

In this study, the authors use two-dimensional numerical simulations based on the Material Point Method (MPM) to investigate the characteristics of Mach cones formed during crack propagation in weak snow layers. First, they identify and characterize Mach cones using a single-layer slab. Next, they conduct numerical experiments assuming an idealized three-layer slab structure and propose empirical equations describing the influence of the layered structure on crack propagation and the shape of Mach cones. Furthermore, these results are compared with those from field tests.

The study and its main findings presented in this paper are generally easy to understand, and I believe that the scientific content is suitable for publication in *The Cryosphere*. However, regarding the relationship between the idealized slab structure considered in this study and the fracture propagation process in actual snowpacks, I would like the authors to clarify the physical interpretation of the numerical results and the range of applicability of the proposed empirical equations.

Specifically, when comparing Figures 4A and 6A, the supercritical crack length can be identified relatively clearly in Figure 4A, whereas in Figure 6A, this feature becomes less clear, and the crack propagation velocity appears to transition rapidly and smoothly into the supershear regime. Does this difference suggest that it is important to consider a more complex layered structure to accurately reproduce crack propagation in actual snowpacks? Alternatively, do the authors believe that the empirical equations proposed in this paper are sufficient to account for the influence of the layered structure when evaluating crack propagation velocity?

In addition, I believe there is room for improvement in the overall structure of the paper and the presentation of figures. The “Manuscript Composition” guidelines from Copernicus Publications (https://publications.copernicus.org/for_authors/manuscript_preparation.html) state the following regarding appendices:

“all material required to understand the essential aspects of the paper such as experimental methods, data, and interpretation should preferably be

included in the main text. Additional figures, tables, as well as technical and theoretical developments which are not critical to support the conclusion of the paper, but which provide extra detail and/or support useful for experts in the field and whose inclusion in the main text would disrupt the flow of descriptions or demonstrations may be presented as appendices.”

In this paper, many methodological explanations and figures are placed in the appendices. Consequently, readers must repeatedly switch between the main text and the appendices to understand the research content, making the paper less reader-friendly. In particular, some of the methods described in the appendices appear to be important for understanding the novelty and key results of this study. Therefore, I would like the authors to consider moving this content into the main text.

Some figures also seem to lack sufficient explanation. For example, in Figures 2B–E, there is no clear explanation of what x_{rel} on the horizontal axis represents. I assumed that the line shown near $a = 25$ m in Figure 2A might correspond to $x_{rel} = 0$. However, since this line itself is not explained, I cannot determine whether my interpretation is correct. Please clearly define the coordinates. Other comments regarding the figures are provided in the “Specific Comments” section below.

Additionally, the current “Discussion” section appears to be closer to a “Conclusions” section, as it mainly summarizes the key findings of the study. Moreover, there is no separate “Conclusions” section. Therefore, I would appreciate it if the authors could reconsider the organization of these sections and clarify their respective roles.

Specific Comments

1. Line 12: The citation “Ava” appears to be incomplete. Please check it.
2. Line 25: Please check whether the citation to McClung (1979) is correct.

3. Lines 31 and 49–50: Nearly identical explanations appear in two places, one in parentheses and the other in the main text. Is there a specific reason for this difference?
4. Line 100: The main text states “ten furthest plasticized,” whereas the caption of Figure 2 states “five furthest plasticized.” Please clarify which is correct.
5. Lines 126–128: The explanation of Figure B4 is insufficient, making the figure difficult to interpret. Is my understanding correct that $x = 25$ m corresponds to “final closure,” while the regions near $x = 19$ m and $x = 5$ m correspond to “the tip of the crack” in the sub-Rayleigh and supershear regimes, respectively? Furthermore, the collapse height appears to be almost zero for $x > 25$ m. Does this mean that no displacement occurs beyond $x = 25$ m? Please clarify the relationship between the positions shown in the figure and their physical interpretations.
6. Line 134: Should the interval be written as $[-0.5, 0.5]$? Please check.
7. Figure 4: The caption states that V_{sc} was averaged over the values at 75 m and 85 m. However, according to Table C1 (which does not explicitly indicate that the calculation conditions correspond to Figure 4), the length of the computational domain appears to be 30 m. Even if the computational domain were larger, Figure 4A suggests that its length is 50 m or less. Please clarify this discrepancy. Furthermore, I could not find a clear statement of the slope angle. Is it 30° ?
8. Line 137: A unit “MPa” appears to be included in Equation (2). Please check whether the units are expressed appropriately in the equation.
9. Line 140: The text states, “we first observe that larger values of γ correspond to longer supercritical crack lengths,” whereas Figure 4C appears to show the opposite trend. Please check the consistency between the text and the figure.

10. Line 178: A Poisson's ratio of 0.3 appears to have been used in the preceding numerical simulations (Table C1), whereas a value of 0.2 is used here. Please explain how this change affects the results.
11. Line 182: The sub-Rayleigh speed is stated to be approximately 60 m s⁻¹, but it is not clear from Fig. 6A how this value was determined. Please clarify how this value was obtained.
12. Line 199: Although this likely does not affect the subsequent results, a sign appears to be missing from the equation. Please check.
13. Throughout the manuscript: Please check the formatting of units. Units should be written in upright (roman) type rather than italic type. In addition, a space should be inserted between numerical values and units.