone, c_s the shear wave speed, and $\dot{a}$ the crack tip speed (derivation provided
in Methods section).

In addition, since the crack propagates more slowly than the pressure waves, we observe a densification of the waves around
the crack tip due to the Doppler effect, forming a so-called “dilatation lobe” ahead of the Mach cone. In contrast, for the flat
sub-Rayleigh case, only a small lobe is observed (Figure 2B). In this case, the crack is slower than both the shear and pressure
115 waves, which therefore produces overlapping shear and dilatation lobes. Figure B2 shows identical Mach cones and lobe fronts
appearing in most components of speed, strain, and strain rate.

Beyond crack-tip speed and shock-front formation, the sub-Rayleigh and supershear regimes also differ markedly in their
deformation patterns within the slab. Figure 3 shows the displacement field for each failure type. In the low angle and slow
failure case (Fig. 3A), the slab movement around the crack tip is dominated by slope perpendicular “downward” displacement.
120 This is characteristic of a mode -I collapsing crack with closing crack faces, also called “anticrack”. In contrast, in the steep-
slope, high-speed case (Fig. 3B), the displacement field is mainly parallel to the slope, thus inducing a shear failure behavior
of the weak layer.

In 1989, Broberg showed that the regime $\dot{a} > c_s$ is not admissible for pure mode I crack propagation. This is not in con-
tradiction with our results. As shown in Figure 3, the fracture transitions from a collapse-driven mode -I anticrack to a shear-
125 dominated mode II crack, for which supershear propagation i.e. $\dot{a} > c_s$ is admissible.

Finally, one can also note that the two regimes also differ by the touch-down distance, i.e. the distance between the tip of the
crack and the location of the final closure of the crack faces, which is approximately $6.5m$ for the sub-Rayleigh regime and
 $19.5m$ for the supershear regime (see Figure B4), following approximately the ratio $\dot{a}^{sub-R}/\dot{a}^{sup-S} \simeq 1/3$.

2.3 Fracture behavior in a layered slab

130 The Mach cone existence depends on the crack propagation regime and its angle depends on the local shear wave speed c_s .
Layered snow slabs are common in a natural snowpack. To assess the influence of such a snow cover, we performed PST
simulations on a slab with three layers, inspired by Habermann et al. (2008) and Monti et al. (2016). The middle layer has
a density $\rho^m = 250 \text{ kg.m}^{-3}$, while the bottom and top layers have densities $\rho^m(1 + \gamma)$ and $\rho^m(1 - \gamma)$, respectively, with
 $\gamma \in [-0.5 : 0.5]$.

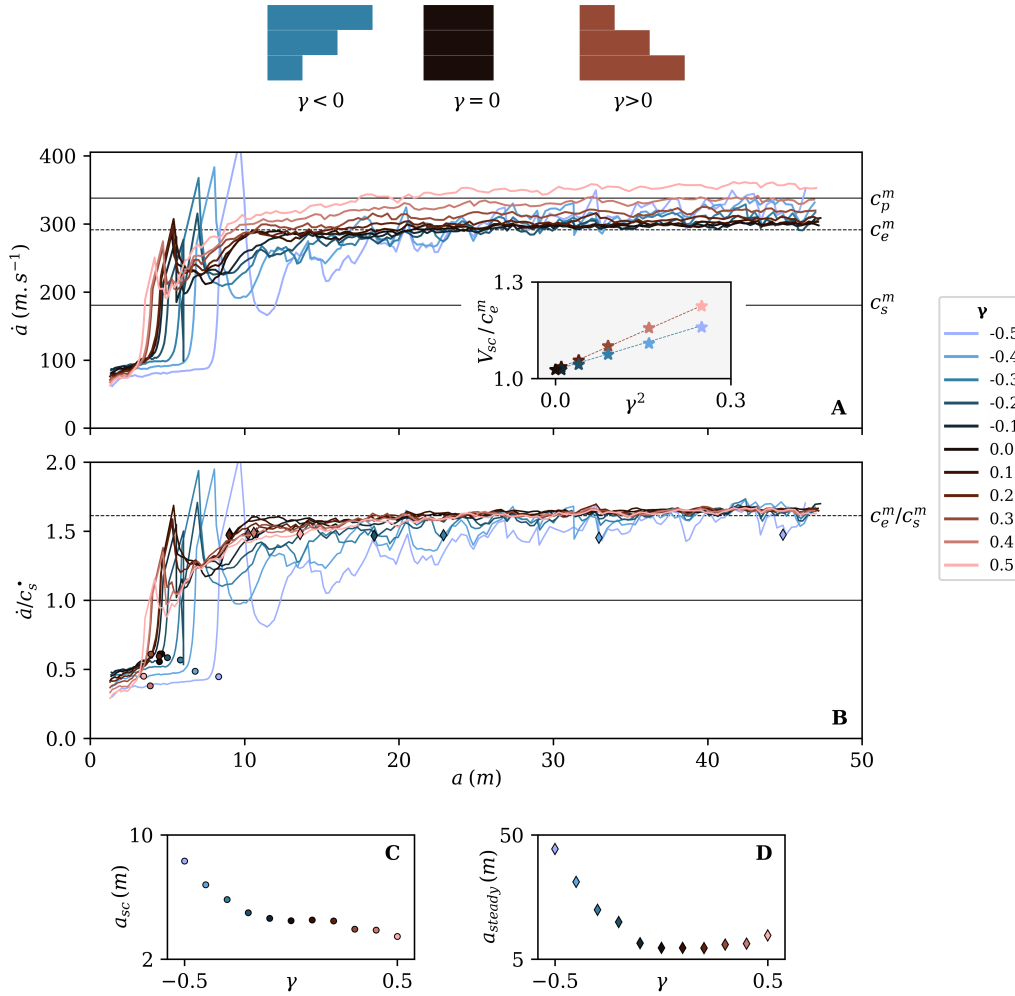


Figure 4. Crack tip speed ($\dot{a}$) over crack tip position (a) for different configurations of layered snow slabs. **(A)** Crack tip speed compared with the wave speeds for the homogeneous mean slab. "m" meaning "mean". The inset shows the relative supershear steady state crack tip speed V_{sc} , averaged between 75 and 85 m, normalised by the elastic wave speed of the equivalent mean slab c_e^m , as a function of the squared layering coefficient γ^2 . Blue and red tones correspond respectively to $\gamma < 0$ and $\gamma > 0$, and black to $\gamma = 0$ (the homogeneous configuration). In the schematic view above **(A)**, the bar length represents relative density for each layer. The dotted lines show the linear regressions of the corresponding points. **(B)** Crack tip speed normalized by the "effective" shear wave speed c_s^* . The black dotted line represents the expected supershear crack tip speed associated to the "effective" shear wave speed c_e^*/c_s^* (Trottet et al., 2022). **(C)** supercritical crack length a_{sc} for each simulation, determined from the raw speed measurement. Discrepancies with the corresponding speed curves come from the smoothing operation. **(D)** Steady state crack length after which the smoothed speed stays within 10% of the steady state speed.



135 The three layers have the same thickness with a total thickness $D = 0.8\text{ m}$, giving all configurations the same overall weight and mean density of $\rho = \rho^m$. The Young's modulus of each layer is derived using Sigrist's formula (Sigrist, 2006)

$$E^{sig} = 967 \text{ MPa} \left(\frac{\rho}{\rho_{ice}} \right)^{2.94} \quad (2)$$

with $\rho^{ice} = 917 \text{ kg.m}^{-3}$ the density of ice. The Poisson ratio is taken as constant and equal to 0.3. We observe in Figure 4A that the layering affects both the transient phase and the asymptotic crack tip speed.

140 Regarding the transient phase, we first observe that larger values of γ correspond to longer supercritical crack lengths. In addition, positive and higher values of γ appear to mitigate the previously mentioned crack tip speed overshoot, resulting in a much smoother crack tip speed evolution along the PST length.

In the steady-state regime, we observe that, in some configurations, the crack propagation speed exceeds the mean pressure wave speed. This apparently nonphysical result suggests that simple averaging of slab properties is insufficient to describe wave-speed control in layered slabs. More precisely, the more spread out the layering, the higher the asymptotic crack speed (inset in Fig. 4A). This suggests that the stiffest and densest layers exert a stronger influence on crack propagation than would be expected from a simple averaging of slab properties. However, the material properties of the stiffest layer cannot account alone for the asymptotic crack speed. The latter differs by a maximum of 22% to that of the homogeneous slab ($\gamma = 0$) whereas the elastic wave speed differs by a maximum of 48%.

