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Spectral Neutrality of Climate Reductions: An Operator Perspective

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Abstract

Climate theory relies on a hierarchy of reductions that simplify the governing equations of radiative transfer and geophysical fluid dynamics. Examples include global averaging in energy balance models, quasigeostrophic filtering, the β-plane approximation, and idealized Kelvin–Rossby mode decompositions. These approximations are typically justified asymptotically and are highly successful within their intended regimes. However, they also modify the operators, domains, boundary conditions, or nonlinear functionals that define the admissible variability of the system.

This paper develops an operator-based framework for evaluating the spectral neutrality of climate reductions. A reduction is termed spectrally neutral if it preserves the operator class, admissible function space, domain topology, boundary conditions, and leading spectral structure of the original problem. Many widely used climate reductions are not spectrally neutral in a global sense, even when they remain locally or asymptotically accurate. Two examples are examined in detail. First, nonlinear radiative averaging in energy balance models is interpreted as a projection from a field equation onto a scalar closure, where averaging and nonlinear radiation operators do not commute. Second, the relation between spherical shallow-water dynamics and the β-plane approximation is reconsidered from the viewpoint of operator equivalence. The spherical Laplace tidal operator defines a compact global eigenvalue problem with discrete Hough spectra, whereas the β-plane formulation defines a different operator on a different domain with distinct admissible eigenfunctions. Boundary-value constraints in ocean basins further illustrate that low-frequency adjustment and teleconnections are governed by the spectrum of the full basin operator rather than by local plane-wave dispersion relations alone. The central issue is therefore not whether classical reductions are useful, but whether they preserve the spectral structure of the underlying climate dynamics. This perspective provides a unified framework connecting radiative closures, geometric reductions, and basin-scale wave adjustment.

Keywords: climate dynamics, operator theory, spectral methods, Rossby waves, beta-plane approximation, energy balance models, teleconnections

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1 Introduction

Climate dynamics is fundamentally a problem in fluid mechanics on a rotating sphere. The governing equations—Navier–Stokes, thermodynamics, continuity, and radiation—define a nonlinear system constrained by geometry, rotation, and boundary conditions. To render this system analytically tractable and numerically feasible, a sequence of approximations has been developed over the past century, including hydrostatic balance, shallow-water reductions, quasigeostrophic filtering, β-plane geometry, and global radiation balance closures (Richardson, 1922; Charney, 1948; Pedlosky, 1987; Vallis, 2017).

These approximations are individually well justified within their intended regimes. Their success is evident in the realism of simulated circulation patterns and the predictive skill of modern models. At the same time, each approximation modifies the mathematical structure of the governing equations, particularly the differential operators that determine admissible eigenmodes, spectral constraints, and stability properties.

This question motivates the notion of *spectral neutrality*. A reduction is spectrally neutral only if it preserves the defining elements of the original eigenvalue problem: the operator class, the topology and compactness of the domain, admissible boundary conditions, and the associated function space. If these properties are modified, then the reduced system defines a different spectral problem, even if it reproduces local asymptotic behavior or bulk statistics.

These questions are closely related to broader issues concerning recoverability, coarse graining, and effective laws in physics. In Lohmann (2026a), irreversible information loss arising from reduced observational representations was discussed in the context of quantum measurement and climate reanalysis. Likewise, Lohmann (2026b) argued that finite observability can generate state-dependent effective laws when unresolved variability is absorbed into reduced dynamical descriptions.

The present study extends these ideas into geophysical fluid dynamics. Here the reduction is geometric and spectral rather than observational: global averaging, planarization, asymptotic filtering, and boundary idealization modify the admissible function space and therefore the spectral structure of the governing operators. The central issue is therefore not only whether a reduction reproduces bulk observables, but whether it preserves the spectral problem associated with the original spherical boundary-value system.

The present discussion is related to broader issues known in operator theory and statistical mechanics, where projections onto reduced function spaces may alter spectral structure, modify admissible eigenspaces, or generate effective dynamics that are not spectrally equivalent to the original operator formulation. Related ideas arise in projection-operator approaches such as the Mori–Zwanzig formalism, where unresolved degrees of freedom induce memory effects and apparent stochasticity in reduced descriptions (Mori, 1965; Zwanzig, 1960, 2001).

