

General Comments

The paper reports about the first-light observations with the Polaribotn imaging polarimeter of auroral emission in the sky at 557.7 nm green line. The authors performed a rough comparison of with a well established polarimeter called Corbel Cru during a simultaneous observation and then discussed more in detail the results obtained with the Polaribotn instrument.

The topic of the paper is for sure of interest being focused mainly on the instrument point of view with some discussion of the results from the scientific point of view to be assessed more in deep in future observation campaign and papers.

However, the paper, as presented, cannot be published due to a major issue related to the methodological and mathematical treatment of polarization. Polarization degree (DoPL) and polarization angle (AoPL) are considered as independent parameters and mathematical manipulation follows this assumption. On the contrary DoPL and AoPL are not independent and cannot be treated separately. An AoPL is measured because a DoPL is measured. Thus, the correct treatment implies the use of Stokes parameters all along the mathematical computations and then results can be displayed in the form of DoPL and AoPL to make the result human readable. Thus, to make the paper acceptable for a publication, a deep revision of the mathematical treatment of polarization is needed.

In addition also the way the average DoPL and AoPL are computed should be revised. The use of normalized Stokes parameters of different ROIs is not appropriate, because it lose completely the information about ROIs intensity while computing the average.

Specific comments:

Line 5 : “ Polaribotn measurements show good agreement with Corbel Cru in both degree and angle of linear polarization”. Consider polarization as a whole. If the agreement would have been in terms of degree and not in angle, there is no agreement at all. Two polarizations would have been different.

Line 6: what does it means “DoLP and AoLP do not respond in the same way to auroral structures:”? DoPL and AoPL are the (linked) parameters the authors are measuring. They are not the response to something. Modify the sentence.

Line 89 - 95: Clarify the sentence “The instrument is not calibrated for absolute radiometric measurements. ”. This other form should be more appropriate: “The instrument is not calibrated for absolute flux measurements.”. The Calibration performed by Bosse et al. 2020 is not the goal of this review, however it is worth noting that calibration cannot be performed on DoPL alone. It must be performed on both DoPL and AoPL, thus Stokes parameters are essential for it.

Line 131: Clarify the geometry of the set-up from the the point of view of the pointing system. What does it means the Corbel Cru has no dedicated pointing system? How do the authors control the pointing direction of the two polarimeters? Add a picture of a drawing describing the set-up.

Line 132-135: How the placement of the 2° circular region was chosen in the Polaribotn FOV? If the pointing direction of the two instruments is not aligned within 2°, the Corbel Cru FOV could be somewhere in the large Polaribotn FOV. Thus, an arbitrary arrangement of 2° regions in it does not ensure that one of them significantly match the Corbel Cru FOV. Such a FOV could be split in two or four 2° regions (see Figure 5 top panel). The Corbel Cru FOV could fall in the

center of four 2° regions causing a large fraction of the signal from a 2° region of the Polaribotn polarimeter to be excluded from the comparison analysis. To make the comparison of two polarimeters of Sect 3 meaningful a better method should be used. For example, a large number of random positioned 2° regions in the Polaribotn FOV could be compared with the Corbel Cru signal to ensure as much as better the correspondence in the FOV. In alternative, a 2° regions could be systematically shifted in x and y to assess this match as well. In conclusion the empirical assessment of a single pattern is not a robust method.

Line 135-136: The DoPL is the only parameter considered to assess the match among the 2° regions in the Polaribotn FOV with respect to the Corbel Cru signal. This is methodologically wrong ! DoPL and AoPL cannot be assessed separately. The assessment have to be performed on both. If a region is compatible in terms of DoPL, but it is not in terms of AoPL it means two regions ae not compatible. An even better and robust approach is based on the comparison of the Stokes parameters Q and U measured in the 2° regions wrt the values measured by the Corbel Cru polarimeter. Since Q and U are independent parameters, they can be used in the same statistical test.