150 The inset of Figure 4A shows that the steady-state supershear crack tip speed scales with the square of the layering coefficient γ , with the slope of this relationship depending on the sign of γ . To account for this effect, we introduce a simple empirical correction to the homogeneous-slab shear-wave speed

$$V^\bullet = \sqrt{2(1+\nu)} c_s^\bullet \quad (3)$$

with

$$155 \quad c_s^\bullet = c_s^m (1 + \zeta \gamma^2) \quad (4)$$

where $V^\bullet$ is the estimated crack tip speed, c_s^m is the shear wave speed of the equivalent homogeneous slab and ζ is a coefficient depending on the sign of γ . The coefficient values were obtained based on a linear regression on the values in the inset of Figure 4A.

$$\zeta = \begin{cases} 0.80 & ; \quad \gamma > 0 \\ 0.55 & ; \quad \gamma < 0 \end{cases} \quad (5)$$

160 This normalization effectively collapses the crack speed curves in the steady-state regime (Figure 4B), leading to a unique asymptotic value around c_e^m / c_s^m , similar to what is expected for homogeneous slabs.

From Figure 4C-D), one can also note that the supercritical crack length a_{sc} and stabilization of the steady state are greatly influenced by the layering, even though the total weight on the weak layer is unchanged. In particular, when a dense layer



overlies a lighter one ($\gamma < 0$), the crack tip speed is particularly fluctuating. This may indicate a greater propensity for crack
165 arrest and slab tensile failure, which could in turn influence the avalanche release size.

2.4 Experimental evaluation

We next compare the numerical results with the long-PST experiment of Bergfeld et al. (2025) which provided the first direct
field evidence for the sub-Rayleigh to supershear fracture transition in snow. The experiment was not long enough to reach
steady-state propagation, and the imaging setup focused on deformation around the weak layer rather than on the dynamics of
170 the full slab, which prevented the detection of a potential shock front. To address this, we reproduced the experiment using a
longer column, enabling us to capture the steady state propagation regime and the associated Mach cone.

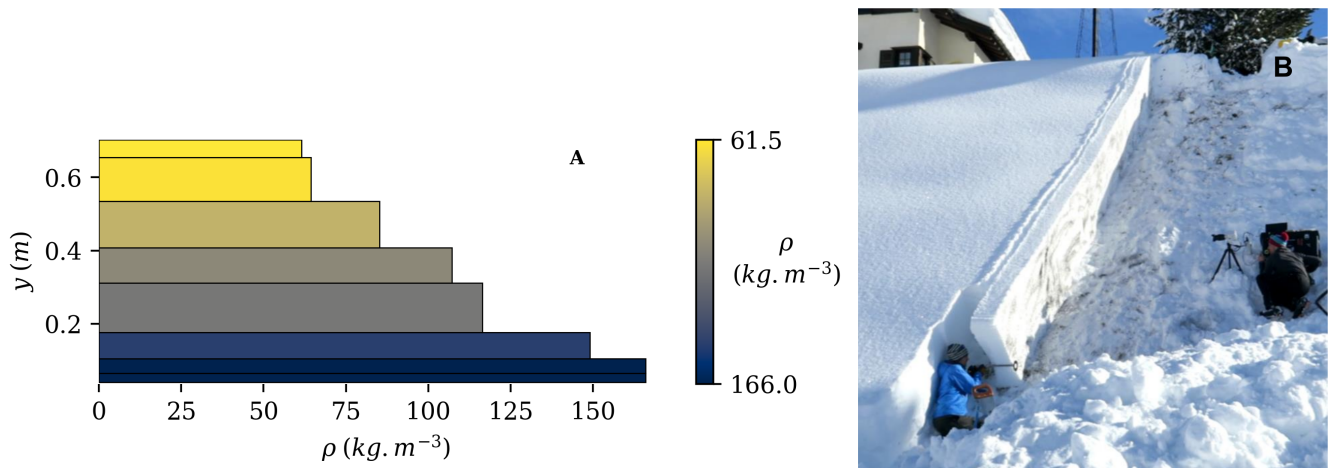


Figure 5. Experimental setup of Bergfeld et al. (2025). (A) Natural slab layering observed in the experimental setup. (B) Picture of the experimental setup. Layer density (x-axis) for given perpendicular distance to the base of the weak layer (y-axis). The detail of layer thicknesses and densities is presented in Table C4.

The experimental snow cover consists of a 127 cm thick column on a 37° slope, composed of 14 layers of disparate heights and densities decreasing from bottom to top of the column. A 1 cm thick weak layer is located at a height of 44 cm. As before, for simplicity and computational efficiency, we model the substratum – i.e. the portion of the column below the weak layer –
175 as perfectly rigid. The weak layer is modelled as a 5 cm elastoplastic layer using the Cohesive Cam Clay model (see Methods). The slab's layers were assigned thicknesses and densities matching the measured snow profile shown in Figure 5 for a mean density of $104 \text{ kg} \cdot \text{m}^{-3}$. They are modelled as elastic, with a Young's modulus derived from Sigrist's (Sigrist, 2006) formula and a Poisson's ratio of 0.2. The details of the snow layers' thickness and density can be found in Table C4. The complete snow profile is presented in Figure B8. We additionally performed a PST simulation on a 15° slope to provide a sub-Rayleigh
180 reference case for comparison with the supershear regime.

For the 37° slope (Figure 6A), the regime transition occurs at approximately 5 m, from a sub-Rayleigh speed of approximately $60 \text{ m} \cdot \text{s}^{-1}$ to a supershear asymptotic speed of $137 \text{ m} \cdot \text{s}^{-1}$ (measured between 45 and 55 meters), about three times



higher than the 15° slope sub-Rayleigh crack speed ($50 m.s^{-1}$). These values agree well with the experimental estimates of a
6 m super critical crack length, a $50 m.s^{-1}$ sub-Rayleigh speed and a $130 m.s^{-1}$ supershear speed. Additionally, the collapse
185 height of the simulated weak layer is around 7 mm (see Figure B5), close to the one observed in the experiment (8 – 9 mm).

We next analyze this propagation regime using a single equivalent shear wave speed representative of the layered slab.
Because Eq. (4) is defined for the idealized three-layer configuration of Figure 4, we introduce a more general description
based on the density gradient across the slab thickness $\nabla\rho$, defined by :

$$\nabla\rho = -\frac{d\rho}{dy} \mathbf{y} = -\nabla\rho \mathbf{y} \quad (6)$$

190 with $\mathbf{y}$ the unit vector in the y direction. With this definition, $\nabla\rho$ is of same sign as γ .

For the three-layer slab of Figure 4, we can write :

$$\nabla\rho = \frac{\rho_t - \rho_b}{d_t - d_b} \quad (7)$$

with ρ_t and ρ_b the densities of the top and bottom layers respectively, and d_t and d_b their respective mean position within the
slab thickness. By construction,

$$195 \quad \begin{cases} \rho_t = \rho^m(1 - \gamma) \\ \rho_b = \rho^m(1 + \gamma) \end{cases} \quad \text{and} \quad \begin{cases} d_t = D \times 5/6 \\ d_b = D \times 1/6 \end{cases} \quad (8)$$

with d_t and d_b respectively the distance of the top and bottom layers from the weak layer, taken perpendicular to the ground.
The distance is taken in the middle of the layers.

This then gives

$$\nabla\rho = \frac{3\gamma\rho^m}{D} \quad (9)$$

200 Equation 7 can then be inserted into equation 4 to make it independent of γ

$$c_s^\bullet = c_s^m \left[1 + 3 \left(\frac{D\nabla\rho}{\rho^m} \right)^2 \right] \quad (10)$$

with

$$3 = \frac{\zeta}{9} = \begin{cases} 0.089 & ; \quad \nabla\rho \geq 0 \\ 0.061 & ; \quad \nabla\rho < 0 \end{cases} \quad (11)$$

We compute the density gradient in Bergfeld's experiment with a linear regression on the layer densities to obtain $\nabla\rho_B =$
205 $185 kg.m^{-4}$ (see Figure B6). Hereafter, the subscript $_B$ denotes quantities related to Bergfeld's experiment. Given that $D_B =$
 $0.663 m$ and $\rho_B^m = 104 kg.m^{-3}$, we obtain $c_{s_B}^\bullet = 90 m.s^{-1}$. We use this value to normalize the crack tip speed in Figure 6B
along with an equivalent elastic wave speed. The normalized speed also converges to $c_{e_B}^\bullet / c_{s_B}^\bullet$ supporting the relevance of this
effective wave speed normalization for layered slabs.



Regarding deformation patterns, Figure 6D shows the presence of a dilatation lobe as well as a supershear Mach cone, both being curved. This is due to the variation of material properties between the layers, which affect the shear wave speed and thus the cone angle, following equation (1). Extending the analogy with earthquakes and wave propagation in layered media, this curvature can be interpreted as a consequence of refraction at the interface between layers of different wave speeds. In particular, the refraction relation between two layers can be derived directly from the expression for the Mach-cone angle:

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{\frac{c_{s2}}{a}}{\frac{c_{s1}}{a}} = \frac{c_{s2}}{c_{s1}} \quad (12)$$

This relation can also be derived from the Snell-Descartes formula for refraction and the geometric properties of the waves, as done in the Methods Section.