The issue raised here is not whether planar eigenfunctions provide useful local asymptotic descriptions, but whether they can be identified with the admissible global eigenspaces of the underlying spherical boundary-value problem. In operator-theoretic terms, eigenfunction expansions are not independent of geometry, topology, and boundary conditions, since these structures determine the spectral decomposition itself (Courant and Hilbert, 1953; Reed and Simon, 1972).

In what follows, geometry is the common thread. It enters energy balance models through

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the global averaging implied by a spherical planet; it enters the dynamical core through rotation on a curved manifold; and it enters wave theory through the domains, boundary conditions, and eigenfunctions supported by the relevant operators. The goal is therefore not to revisit the empirical success of standard reductions, but to separate their local asymptotic motivation from their global operator consequences: eigenstructure, degeneracies, admissible mode families, and spectral measures.

This paper develops this idea in three steps. First, nonlinear radiative averaging in energy balance models is interpreted as a projection from a field equation onto a scalar closure, where averaging and nonlinear radiation operators do not commute. Second, the relation between spherical shallow-water dynamics and the β-plane approximation is reconsidered from the viewpoint of operator equivalence. Third, basin-scale adjustment is interpreted as a boundary-value problem in which the relevant modes are constrained by coastlines, stratification, and dissipation.

The novelty of the present study lies in the unified interpretation of classical climate reductions as operator reductions whose spectral consequences depend on whether the underlying eigenvalue problem is preserved. The central issue is therefore not whether local reductions are useful, but whether their eigenfunctions and spectra can be interpreted as globally equivalent to those of the unreduced spherical boundary-value problem.

This question is increasingly relevant for modern reduced-order and operator-based approaches to climate variability, where projected dynamics, modal decompositions, and effective closures are widely used to interpret large-scale variability and predictability (Majda et al., 2003; Mezić, 2005).

2 Operator Reductions and Spectral Neutrality

The central idea of this paper is that many climate approximations can be interpreted as reductions acting on operators, function spaces, or admissible mode families. Such reductions are often asymptotically justified, but they are not necessarily spectrally neutral. The central question addressed here is structural rather than empirical:

> When does a climate reduction preserve the spectral structure of the underlying dynamical operator, and when does it define a different spectral problem?

This issue is related to a broader question in theoretical physics: how much information about the underlying system survives coarse graining, projection, or observational reduction. Similar questions arise in quantum measurement theory, climate reanalysis, and state-dependent effective laws, where incomplete observability modifies the effective dynamics inferred from data (Lohmann, 2026a,b). In Lohmann (2026a), recoverability and irreversible information loss are discussed in the context of physical inference and climate reanalysis, emphasizing that reduced observational representations need not preserve the full dynamical structure of the underlying system. Likewise, Lohmann (2026b) argues that effective laws inferred from finite observability may become state dependent because unresolved dynamics are absorbed into reduced representations.

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In climate dynamics, the problem appears geometrically and spectrally. Averaging, planarization, asymptotic filtering, and boundary idealization may simplify the governing equations while simultaneously modifying the admissible spectral structure of variability.

Consider an evolution equation

$$\partial_t u = Lu, \tag{1}$$

where L is an operator acting on a function space $\mathcal{H}$. A climate reduction introduces a projection, averaging operator, asymptotic truncation, or coordinate reduction

$$P : \mathcal{H} \to \mathcal{H}_r, \tag{2}$$

leading formally to a reduced operator

$$L_r = PLP^*, \tag{3}$$

or, more generally, to a closure acting on a reduced set of variables. In general, the reduction and the dynamics need not commute:

$$[P, L] = PL - LP \neq 0. \tag{4}$$

When this commutator is nonzero, the reduced dynamics need not preserve the spectral structure of the original operator.

Proposition 1 (Non-equivalence of spherical and β-plane operators)**.** *Let L_{S^2} denote the linear shallow-water or quasigeostrophic wave operator defined on the rotating sphere S^2, subject to regularity conditions at the poles. Let L_β denote the corresponding β-plane operator defined on $\mathbb{R}^2$, a channel, or a periodic rectangle.*

In general, the spherical operator L_{S^2} and the corresponding β-plane operator L_β are not unitarily equivalent because they are defined on different domains with different admissible function spaces and boundary conditions. Consequently, their eigenspectra and admissible modal structures differ except in explicitly localized asymptotic limits. In particular, L_{S^2} defines a compact global boundary-value problem with discrete Hough or spherical eigenmodes, whereas L_β admits plane-wave or boundary-imposed modal structures. Therefore, β-plane eigenfunctions cannot generally be identified with global spherical eigenfunctions unless an explicit asymptotic localization limit is imposed.