Line 135-136: The estimate of the common FOV is obtained by comparing the intensities and then the polarization of the two polarimeters. Moreover, both parameters are used to select the proper FOV *and* to assess the “validation” of the Polaribotn polarimeter response wrt the Corbel Cru one. Mixing two goals by using the same parameters is in principle not appropriate. However, it is not obvious to separate two goals. I suggest to report in a table the ranking of more significant FOVs in terms of polarization compatibility (see previous comment) to make clear how much such a selection is strong. In place of a table, a picture could be added to show graphically the match and the significance of most relevant FOVs.

Line 138: It seems the author used Pearson R (or simple linear correlation) test to assess the compatibility of the response of two polarimeter. Such a correlation analysis is a statistical method used to assess a possible linear association between two variables. Author should clarify how the R parameter is computed with respect to the two time series. If R is computed over the absolute values of the two time series, strong variations in the same direction would push R towards high values, unless smaller variations disagree. This is the case of an emission component that abruptly appears and dominate the Polaribotn FOV as it could be the auroral arc appearance. A more appropriate statistical test for comparing specifically time series of measurements should be applied to intensity and polarimetric data.

Line 185: Quantify at which level is the integrating sphere spuriously polarized. At least from the theoretical point of view. In “Depolarization measurements of an integrating sphere” by Stephen C. McClain APPLIED OPTICS, Vol. 34, No. 1, 1 January 1995, a residual polarization of 0.4% was reported for that specific model of integrating sphere at 550 nm. A residual polarization sum to the measured polarization and to the spurious polarization induced by the instrument that should be also characterized. In the paper under review polarization degrees below 0.4% are reported. At which level they are affected by a miss-calibration due to the integrating shere residual polarization? Moreover a miss-calibration of 0.4% is not negligible even for a polarization of 2-3% as measured in the observation.

Moreover, I suggest to read the paper <https://doi.org/10.3847/1538-3881/ac397f> Ranking et al. 2022, The Astronomical Journal, 163:39 (13pp). The authors show a methodology to quantify the instrument systematic effect on polarization independently from the intrinsic residual polarization of the "unpolarized" source. The paper is related to a polarimeter in the X-rays. Unless the differences in the detection technique (i.e. photon by photon vs flux) the principle by

using Stokes parameters should be applicable also to this case. In particular focus on Sect. 3. Equation 21 and 22 corresponds to eq. 8 in the paper under review: q_{sm} and u_{sm} are q and $u_{inst} \cdot Q$ and U_{true} . If two measurements of the calibration source are performed by rotating the detector by 90° wrt the beam axis, stokes parameters for the two measurements are reported in eq. 27 and 28. thus, eq. 29 and 30 allow to compute the spurious effect intrinsic of the detector, while eq. 33 and 34 allow to compute the intrinsic polarization of the calibration source (integrating sphere). With this methodology it is possible to put in Eq 8 of the paper under review only the spurious instrument polarization. If the authors are able to perform the measurement and include this methodology in the paper would be of great interest.

Line 207-214 and Figure 3: Which are the uncertainties of data points? DoPL of 0.3% +/- what? At the beginning of Sect 5.1 the polarization signals are interpreted in correlation with intensity, but in the figure it is not plotted. Plot also intensity of both polarimeters as done for polarization. Which is the significance of data reported in figure 3 if no uncertainties are reported? It seems Polaribotn AoPL flips rapidly by +/- 75 deg or so when DoPL is below 0.5%. See 18:35 and after 18:55. My interpretation is that DoPL is not significant at that level and the angle is not constrained. Thus, uncertainty on DoPL should include 0° . The Corbel Cru polarimeter shows some smaller scale flips when DoPL goes below 0.25%. Corbel Cru seems more sensitive. I suggest to plot also intensity and report the uncertainties in the plot. Clarify what the reader sees related to the sensitivity of two polarimeters. In particular the fast flips.