Such curved Mach cones are also visible in the previous 3-layered slab simulations. Because the cone angle depends on the local shear-wave speed, the resulting Mach cone becomes convex for $\gamma > 0$ and concave for $\gamma < 0$. This can be seen in Figure B3 where theoretical Mach cones are plotted on close-ups of the simulations of Figure 4.

Although the shock front becomes sharp when we reach a quasi-steady state, it remains poorly defined during the transition phase, as can be seen in Figure 6C, where we show a close up of the slab for a crack propagation distance of $8.5m$. This distance was chosen to portray the relative position of the camera from Bergfeld's experiment, which was positioned $2.5m$ after the super critical crack length. In this simulation, the cone is not clearly identifiable until about $12m$, approximately $7m$ after the transition (see Supplementary Movie 1). In addition, the close field camera only focused on a small region around the weak layer (delimited by the green box in Figure 6C-D), and not the entire slab thickness.

In summary, the numerical twin provides further support for the supershear character of the crack observed by Bergfeld et al. (2025). It also yields a practical effective wave-speed formulation for layered slabs and reveals an intersonic curved Mach cone during crack propagation. Finally, it clarifies why this feature could not be observed in the original experiment: capturing it would require a longer PST column and high resolution imaging of the full slab thickness.

3 Discussion

In 2022, Trottet et al. reported and theoretically explained the transition between sub-Rayleigh and supershear fracture regimes in the release of snow slab avalanches using MPM numerical simulations and Burridge's (Burridge, 1973) and Andrews' (Andrews, 1976) work. Indirect validation was carried out using avalanche surface observations from video recordings. In 2024, Bobillier et al. further supported these predictions using Discrete Element Method (DEM) simulations. Bergfeld et al. (2025) provided the first direct experimental proof of this phenomenon with a 10 meter-long PST experiment. Here, we further consolidate this theory with new MPM simulations revealing intersonic Mach cones during supershear propagation of slab avalanches, as well as a numerical twin of Bergfeld's experiment supporting the supershear nature of the crack propagation that was observed. We further demonstrate how slab layering influences crack propagation and the Mach cone shape, which can become curved—either concave or convex—depending on the density gradient, due to wave refraction at layer interfaces. Additionally, we provide practical guidance for future experiments to better capture supershear regimes and potentially observe



shock fronts during crack propagation in large-scale experiments. Together, these results strengthen the evidence for supershear crack propagation in snow and show that slab stratification must be considered when interpreting shock-front geometry.

The numerical model employed in this study was intentionally kept simple to reduce computational cost and memory requirements. Its two main limitations are the assumption of a rigid substratum and a purely elastic slab. Neglecting the cushioning effect of the substratum on the crack affects the super critical crack length a_{sc} (see Figure B7). However, it does not affect the steady state crack tip speed. Additionally, including slab plasticity, would allow tensile fractures to develop, an important feature for reproducing "en-echelon" fracture or crack arrest possibly caused by a slab fracture. In particular, it could improve the numerical twin of Bergfeld's experiment by reproducing the tensile failure observed in their test, which still did not result in crack arrest. Such "en-echelon" fractures with continued crack propagation were simulated in Trottet et al. (2022).

Because the present study is two-dimensional, it resolves propagation only along the slope direction. On natural slopes, crack propagation also develops across the slope, where a supershear transition is not expected. Mach cones should therefore primarily arise in the downslope and upslope propagation directions, whereas cross-slope propagation should mainly generate dilatational lobes.

These results further suggest that the transition from sub-Rayleigh to supershear fracture might be detectable using ground-based seismo-acoustic measurements. In that case, the supershear regime should be characterized by a bulk-wave arrival followed by a sharp, high-energy shear shock front, whereas the sub-Rayleigh regime should exhibit only the bulk-wave signature. This approach could be applied to large slopes (>10 m) and performed entirely remotely, offering a practical alternative to PST experiments, which are challenging to conduct at scale and pose significant avalanche risk (Bergfeld et al., 2025).

Data availability. All experimental data are available at the following public link <https://doi.org/10.16904/envidat.679>

Video supplement. Supplementary Movie 1 represents an animated version of Figure 6.

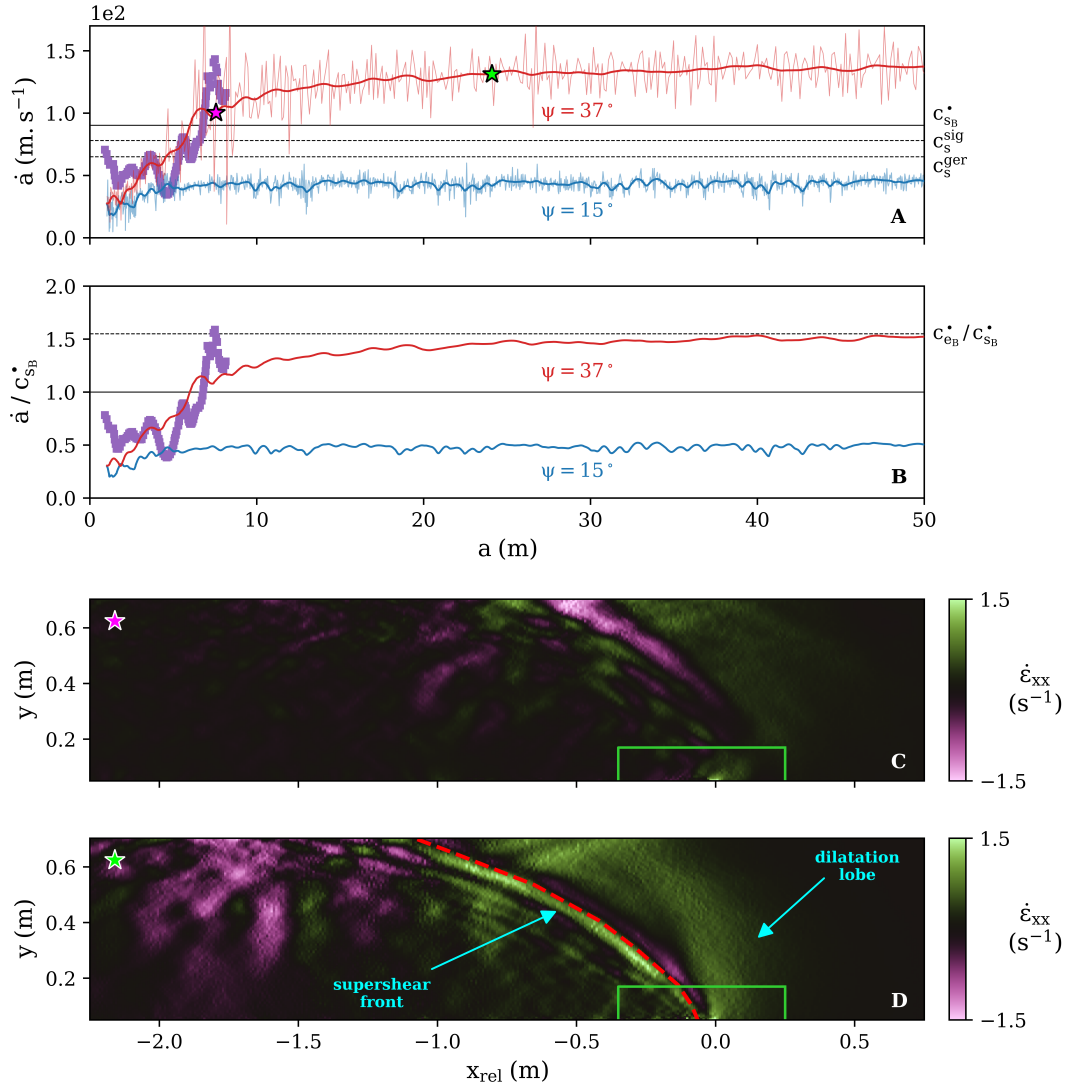


Figure 6. Numerical twin of the experiment from (Bergfeld et al., 2025). Identical 60 m long PST experiment at slope angles of 15° and 37° , for which only the first 50 meters are displayed. **(A)** Crack tip speed ($\dot{a}$) over crack tip position (a) along the x -axis. The low and high opacity lines correspond to the original and smoothed crack tip speed, respectively. The crack tip position is measured by taking the average x position of the five furthest plasticized particles. $c_s^{sig} = 78 \text{ m.s}^{-1}$ and $c_s^{ger} = 65 \text{ m.s}^{-1}$ are two equivalent shear wave speeds evaluated in Bergfeld et al. (2025) for this set-up, according to Sigrist's (Sigrist, 2006) and Gerling's (Gerling et al., 2017) formulas. **(B)** Crack tip speed normalized by the equivalent shear wave speed $c_{sB}^* = 90 \text{ m.s}^{-1}$. For **(A)** and **(B)**, the crack tip speed measurements of Bergfeld's experiment are plotted in purple squares. **(C)** and **(D)** Close up views of the slope parallel strain rate in the slab for two close up positions indicated by the pink and green stars in **(A)**. The green rectangle illustrates the field of view of the camera in Bergfeld's experiment. The red dotted line in **(D)** represents the theoretical Mach cone.