The common structure underlying the examples discussed below is illustrated schematically in Fig. 1. In each case, a reduction or closure replaces the original spherical operator by a modified effective operator acting on a reduced admissible function space.

The proposition is not intended to claim that the β-plane is invalid. Rather, it separates local asymptotic usefulness from global spectral equivalence. A tangent-plane approximation may approximate local propagation properties while still changing the object being diagonalized.

Definition: Spectral neutrality

A reduction is termed *spectrally neutral* if it preserves:

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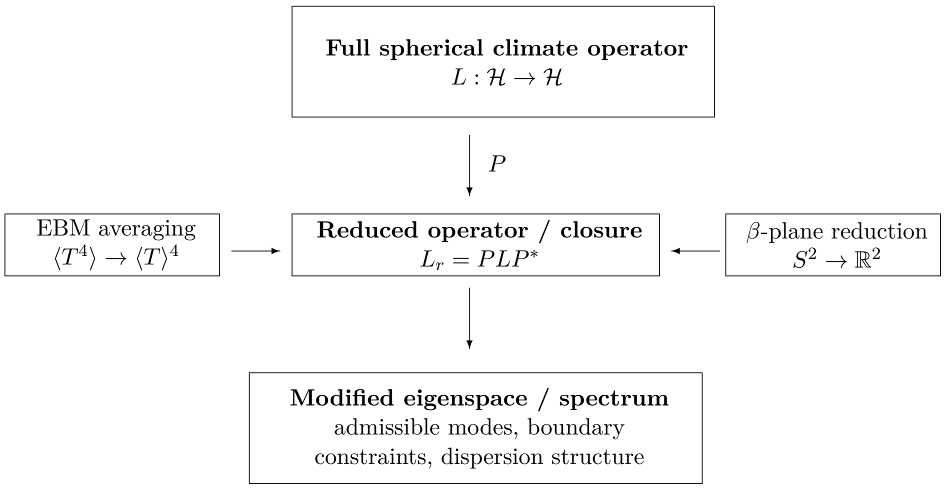

Figure 1: Schematic illustration of climate reductions as operator reductions. A full spherical climate operator L acting on a function space $\mathcal{H}$ is replaced by a reduced operator or closure through a projection or asymptotic reduction P. The resulting operator generally acts on a modified admissible function space and may therefore alter the eigenspectrum, boundary constraints, or mode structure of the system. The examples discussed in this paper, including nonlinear radiative averaging in energy balance models and the β-plane approximation, are interpreted within this common reduction framework.

- the operator class,
- the topology and compactness of the domain,
- admissible boundary conditions,
- and the leading eigenspaces or spectral measure.

Spectral neutrality is stronger than asymptotic consistency. A reduction may remain locally accurate while still altering the global eigenstructure. Conversely, if a reduction changes the operator, collapses nonlinear functionals to scalar closures, or imposes different boundary conditions, then it defines a modified spectral problem.

3 Nonlinear Radiative Averaging as Operator Reduction

Zero-dimensional energy balance models (EBMs) play a central conceptual role in climate theory (Budyko, 1969; Sellers, 1969; North et al., 1981; Lohmann, 2020). The canonical steady-state relation,

$$(1-\alpha)\frac{S}{4} = \varepsilon\sigma T^4, \tag{5}$$

yields the equilibrium temperature

$$T = \left(\frac{(1-\alpha)S}{4\varepsilon\sigma}\right)^{1/4}. \tag{6}$$

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However, incoming shortwave forcing depends on geometry and albedo, while outgoing longwave emission depends nonlinearly on the local temperature field; both are therefore spatially heterogeneous on a sphere. Writing

$$\mathrm{OLR}(\mathbf{x},t) = \varepsilon\sigma T(\mathbf{x},t)^4, \tag{7}$$

an exact global balance can be expressed as

$$\left\langle (1-\alpha(\mathbf{x},t))\frac{S(\mathbf{x},t)}{4} \right\rangle = \left\langle \varepsilon\sigma T(\mathbf{x},t)^4 \right\rangle, \tag{8}$$

where $\langle\cdot\rangle$ denotes an area average over the planetary surface. The factor $1/4$ encodes the disk-to-sphere area ratio; the brackets encode the global geometry.