Clarify the intervals in which the comparison assessment is performed. It is probably meaningless to do it in time intervals where the polarization measurement is not significant. The authors are comparing statistical fluctuations of a detector with a significant measurement of another.

I suggest to find a better estimator than the simple correlation (see comment Line 138). Moreover, a correlation performed on the DoPL alone is not a good choice, because DoPL cannot be considered separately wrt AoPL. A robust assessment should be based on Stokes Parameters Q and U . Any manipulation of data should be done in the Q and U space and then converted to polarization degree and angle.

Line 219: Why AoPL become noisier? AoPL cannot be interpreted separately wrt DoPL. They are not independent parameters. The fast flipping of the AoPL depends on the fact that in the short integration time of the data points the polarization measurement is not significant. DoPL measurement is compatible with zero within the uncertainty, thus the AoPL is not constrained and the data points flip rapidly. There is not a deterioration of the agreement between two instruments. It is less sensitive than the other one. Clarify this in the text.

If authors write that "The empirical definition of the common FoV" affect the results, this means significantly different regions are under observations. Main effect I see is due to the difference in sensitivity of two polarimeters.

Line 222-223: The sentence "This suggests that the two instruments do not weight the scene in exactly the same way" is too much generic. What does it means "weight the scene"?

Line 228-230: Polarization degree and angle are not independent parameters. It is meaningless to compute them differently to assess the polarization behaviour of the sky source. All the mathematical manipulations have to be performed in the space of Stokes parameters and only at the end the conversion in terms of DoPL and AoPL can be done.

The assumption to properly handle Stokes Parameters with usual maths assumes such parameters to have a Gaussian distribution. Thus, the integrated polarization in the entire FOV (assumed as the composition of the ROIs) should be computed by summing Q and U fluxes and dividing by the total I of each ROIs. And only after that it is possible to convert it into DoPL and AoPL, with no additional mathematical manipulation. This procedure allows to properly handle the Stokes Parameter maths and to properly take into account the fact that different ROIs have a different fluxes and must be properly weighted in the total polarization. A region with lower flux will contribute less in the computation of the integrated polarization wrt a region with a higher flux.

Conversely, to compute the polarization in each ROI_i just divide Q_i and U_i by the corresponding I_i and then convert it to DoPL and AoPL. Do not use these values for further mathematical computations such as the computation of the average polarization.

Related to the computation of the uncertainties consider that DoPL uncertainty is strongly asymmetric if S/N is low. Similarly, the uncertainty on AoPL is wide as well.

A very important point is related to the estimation of the uncertainties of DoPL and AoPL starting from Q, U and the S/N of the measurement.

The 1D errors on DoPL and AoPL can be derived from:

- Placing Confidence Limits on Polarization Measurements, John E. Vaillancourt, Publications of the Astronomical Society of the Pacific, 118: 1340–1343, 2006 September

- On the statistical behaviour of the position angle of linear polarization, Naghizadeh-Khouei, J. ; Clarke, D. . Astronomy and Astrophysics, Vol. 274, p. 968 (1993)

To have an overview of the topic see chapter 5 - Polarization Statistics and Data Treatment, book title Stellar Polarimetry by David Clarke, edited by WILEY-VCH Verlag GmbH & Co. KGaA

Additionally, related to FOVs shape, if in the first part of the paper the authors assess a validation of the Polaribotn polarimeter wrt the Corbel Cru by comparing intensity and polarization in the 2° FOV. In the second part why the 2° FOVs are used to compute the polarization in the 20°x17° FOV of the Polaribotn polarimeter? A large fraction of the sky is kept out of the analysis because of the circular FOV shape does not fill it completely.

From the scientific point of view it would be more efficient to divide the FOV in regions that completely cover the sky FOV. For example squared regions. In such a way all the data acquired would be exploited.