Appendix B: Methods

A1 Mach cone angle

A Mach cone is a consequence of the Doppler effect: the apparent frequency, and therefore the spacing of emitted wavefronts, depends on the relative motion between the source and the observer. For an observer looking at the moving object perpendicular to its movement from an infinite distance, as in Figure A1, it can be interpreted as a contraction of the wavelengths in front of the object and a dilatation at its back.

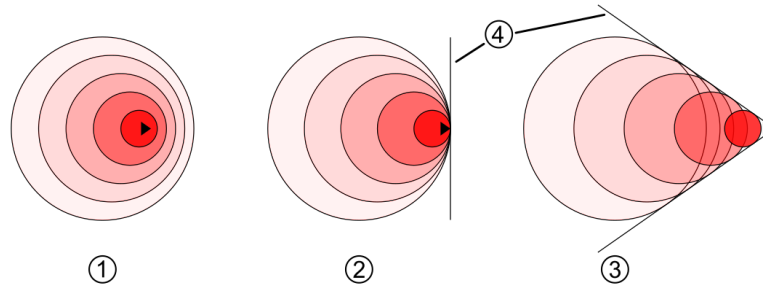


Figure A1. Doppler effect for three speed regimes. (1) Subsonic, (2) sonic, (3) supersonic. The sonic and supersonic regimes create a Mach cone (4). Credit: Pbroks13 - Own work, CC BY-SA 3.0, <https://commons.wikimedia.org/w/index.php?curid=5060487>

However, when the object moves faster than the waves, the former stays in front of the latter. We can demonstrate (in 2D here) that in this situation, the emitted wavefronts pile up along a unique common tangent, thereby forming a shock front.

Consider a point source moving along a line at a constant speed of $\dot{a}$, represented in Figure A2. The object emitted a wave of speed $c_s < \dot{a}$ at the position $O_1(a = \dot{a}t_1)$ at a time t_1 and is now at the position $O_t(a = \dot{a}t)$ at a time t . Between times t_1 and t , the wave and the object have moved by distances of $c_s(t - t_1)$ and $\dot{a}(t - t_1)$ respectively. We select a tangent of the wave at a point P that passes through O_t (which is always outside the circle given $c_s < \dot{a}$). By construction, the tangent in P is

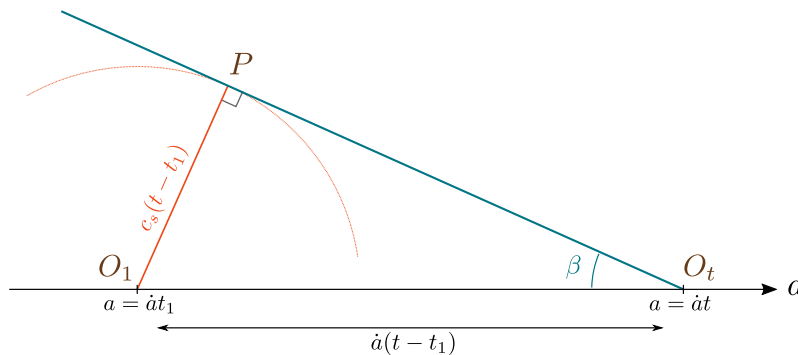


Figure A2. Single wave emitted by a point moving in a straight line and at a constant speed.



perpendicular to the ray of the wave. Thus, in the right-angled triangle O_1O_tP , the angle between the tangent segment $[O_tP]$ and the segment in the direction of movement $[O_1O_t]$ is given by

$$\sin \theta = \frac{\overline{O_1P}}{\overline{O_1O_t}} = \frac{c_s(t - t_1)}{\dot{a}(t - t_1)} = \frac{c_s}{\dot{a}} \quad (\text{A1})$$

This relation is valid for all values of $t > t_1$. All waves emitted by the object between t_1 and $t > t_1$ have this tangent in common. The uniqueness of the common tangent is given by the angle θ which is the same for all tangents of all waves considered above.

A2 Numerical Model

Simulations are performed with the Material Point Method (MPM), a hybrid Eulerian–Lagrangian continuum approach in which the material is discretized into Lagrangian particles carrying mass, momentum, and state variables, while the governing equations are solved on a background Eulerian grid. This combination allows accurate treatment of large deformations without mesh distortion, making MPM particularly suitable for the simulation of crack propagation in snow slab avalanches. More details about the MPM scheme can be found in Gaume et al. (2018).

Regarding the constitutive models, the slab is modeled as elastic, whereas the weak layer is modeled as elastoplastic. To model elasticity, we use Hencky's finite strain elasticity

$$\underline{\underline{\tau}} = \Lambda \text{tr}(\underline{\underline{\epsilon}}^e) \underline{\underline{I}} + 2G \underline{\underline{\epsilon}}^e \quad (\text{A2})$$

$$\underline{\underline{\epsilon}}^e = \frac{1}{2} \ln(\underline{\underline{F}}^e \underline{\underline{F}}^{eT}) \quad (\text{A3})$$

with $\underline{\underline{\tau}}$ the Kirchhoff stress, $\underline{\underline{\epsilon}}$ the Hencky strain, $\underline{\underline{\epsilon}}^e$ the elastic Hencky strain and Λ and G elasticity coefficients, analogous to Lamé's coefficients in small strains. Plasticity is introduced with the Cohesive Cam-Clay model, an adaptation of the Modified Cam-Clay model, itself derived from the original Cam-Clay framework..

The Cam-Clay model, initially developed for clays and later extended to granular materials is based on the critical state theory (Andrew Schofield and Peter Wroth, 1968; Muir Wood, 1992), which assumes that, at large deformations, the material reaches a critical state characterized by constant volume fraction during continued deformation.

The Cohesive Cam-Clay model relies on an elliptically shaped yield function, a flow rule, and a hardening law. The yield surface is given by

$$Y(p, q) = q^2(1 + 2\beta) + M^2(p + \beta p_0)(p - p_0) = 0 \quad (\text{A4})$$

where p and q are respectively the volumetric and deviatoric stresses, $M > 0$ is the slope of the so-called critical state line, $p_0 > 0$ the pre-consolidation pressure and β the tension/compression ratio, interpretable as a cohesion ratio.

An associative plastic flow rule is used

$$\dot{\underline{\underline{\epsilon}}}^P = \dot{\lambda} \frac{\partial Y}{\partial \underline{\underline{\tau}}} \quad ; \quad \dot{\lambda} \geq 0 \quad (\text{A5})$$



with ϵ_V^P the plastic strain and λ a Lagrange multiplier.

The hardening law proposed by Gaume et al. (2018), derived from Ortiz and Pandolfi (2004), assumes the hardening to be a function of the volumetric plastic deformation only :

$$305 \quad p_0 = K \sinh(\xi \max(-\epsilon_V^P, 0)) \quad (A6)$$

where K is the bulk modulus and ξ is the hardening factor. In finite strain theory, we assume $\underline{\underline{F}} = \underline{\underline{F}}^E \underline{\underline{F}}^P$. Thus, we can define a plastic jacobian $J^P = \det(\underline{\underline{F}}^P)$ and write $\epsilon_V^P = \log(\det(\underline{\underline{F}}^P))$. To represent the brittle collapse of the weak layer as well as residual friction, Gaume et al. (2018) adapted the previous law to allow for softening under compressive loading using a so-called *anticrack plastic strain*

$$310 \quad \dot{\eta} = \begin{cases} \alpha |\dot{\epsilon}_V^P| & ; \quad t \leq t_c \\ \dot{\epsilon}_V^P & ; \quad t > t_c \end{cases} \quad (A7)$$

where α is the *softening factor*, controlling the energy dissipated during fracture, and t_c is the time of total softening, corresponding to $\dot{\epsilon}_V^P = 0$ and $p_0 = 0$. Hence, η replaces ϵ_V^P in equation (A6).

In the present work, we simplify equation (A7) by assuming instantaneous softening of the weak layer, i.e. by setting p_0 to zero at the first occurrence of plasticity. It is equivalent to considering equation (A7) with $\alpha \rightarrow +\infty$ and $t_c = t_0$ with t_0 the time of
315 first occurrence of plasticity.