Because x^4 is convex for $x \geq 0$, Jensen's inequality implies

$$\langle T^4 \rangle \geq \langle T \rangle^4. \tag{9}$$

Thus, radiative balance is a nonlinear functional of the temperature field, and the fourth-root relation corresponds to a scalar closure. In climate theory, replacing $\langle T^4 \rangle$ by $\langle T \rangle^4$ amounts to assuming that temperature spatial variance (from resolved gradients and subgrid fluctuations) is negligible or effectively absorbed elsewhere (for instance, via a tuned emissivity). This is often acceptable in calibration exercises, but it becomes conceptually relevant when interpreting feedbacks and stability, because the linearized radiative tendency depends on higher moments of T. From the operator perspective summarized in Fig. 1, global averaging acts as a projection from a spatially distributed radiative field onto a reduced scalar closure.

A time-dependent global energy balance reads

$$C\frac{dT}{dt} = \langle (1-\alpha)S \rangle - \varepsilon\sigma\langle T^4 \rangle, \tag{10}$$

where C denotes an *effective* heat capacity representing the vertically integrated thermal inertia of the coupled atmosphere–ocean system. This inertia is large compared to high-frequency variations in the forcing. Because Earth rotates rapidly, diurnal fluctuations are dynamically filtered, and the dominant temperature response is governed by the slowly varying component of the radiation balance. As emphasized by Lohmann (2020), meaningful temperature estimates from EBMs require an explicit recognition of this timescale separation; otherwise, solutions may misrepresent the physically relevant regime.

A central point is therefore that the textbook fourth-root relation derived from a steady, globally averaged balance is not a purely radiative identity. Its validity depends on the dynamical suppression of sub-daily variability by rapid rotation and sufficiently large effective heat capacity. In the fast-rotation / large-C limit, the area-weighted global equilibrium temperature becomes

$$\widetilde{T}_{eq} = G\left(\frac{(1-\alpha)S}{4\varepsilon\sigma}\right)^{1/4}, \tag{11}$$

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with the dimensionless factor

$$G = \sqrt{\frac{\pi}{2}\,\frac{\Gamma(9/8)}{\Gamma(13/8)}} \approx 0.989. \tag{12}$$

Although the numerical deviation of G from unity is small for present-day Earth, this explicit solution clarifies that the classical fourth-root expression is an *asymptotic* result of the time-dependent problem, not a general equilibrium identity. Its apparent simplicity reflects a particular dynamical regime in which temporal variability is strongly averaged.

Crucially, the effective heat capacity is not a fixed material constant. It emerges from ocean mixed-layer depth, vertical mixing, and turbulent heat transport. Variations in stratification, overturning circulation, or sea-ice cover alter C, thereby modifying the amplitude and phase of both diurnal and seasonal cycles. Turbulence and mixing thus enter the global temperature problem not only through horizontal redistribution of heat, but through the dynamical filtering of temporal variability.

This dependence becomes particularly relevant in paleoclimate contexts. Changes in ocean circulation and mixing state can modify seasonal amplitude and transient response even under identical mean radiative forcing (Lohmann et al., 2022). The global mean temperature is therefore not solely a function of radiative parameters, but of the dynamical buffering capacity of the climate system. The classical fourth-root estimate holds only insofar as the system behaves as a rapidly rotating, strongly buffered integrator.

4 Geometric Reduction and Global Wave Operators

The primitive equations on the sphere can be written in covariant form (Müller, 1997). In this formulation, geometry and rotation enter through differential operators defined on a compact curved manifold. The Coriolis term, metric factors, and boundary conditions together determine the admissible eigenfunctions of the system.

The β-plane approximation may be interpreted as an operator reduction: a globally covariant operator on the sphere is replaced by a locally Cartesianized operator defined on a tangent plane. This replacement alters not only the coefficients (through $f = f_0 + \beta y$), but also the domain, boundary conditions, and therefore the spectral structure of the problem. As indicated schematically in Fig. 1, the reduction replaces a globally covariant spherical operator by a locally Cartesian operator with different admissible eigenfunctions and boundary constraints.