Line 234: Equation number missing. AoPL_{mean} reported is not the mean angle of polarization. In the paper formula $u_i = U_i / I_i$ the $\langle u \rangle = 1/N \sum(u_i)$. The normalized Stokes parameters do not take into account the flux difference of each ROI if they are directly used to compute the average polarization angle. A high flux ROI weights in the computation like a low flux ROI. Thus, AoPL as computed does not consider where the emission is concentrated, that is also the region of the arc where the physics has to be assessed.

$$\text{AoPL}_{\text{mean}} = 1/2 \arctan \left(\frac{(U_1 + U_2 + \dots) / (I_1 + I_2 + \dots)}{(Q_1 + Q_2 + \dots) / (I_1 + I_2 + \dots)} \right)$$

Similarly for DoPL with its formula.

Figure 4: It is meaningless to describe the polarimetric behaviour by associating the average AoPL with the median DoPL, moreover computed by using the normalized Stokes parameters of ROIs. They lose the information about the flux of each ROI.

Line 251: Equation number missing. Clarify how $\langle q \rangle$ and $\langle u \rangle$ are computed. If they are

$$\langle q \rangle = 1/N_{roi} (Q_1/I_1 + Q_2/I_2 + \dots)$$

$$\langle u \rangle = 1/N_{roi} (U_1/I_1 + U_2/I_2 + \dots)$$

DoPL does not take into account properly the fluxes of different regions to compute the mean polarization. A low flux region is weighted as well a high flux region. A region barely emitting weights in the computation as well a region strongly emitting in the auroral arc.

Proper formula is

$$U_{tot} = U_1 + U_2 + \dots$$

$$Q_{tot} = Q_1 + Q_2 + \dots$$

$$I_{tot} = I_1 + I_2 + \dots$$

$$\text{DoPL mean} = \sqrt{(Q_{tot}^2 + U_{tot}^2)/I_{tot}} = \sqrt{(U_{tot}/I_{tot})^2 + (Q_{tot}/I_{tot})^2}$$

Another option is to do the weighted mean of the normalized stokes parameters by the ROI flux to compute the DoPL. Polarization maps are widely used in different wavelength. Concepts of mean polarization or polarization in ROIs are already developed with the corresponding maths tools.

I suggest looking at some example:

- X-rays

In Kislat et al 2015, *Astroparticle Physics* 68 (2015) 45–51, "Analyzing the data from X-ray polarimeters with Stokes parameters", look at formulae 10a, b, c, 11 a,b and 21. Except for a factor $2/\mu$ strictly related to the physics of the X-ray polarimeter, by substituting Eq. 10 into 11 and then into 21.

I suggest also to give a look at the paper

Vink et al. 2022 *The Astrophysical Journal*, 938:40 (14pp), 2022 October 10

in which the analysis of an extended astrophysical source is performed. Except for the peculiarities of the X-ray analysis related to the specific detector, the polarization is computed in regions of the image, but also the average polarization is computed not averaging the mean stokes parameter of each region (ROI) as defined at the beginning.

-Visible light

In Skolidis et al. 2018 A&A 616, A52 (2018) the weighted mean q and u are explicitly mentioned.

Figure 5 bottom panel: The average polarization is not the average of the polarization of each ROI. The flux weighting must be taken into account (see previous comment). In the bottom figure circular plots the uncertainty on DoPL is not the one in the figure. You can plot the errors on Q and U of the measured DoPL and AoPL. I suggest to look at figure 6 in Muleri et al. 2022, Analysis of the Data from Photoelectric Gas Polarimeters

https://doi.org/10.1007/978-981-16-4544-0_143-1

Line 270: By computing in the correct way the mean Q and U it is possible to represent in bottom panels of figure 5 the proper Gaussian errors of the Stokes parameters and verify if the $(0,0)$ is included in the uncertainties.

Line 356-358: The comparison of the two polarimeters should be done comparing both DoPL and AoPL (see Sec. 3) because these two parameters are not independent

Technical corrections:

In general use larger font size for axes titles and when possible for values.