A3 Shear wave refraction

A shear wave passing through different materials can be studied as a ray, for which the laws of geometrical optics can be applied. One such law is refraction. When a ray crosses an interface between two materials of different wave properties, it is refracted according to Snell-Descartes' law, which relates the incident and refracted ray angles with the wave speeds in the
320 materials according to the following relation :

$$\frac{\sin r}{\sin i} = \frac{c_r}{c_i} \quad (A8)$$

with r and i respectively the refracted and incident angles and c_r and c_i the wave speed in the final and initial material. In the case of two layers in the snow slab, for which a schematic example is presented in Figure A3, this relation becomes

$$\frac{\sin r}{\sin i} = \frac{c_{s2}}{c_{s1}} \quad (A9)$$

325 If we consider the shear ray perpendicular to the shock front and take into account that the normal at the incidence point is also perpendicular to the slope (Figure A3), we can write

$$\theta_1 + 90^\circ = i + 90^\circ \quad (A10)$$

Thus $\theta_1 = i$ and $\theta_r = r$ and equation (A8) becomes

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{c_{s2}}{c_{s1}} \quad (A11)$$

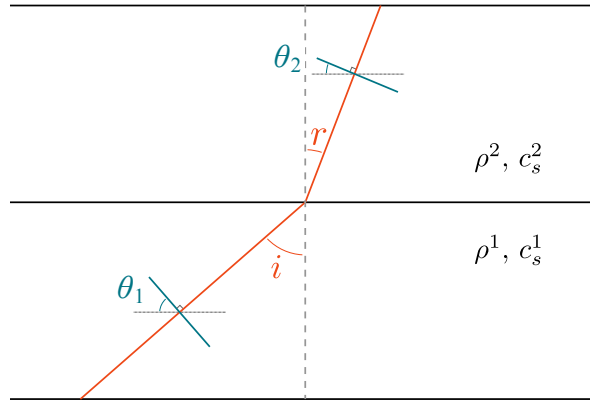


Figure A3. Refractive behaviour of a shear wave, analogous to geometrical optics. The black lines delimit the two layer crossed by the shear ray (orange), perpendicular to the shock front, for which the direction is represented by the blue lines.

330 A4 Identification of Mach cones

The Mach cones in the slab are identified using a python program. Figure A4 gives an example of the identification algorithm.

1-The bottom and top parts of the slab are removed. 2-A layer of thickness $3dx$ is isolated from the rest of the selected slab area. 3-The x position of the extremum value of either $\dot{\epsilon}_{xx}$ or $\dot{\epsilon}_{yy}$ is calculated and associated with the average y position of the layer. 4-The same operation is executed on the whole sample, with a loop sliding the selected layer by $1dx$ at every iteration.
335 5- A first coarse filtering is operated to remove out-of-bounds points. The points are filters by their distance to the theoretical cone. 6- A first linear regression of the remaining sample is made, and used to operate a second and finer filtering, removing diverging points. This last process can be repeated up to 4 times to ensure that all and only diverging points are removed. Finally, a last linear regression is made to obtain the slope angle and the Pearson correlation factor ($r^2 > 0.993$).

Please note that, in Figure A4, the identification is made using $\dot{\epsilon}_{xy}$, a more noisy and imprecise quantity than $\dot{\epsilon}_{xx}$ or $\dot{\epsilon}_{yy}$, to
340 provide with a more visual example. In reality, as mentioned before, the identification was operated using either $\dot{\epsilon}_{xx}$ or $\dot{\epsilon}_{yy}$, which feature sharper fronts and produce significantly fewer outliers. See Figure B2 for comparison of cone sharpness between $\dot{\epsilon}_{xy}$ and $\dot{\epsilon}_{xx}/\dot{\epsilon}_{yy}$. Moreover, not all points are shown on Figure A4 for the purpose of clarity.

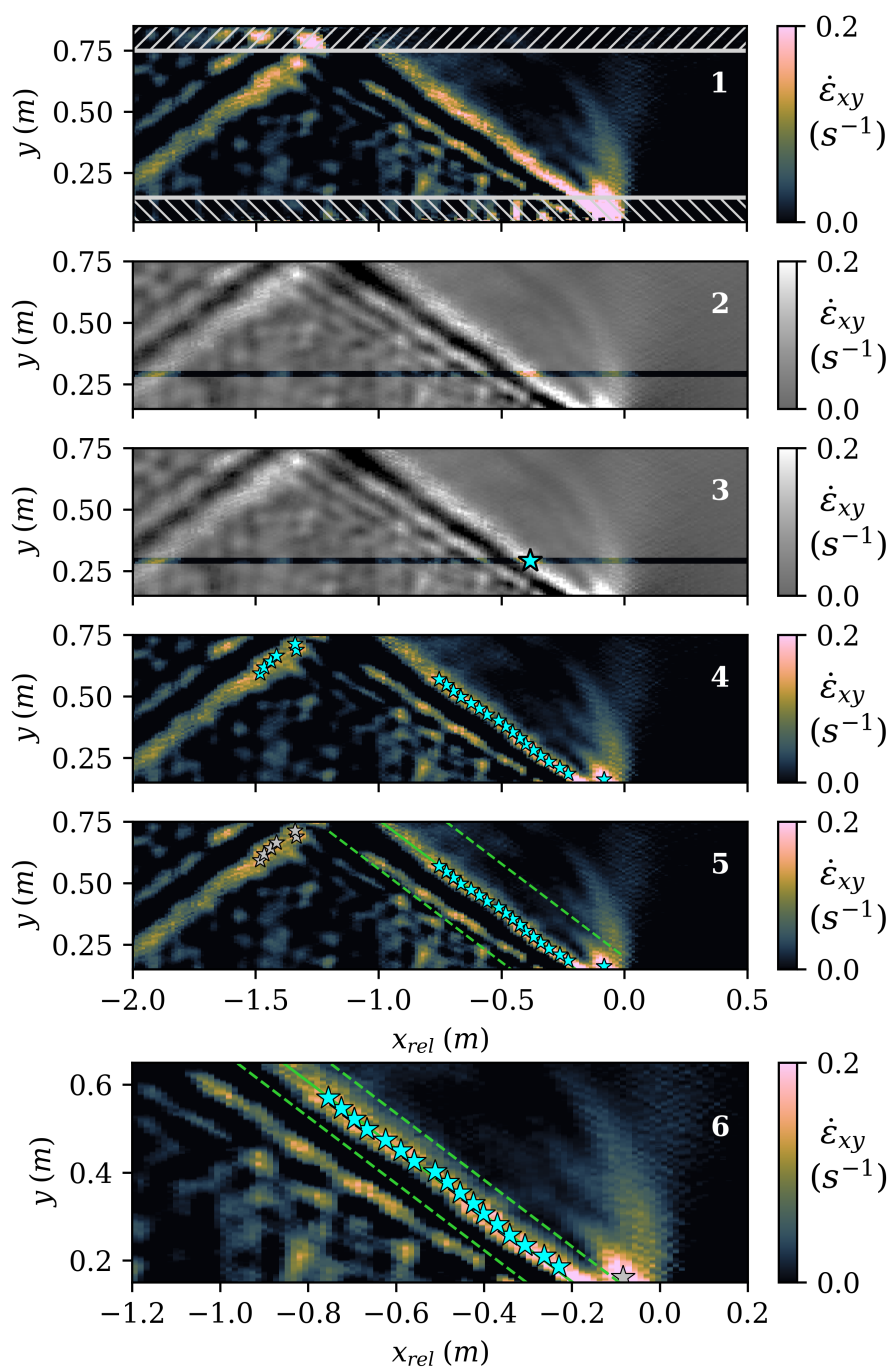


Figure A4. Mach cone identification example

according to the Method described in Appendix A4.