Accordingly, the β-plane should be viewed as more than a coordinate convenience. It changes the object being diagonalized. On the sphere, eigenfunctions are constrained by compactness and regularity at the poles; on a planar domain, admissible modes depend on imposed boundary conditions or, in the unbounded case, form a continuous spectrum. This distinction becomes structurally relevant when global modes, teleconnections, or basin-scale adjustments are considered.

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4.1 Rossby waves on a β-plane

On a β-plane, quasigeostrophic dynamics take the form

$$\partial_t(\nabla^2\psi - \psi/L_D^2) + \beta\partial_x\psi = 0, \tag{13}$$

which admits Fourier solutions

$$\psi \propto e^{i(kx+ly-\omega t)},$$

with dispersion relation

$$\omega = -\frac{\beta k}{k^2 + l^2 + 1/L_D^2}. \tag{14}$$

The β-plane formulation admits plane-wave solutions whose spectrum is continuous in (k, l) for an unbounded domain and whose discretization, when present, is imposed by the chosen planar boundary conditions rather than by intrinsic spherical compactness. This framework has proven highly successful for understanding anisotropic turbulence, spectral cascades, and jet formation (Rhines, 1975). However, its spectral properties derive from planar geometry rather than from the compact spherical manifold. In bounded basins, the reduction structure shown in Fig. 1 becomes especially important because admissible variability is determined jointly by geometry, stratification, and boundary-value constraints.

4.2 Spherical shallow-water dynamics and the Laplace tidal operator

On the sphere, shallow-water dynamics reduce to the Laplace tidal equation (Longuet-Higgins, 1968),

$$\mathcal{L}_M(\alpha)\psi(\varphi) = \lambda\psi(\varphi), \tag{15}$$

with spheroidal (Hough) eigenfunctions (Flammer, 1957). Here $\alpha = (2\Omega a/c)^2$ is the Lamb parameter, M the zonal wavenumber, and λ the separation constant fixed by polar regularity.

In the non-rotating limit, the eigenfunctions reduce to spherical harmonics

$$\psi(\varphi, \lambda, t) = \Re\left\{A\, Y_\ell^m(\varphi, \lambda)\, e^{-i\omega t}\right\}, \tag{16}$$

with eigenvalues $\ell(\ell+1)/a^2$. Rotation modifies the operator but preserves the compact spherical domain, so that the spectrum remains discrete and globally defined. The eigenfunctions are therefore geometry-adapted structures rather than plane waves (see Appendix A).

Figure 2 illustrates this distinction for a midlatitude band. The solid curve shows an illustrative Rossby-like Hough mode obtained from the α-form tidal eigenproblem using a Legendre–Galerkin expansion. The dashed curve shows the lowest-order β-plane channel eigenmode $\psi_n(y) = \sin(n\pi(y + L_y/2)/L_y)$ under rigid-wall boundary conditions, implying meridional wavenumber quantization $l_n = n\pi/L_y$. Even within identical latitude bounds, the meridional structures differ, because the spherical and planar formulations diagonalize different boundary-value operators. The comparison is not intended as a mode-by-mode identification, but to demonstrate that geometry and boundary conditions determine the admissible function space of variability.

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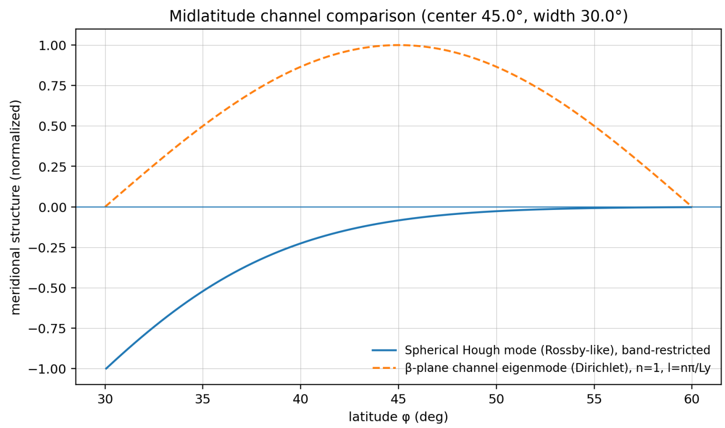

Figure 2: Meridional structure of a Rossby-like Hough mode obtained from the α-form Laplace tidal eigenproblem (solid) compared with the lowest-order β-plane channel eigenmode under Dirichlet boundary conditions (dashed). The figure illustrates differing admissible eigenspaces arising from different operators.