Appendix B: Supplementary figures

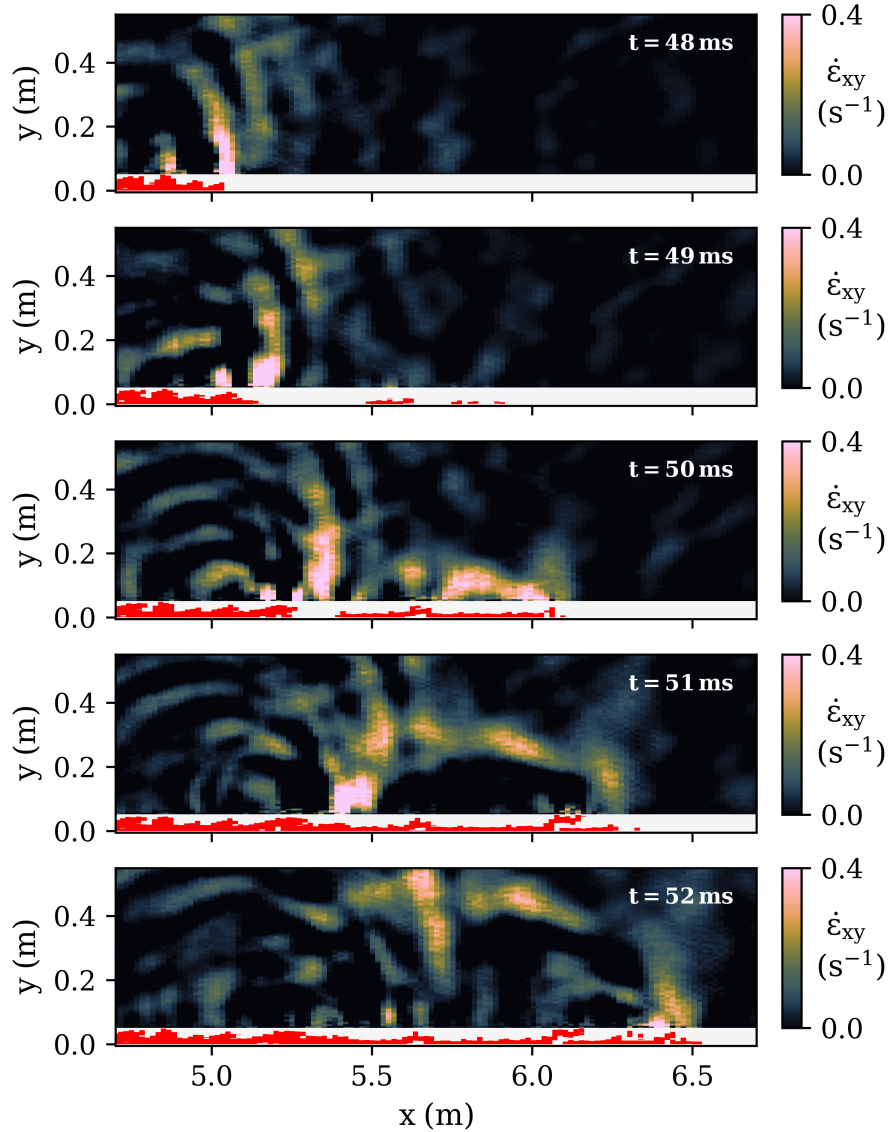


Figure B1. Daughter crack emergence at the transition between sub-Rayleigh and supershear regime. The weak layer is represented in white for the "intact" part and red for the crack. The upper part is the slab. $t = 48 \text{ ms}$, crack propagating at a sub-Rayleigh speed. $t = 49 \text{ ms}$, first appearance of a daughter crack approximately 1 m ahead of the parent crack. $t = 50 \text{ ms}$, daughter crack propagates in both directions. $t = 51 \text{ ms}$, parent and daughter crack connecting. $t = 52 \text{ ms}$ The new crack propagates at an increased speed, leaving behind the fading sub-Rayleigh lobe front.

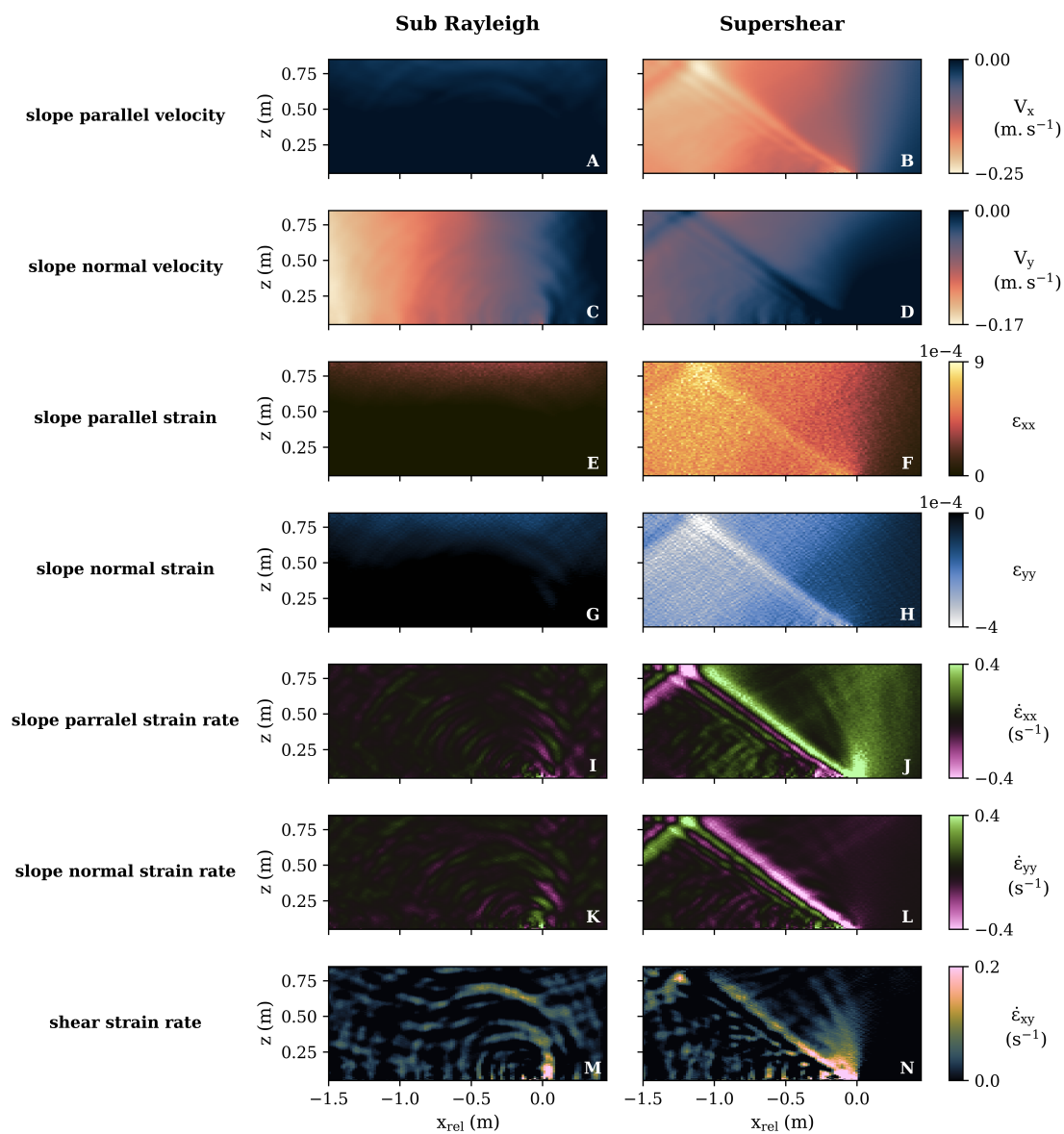


Figure B2. Additional components of movements in the slab for sub-Rayleigh and supershear failure. The close-ups are identical to those of Figures 2 and 3, the sub-Rayleigh and supershear close-ups corresponding to the low ($\psi = 20^\circ$) and high ($\psi = 40^\circ$) slope angle simulations.

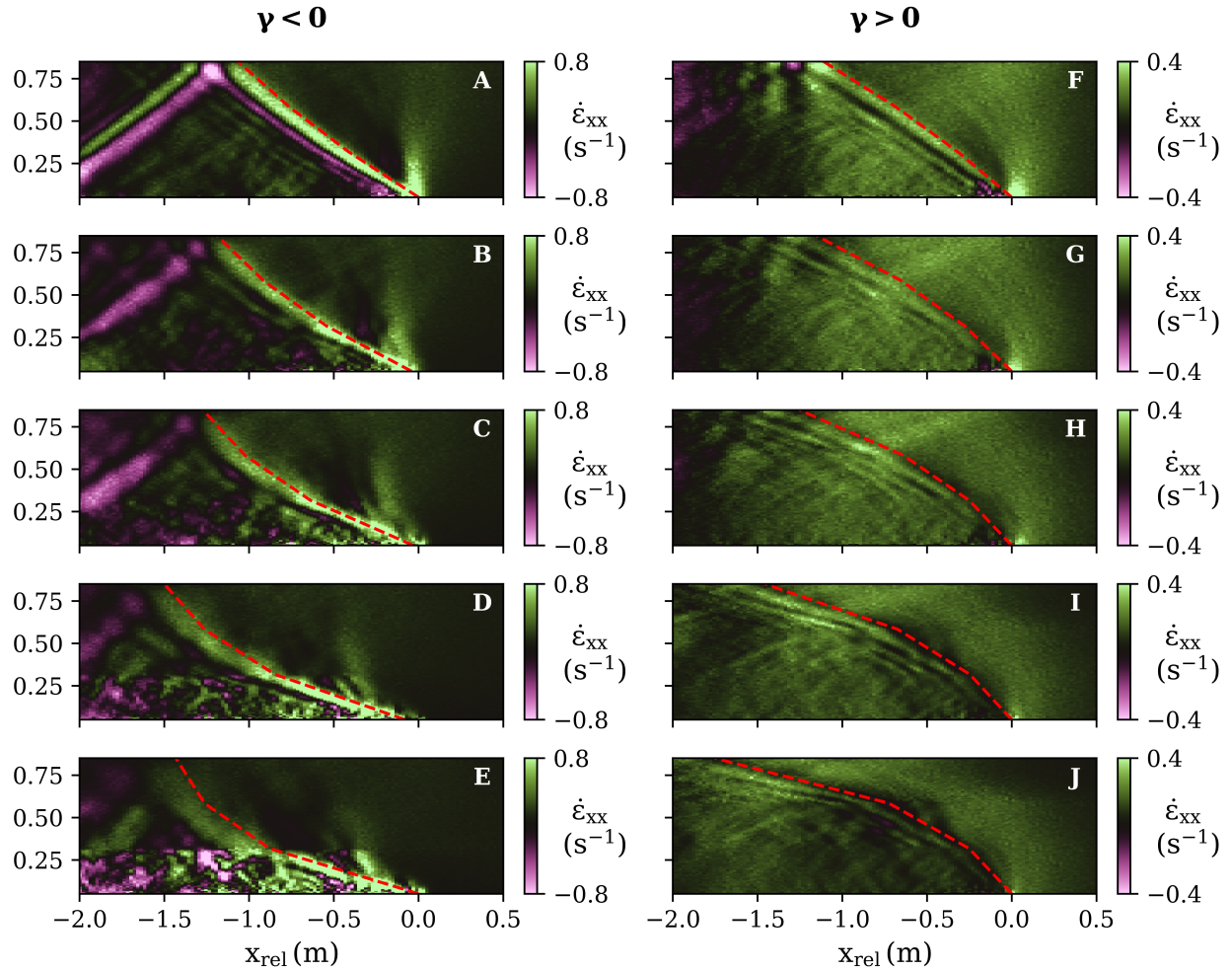


Figure B3. Concave and convex cone fronts at crack position $a = 45\text{ m}$ in the 3-layered slab simulations presented in Figure 4 (A-E) $\gamma = -0.1$ to -0.5 respectively. (F-J) $\gamma = 0.1$ to 0.5 respectively. The dotted line represents the theoretical shape of the Mach cone, computed from the wave properties of each layer and the average crack speed between $a - 1.5\text{ m}$ and $a + 0.5\text{ m}$.

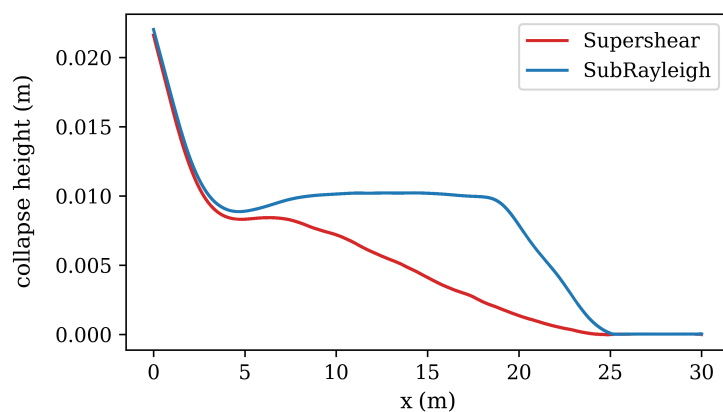


Figure B4. Collapse heights of the weak layer for the simulations presented in figures 2 and 3. The collapse height is derived by measuring the total vertical displacement of the layer of slab directly above the weak layer. The layer thickness for measurement is of $3dx$.

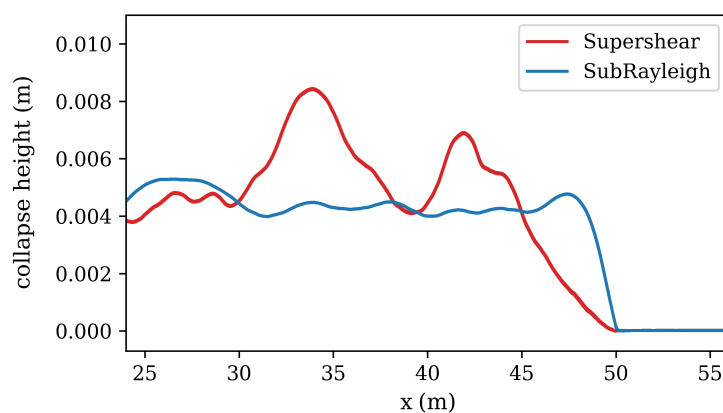


Figure B5. Collapse height of the weak layer for the numerical twin of Bergfeld's experiment (Figure 6). The collapse height is derived by measuring the total vertical displacement of the layer of slab in contact with the weak layer. The layer thickness for measurement is of $3dx$.

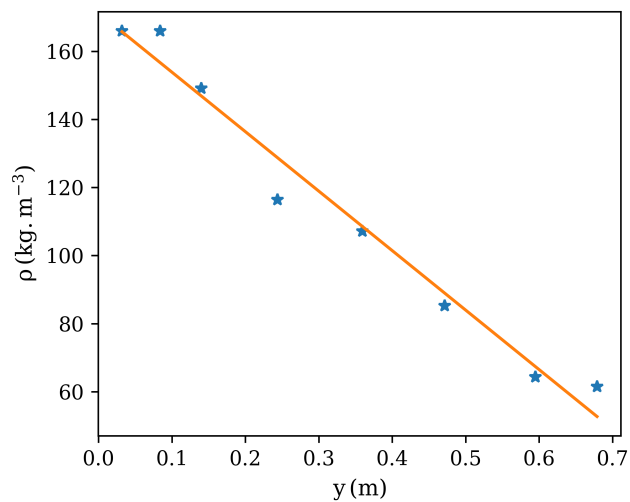


Figure B6. Linear regression on the layer densities in Bergfeld's experiment. The layer heights are taken in the middle of said layers. The correlation coefficient for the regression is $r^2 = 0.97$.

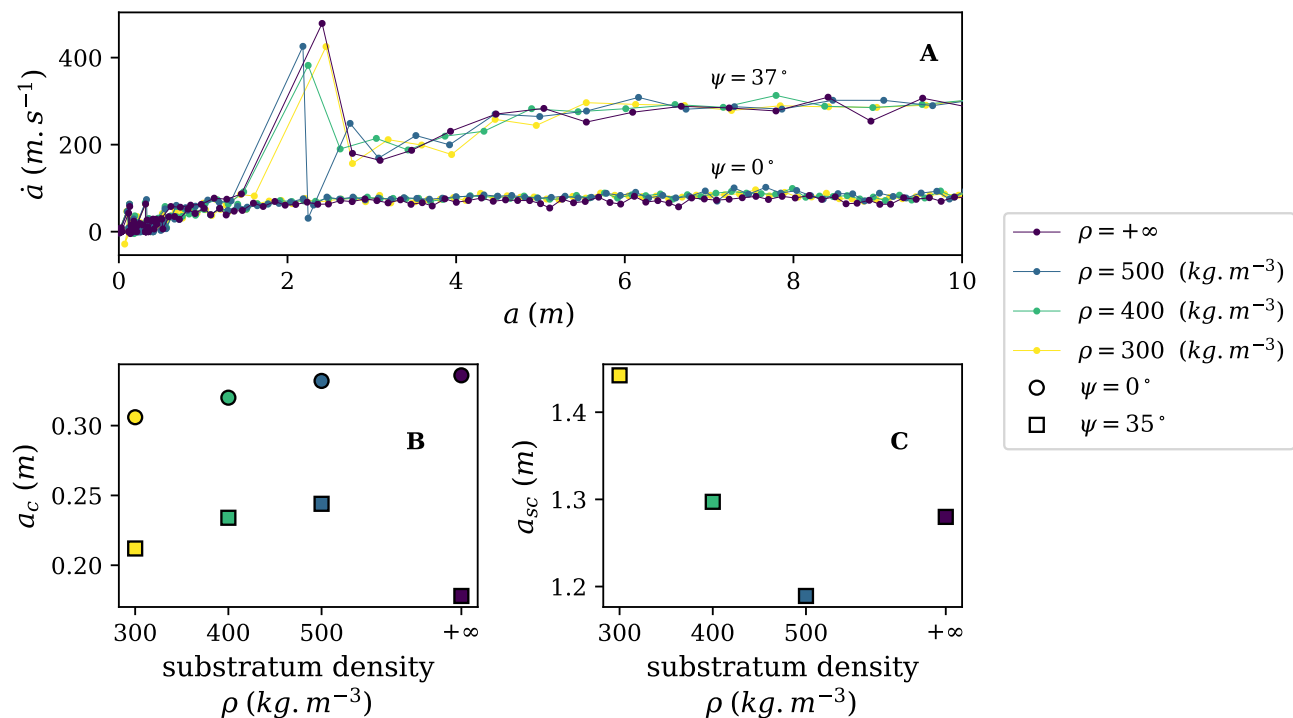


Figure B7. Influence of substratum on crack tip propagation behavior. (A) Crack speed according to crack tip position for 0° and 37° slope angles for different substratum densities, leading to respectively sub-Rayleigh and sub-Rayleigh transitioning to supershear regimes. (B) Critical saw cutting length before self propagation of the failure, for the simulations in A. (C) Supercritical crack length before transition to supershear regime, for the simulations in A. $\rho = +\infty$ corresponds to the absence of a substratum. The a_c and a_{sc} values with no substratum appear as outliers. This comes from the difference of contact interaction. Without a substratum, the slab rests on the ground with a friction interaction, instead of being led by a constitutive law in the presence of a substratum. Simulation parameters can be found in Table C3.