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4.3 A minimal illustration: β -plane versus spherical eigenfrequencies

To quantify how a local β-plane surrogate can deviate from spherical eigenstructure, consider barotropic nondivergent dynamics. On the sphere, a spherical harmonic mode (ℓ, m) has the Rossby–Haurwitz frequency

$$\omega_{\mathrm{sph}} = -\frac{2\Omega m}{\ell(\ell+1)}. \tag{17}$$

On a β-plane in the barotropic limit ($L_D \to \infty$),

$$\omega_\beta = -\frac{\beta k}{k^2+l^2}, \qquad \beta = \frac{2\Omega\cos\varphi_0}{a}. \tag{18}$$

A geometrically consistent local identification $k = m/(a\cos\varphi_0)$ together with $k^2 + l^2 \simeq \ell(\ell+1)/a^2$ reproduces the spherical frequency. However, a simplified planar identification sometimes used in heuristic scaling arguments uses $k = m/a$, neglecting the metric factor $\cos\varphi_0$. This yields

$$\omega_{\beta,\mathrm{flat}} \simeq \cos\varphi_0\,\omega_{\mathrm{sph}}, \tag{19}$$

i.e. a relative frequency bias $\cos\varphi_0 - 1$.

For the Earth ($\Omega = 7.292\times10^{-5}\,\mathrm{s}^{-1}$, $a = 6.371\times10^6\,\mathrm{m}$), a representative large-scale mode $(\ell, m) = (5, 4)$ has $\omega_{\mathrm{sph}} = -1.945\times10^{-5}\,\mathrm{s}^{-1}$ (period $\approx$ 3.74 days). At $\varphi_0 = 45°$, the flat β-plane mapping gives $\omega_{\beta,\mathrm{flat}} = -1.375\times10^{-5}\,\mathrm{s}^{-1}$ (period $\approx$ 5.29 days), corresponding to a $\sim$ 29% frequency error. This simple estimate illustrates that geometric identification between spherical and planar operators can induce order-one deviations for midlatitude, basin-scale variability, even before invoking full tidal eigenfunctions.

5 Boundary Operators and Basin Adjustment

The operator perspective developed above becomes particularly transparent when the domain is no longer the full sphere but a bounded ocean basin. In this setting, the governing equations define a boundary-value problem, and the admissible modes are constrained not only by planetary geometry but also by coastlines and stratification. The relevant variability therefore reflects the eigenstructure of the basin operator, rather than a freely propagating plane-wave spectrum. This viewpoint provides a natural framework for interpreting trapped Rossby waves, coastal Kelvin modes, and basin-scale adjustment.

5.1 Trapped Rossby waves and covariant tidal theory

The distinction is visible already in the Laplacian. On the sphere the operator contains metric factors proportional to $\cos\varphi$, whereas on a β-plane it is replaced by the Cartesian Laplacian. This substitution changes the operator and therefore the admissible eigenfunctions. In covariant (bi-)shallow-water theory, the governing equations can be written in a form that keeps geometric structure explicit (Müller, 1997). For layered spherical shallow-water dynamics, the coupled vorticity equations reduce to a higher-order spheroidal eigenvalue problem (Müller and Maier-Reimer, 2000). In the analyzed configurations, quasigeostrophic Rossby eigenfrequencies remain real, i.e. no exponential growth arises from the corresponding eigenvalue problem

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itself. These results illustrate that spectral properties depend sensitively on operator structure. β-plane instability analyses and spherical tidal analyses therefore address related but not identical mathematical problems.

The present argument concerns global spectral structure rather than the local asymptotic validity of β-plane dynamics. Their utility for regional dynamics, anisotropic turbulence, and wave propagation is well established. The deeper issue is interpretational. Local tangent-plane reductions are often implicitly treated as if they approximate the global eigensystem itself. From an operator viewpoint, however, the reduction replaces a compact spherical boundary-value problem by a different spectral problem with different admissible function spaces and symmetry structure. The resulting modal structure is therefore not merely an approximation to the spherical one, but belongs to a different spectral problem.

5.2 Observational perspective on basin-scale variability

Satellite altimetry shows that large-scale westward-propagating variability in the ocean is often only weakly dispersive and frequently organized into coherent basin-scale structures rather than idealized freely propagating plane waves (Chelton and Schlax, 1996; Chelton et al., 2007, 2011). The purpose of mentioning these observations here is not to revisit the distinction between Rossby waves and nonlinear eddies, but to emphasize that the observed variability is shaped by the admissible structures supported by the underlying operator and domain geometry.

From the operator perspective developed above, this point is natural. Classical Rossby dispersion relations arise from diagonalizing a locally Cartesian β-plane operator, typically on an unbounded or idealized domain (Gill, 1982; Pedlosky, 1987). By contrast, spherical shallow-water dynamics and basin adjustment problems correspond to boundary-value operators defined on compact or bounded domains with discrete admissible mode families (Longuet-Higgins, 1968; Müller, 1997; Müller and Maier-Reimer, 2000). Basin geometry, stratification, and boundary conditions therefore influence the spatial structure and propagation characteristics of low-frequency variability. These observations are consistent with the broader interpretation advanced here: spectral structure depends not only on local asymptotic dynamics, but also on the operator, domain topology, and boundary conditions defining the admissible eigenfunctions.

5.3 Coastally trapped waves with β and dissipation: a boundary-value view

From an operator viewpoint, coastally trapped low-frequency waves correspond to boundary-value problems in which the admissible spectrum depends explicitly on dissipation and boundary representation. The Kelvin–Rossby mode split is commonly introduced through idealized eigen-solutions: Kelvin waves as boundary-trapped gravity modes and Rossby waves as interior PV modes (Gill, 1982). At low frequency on a β-plane, however, the existence and trapping of boundary waves becomes a *boundary-value* issue: β introduces a meridional PV gradient that can destroy midlatitude Kelvin trapping below critical frequencies, and dissipation and numerical discretization determine what boundary-trapped structures remain.

A key theoretical result is that, at ocean boundaries, there exist *critical frequencies* below which classical coastally trapped Kelvin-wave solutions cease to exist on a β-plane (Clarke

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and Shi, 1991). This result is consistent with earlier analyses of equatorial-wave reflection and boundary conversion (Clarke, 1983; Clarke and Shi, 1993), and with later treatments of meridional circulation anomalies along western and eastern boundaries (Marshall and Johnson, 2013). This means that for interannual and longer periods, one should not assume that a boundary-propagating signal necessarily corresponds to an inviscid deformation-radius Kelvin eigenmode. Instead, low-frequency coastal adjustment may project onto boundary-controlled structures whose existence and width depend on friction/viscosity and on how the boundary conditions are represented.

This issue is not only analytical but also numerical. Finite-difference models may represent Kelvin-like boundary waves with substantial dispersion and dissipation errors if the deformation radius or the relevant boundary structure is poorly resolved (Hsieh et al., 1983). In barotropic tide and shelf-sea contexts, intercomparisons between finite element and finite volume approaches further demonstrate that boundary representation and numerical operators can substantially affect wave characteristics (Maßmann et al., 2010). These results reinforce the operator-based message of this paper: the "mode" supported by a model is shaped by the operator (continuous or discrete) and its boundary conditions, and thus the Kelvin–Rossby split should be interpreted as an asymptotic classification rather than a universal global eigendecomposition. From a modern reduced-order perspective, these results illustrate that the admissible low-frequency dynamics depend sensitively on how boundaries, unresolved dissipation, and closure assumptions are represented in the effective operator (Majda et al., 2003; Mezić, 2005).

5.4 Basin-scale adjustment and global teleconnections

In a stratified ocean basin, the large-scale adjustment to deep-water formation or buoyancy forcing can be interpreted as the excitation of boundary-constrained Rossby and Kelvin modes (Kawase, 1987; Huang et al., 2000). These signals are not freely propagating plane waves but are shaped by basin geometry, stratification, and boundary conditions. The relevant eigenstructure is therefore set by the basin operator and its constraints.

This perspective is reflected in studies of thermohaline variability and Atlantic overturning adjustment. Theoretical and numerical analyses (Johnson and Marshall, 2002a,b, 2004) demonstrate that surface signals associated with overturning anomalies are mediated by equatorial and boundary waveguides, whose spatial structure and phase propagation are governed by boundary-value constraints rather than by local β-plane dispersion alone.