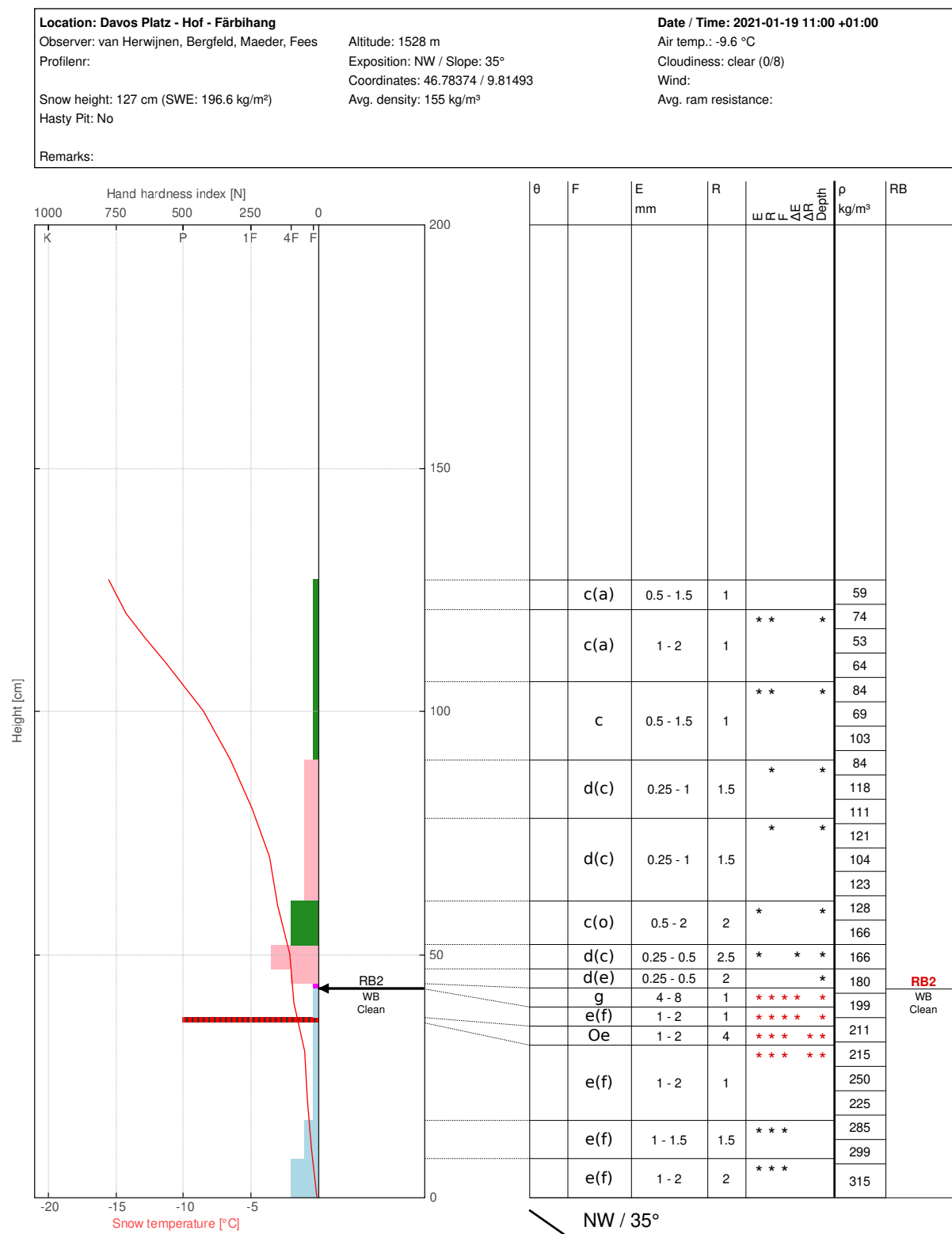


Figure B8. Snow profile of the laboratory PST experiment of Bergfeld et al. (2025) The weak layer is indicated by the black arrow and "RB2" legend.



Appendix C: Supplementary tables

Table C1. MPM simulation parameters for the slab simulations presented in Figures 2 and 3 and Figures B1 to B4.

MPM Simulation Parameters		Slab	Weak Layer
Geometry	Thickness (m)	0.8	0.05
	Length (m)		30
	Slope angle (°)		0, 30
Material properties	Poisson's ratio ν		0.3
	Density ρ ($kg.m^{-3}$)	250 ¹	100
	Young's modulus E (MPa)	21.2* ²	3.0
	Initial consolidation pressure p_0 (kPa)	-	35
	Tension/compression ratio β	-	0.3
	Friction coefficient M	-	1.2
	Hardening factor ξ	-	0.005
Numerical model	Element size dx (cm)		0.8 ³
	Mean number of particles per cell		4
	CFL		0.4 - 0.6
	Frame rate (s^{-1})		1000

* Derived according to Sigrist's formula $E = 1.89\rho^{2.94}$.

¹ Except for Figure 4 where $\rho \in [125, 375]$

² Except for Figure 4 where $E \in [2.76, 69.84]$

³ Except for Figure B2 where $dx \in [0.5, 2]$



Table C2. MPM simulation parameters for Bergfeld’s numerical twin in Figure 6 and Figure B6.

MPM Simulation Parameters		Slab	Weak Layer
Geometry	Thickness (m)	0.66	0.05
	Length (m)	60	
	Slope angle (°)	15 – 37	
Material properties	Poisson’s ratio ν	0.2	
	Density ρ (kg m ⁻³)	104.5 ¹	100
	Young’s modulus E (MPa)	1.61 ^{*1}	2
	Initial consolidation pressure p_0 (kPa)	-	25
	Tension/compression ratio β	-	0.45
	Friction coefficient M	-	1.35
	Hardening factor ξ	-	0.0024
Numerical model	Element size dx (cm)	0.8	
	Mean number of particles per cell	4	
	CFL	0.6	
	Frame rate (s ⁻¹)	1000	

* Derived according to Sigrist’s formula $E = 1.89\rho^{2.94}$.

¹ For the equivalent mean slab of density $\rho^m = 104.5 \text{ kg.m}^{-3}$, see Table C4.



Table C3. MPM simulation parameters for Figure B7.

MPM Simulation Parameters		Slab	Weak Layer
Geometry	Thickness (m)	0.5	0.05
	Length (m)	20	
	Slope angle (°)	0 - 35	
Material properties	Poisson's ratio ν	0.3	
	Density ¹ ρ (kg m ⁻³)	300 - 500	100
	Young's modulus* ² E (MPa)	36.2 - 162.7	2.0
	Initial consolidation pressure p_0 (kPa)	-	20
	Tension/compression ratio β	-	0.2
	Friction coefficient M	-	1.0
	Hardening factor ξ	-	0.008
Numerical model	Element size dx (cm)	1.5	
	Mean number of particles per cell	4	
	CFL	0.4	
	Frame rate (s ⁻¹)	500	

¹ ρ values include 300, 400, 500.

² E values include 36.2, 84.4, 162.7.

Table C4. Layering parameters for the numerical twin of Bergfeld's laboratory experiment (Figure 6), from bottom (Layer 1) to top (Layer 8). The total thickness is 663 mm

	Thickness (mm)	Density (kg.m ⁻³)	Young's modulus (MPa)
Weak Layer	50	100	5.0
Layer 1	24	180	8.07
Layer 2	40	166	6.36
Layer 3	72	149	4.64
Layer 4	136	116	2.24
Layer 5	96	107	1.76
Layer 6	128	85	0.90
Layer 7	120	64	0.39
Layer 8	48	61	0.34



345 *Author contributions.* A.P. performed the numerical simulations and wrote the paper. B.B and A.H. performed the experiment. J.G. supervised the work of A.P.

Competing interests. The authors declare no competing interests.

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