Likewise, the long basin-crossing times of low-frequency Rossby modes (Primeau, 2002) and global thermocline "seiching" (Cessi et al., 2004) indicate that low-frequency variability may reflect discrete basin modes instead of a continuous plane-wave spectrum. In such cases, teleconnections emerge naturally from domain compactness and the coupling of zonal and meridional structure imposed by the operator.

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6 Conclusions

Across radiation balance, rotating shallow-water dynamics, and basin-scale wave adjustment, a common structural theme emerges: climate reductions modify the operators that determine admissible variability and spectral structure. Within this framework, the central question is whether a reduction preserves the spectral problem defined by the original operator. If the reduction modifies geometry, boundary conditions, or admissible function spaces, then it defines a different eigensystem, even when local asymptotic behavior remains accurate. Spectral neutrality requires preservation of the structural elements defining the original eigenvalue problem. In particular, neutrality requires that

- the underlying differential operator is retained, rather than replaced by a reduced or Cartesian surrogate;
- the domain topology and compactness are preserved, so that admissible eigenfunctions belong to the same function space;
- boundary conditions remain dynamically equivalent, such that no new spectral constraints or artificial degeneracies are introduced;
- nonlinear functionals are not collapsed to scalar closures in a manner that removes variance-dependent feedbacks.

If any of these conditions is violated, the approximation defines a modified eigenvalue problem. In that case, changes in variability need not be merely quantitative adjustments of dispersion relations, but may reflect altered mode families and coupling structure.

If a reduction modifies the operator, the admissible function space, or the boundary-value structure, then it defines a different spectral problem. In that case, changes in variability need not represent perturbations of the original eigensystem, but may instead reflect the emergence of different admissible mode families and coupling structures. The distinction developed here is therefore not between "valid" and "invalid" approximations, but between local asymptotic consistency and global spectral equivalence. From this viewpoint, geometry, compactness, and boundary conditions are not secondary technical details. They are part of the dynamical structure that determines which modes, teleconnections, and variability patterns are admissible in the first place.

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Appendix A: Spherical Eigenstructure and Basin Adjustment

This appendix provides the formal operator statement underlying the contrast between β-plane dispersion relations and spherical eigenstructure discussed above.

We consider linear shallow-water dynamics on a rotating sphere of radius a with constant axial rotation Ω and gravity-wave speed

$$c = \sqrt{gH}.$$

A natural non-dimensional control parameter is the Lamb parameter

$$\alpha = \left(\frac{2\Omega a}{c}\right)^2.$$

Assuming zonal structure $\propto e^{iM\lambda}$, the meridional dependence $\Theta(\mu)$ with $\mu = \sin\varphi$ satisfies the Laplace tidal eigenproblem (Longuet-Higgins, 1968; Flammer, 1957)

$$L_M(\nu)\,\Theta(\mu) = -\Lambda\,\Theta(\mu), \qquad \mu \in [-1, 1],$$

subject to regularity at the poles. Here $\nu = \omega/(2\Omega)$ is the nondimensional frequency, and L_M is a self-adjoint operator. The spectrum is therefore discrete, and the eigenfunctions are the Hough (spheroidal) modes.

In the quasigeostrophic Rossby-wave limit ($|\nu| \ll 1$), the leading-order dispersion relation may be written as

$$\nu = -\frac{\alpha M}{\varepsilon(N, M; \alpha)},$$

where $\varepsilon(N, M; \alpha)$ denotes the spheroidal eigenvalue associated with meridional mode index N. The structural distinction can be summarized as follows: on the sphere, Rossby-type variability is constrained by a global boundary-value problem with discrete mode families. On a β-plane, by contrast, one diagonalizes a Cartesian operator and obtains a continuous dispersion relation in (k, l) prior to imposing lateral boundary conditions (Pedlosky, 1987; Vallis, 2017). The transition from planar Fourier modes to spherical Hough modes is therefore not merely a coordinate transformation, but a change of operator, domain, and admissible spectrum.

Code and data availability

No new observational or numerical datasets were generated for this study. The manuscript is based on theoretical analysis and previously published literature. The calculations and figures presented in the manuscript were produced using standard scientific computing tools and are available from the author upon reasonable request.

Author contributions

G.L. developed the conceptual framework, performed the analysis, prepared the figures, and wrote the manuscript.

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Competing interests

The author declares that there are no competing interests.

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