Full Reply on RC1

In the text below our response components are expressed in the form of *italic Times New Romans.*

I regret I cannot recommend acceptance of the paper, even after a possible revision. That is so not because I consider there is no scientific value in the paper (I think on the contrary that it contains material that may eventually lead to at least one publication), but because a large part of its content is largely exaggerated or even unjustified. For instance, the sentence in the abstract (ll. 30-31) This (meaning the increasing difficulty to enhance NWP models forecasting capabilities) is caused largely by catastrophe/tipping phenomena in the atmosphere as well as the multiplicity of NWP equation solutions is not supported by any objective evidence presented in the paper.

We are very grateful to you for the time to carefully evaluate our work and particularly for your kind suggestion to write two separate papers as well as your encouragement by pointing out that our paper contains material which may eventually lead to at least one publication.

Here, we would first show our apology for our unusual selected topic and especially the poor English writing in the MS. The most impressive criticism by you is about the statements in the abstract，which is the relevant point stressed particularly in Editor's comments as well. As a result, in the revised MS it is fully removed with its "replacer" also based on the bullet point (1) in Referee #2's Comments (see the revised MS and the full reply below for further details).

The paper, as I understand it, consists of two distinct parts. One part (Section 3, entitled Literature Review) presents a number of studies of 'tipping points' or 'catastrophes' in the atmospheric/climatic system (I will not here enter into the semantic discussion relative to the meaning of those two terms, and will also use the word 'bifurcation'). That presentation is however incomplete and unstructured. It is incomplete in that it does not mention possible tipping points that have been widely discussed in the literature, such as for instance the collapse of the Atlantic Meridional Overturning Circulation (AMOC, see, e.g., van Westen et al., 2024), or the dieback of the Amazon Rainforest (Melnikova et al., 2025). As for the structure of the presentation, ll. 360-367 of the paper list a number of illustrations of tipping points that are then discussed in some detail. That list is somewhat inconsistent, in that some items in it (1 and 6) refer to specific physical phenomena, while others (5 and 7) refer to much more general possible bifurcation processes in dynamical systems.

On the completeness of Literature Review coverage, we have mentioned in our AC2 to you for the preprint of egusphere-2026-2981 with the emphasis on:

(1) Our present MS involves a subtitle showing its substantial theme being related to catastrophe theory and its applications, and, as far as this point is concerned, the coverage of the current MS is complete, which might be implied by the statement of "Over the scope covered by this article (since 1975), the results related to applications of CT to the atmospheric sciences might be briefly summarized as follows." (ll. 210-212, egusphere-2026-2981).

(2) As for tipping points, though it is discussed as the "supplementary" counterpart in contrast to catastrophes in catastrophe theory in the current MS, its completeness is certainly important as well. In the revised MS, we include some additional articles covering (say) these mentioned in your comments. In the present revised MS, we also state that numerous illustrations

may be found in (say) "the comprehensive document 'Global Tipping Points Report 2025' by Lenton, Milkoreit and Willcock, et al (2025); due to space constraints, we will not elaborate further here". By the way, we have modified the concerned description in the revised MS in order to avoid the issue on "list inconsistence" as pointed out by you. Thank you!

The content of Section 3 could be the basis of a general review paper on the broad question of bifurcations (tipping points, catastrophes) in the atmosphere/climate system. Such a review paper should, in addition to summarizing the status of knowledge on that broad question, and according to the NPG requirements, outline future directions of research.

Yes, in addition to a comprehensive literature review, we also outline future directions of research in the final section in the revised MS. For further details please see the relevant text in the revised version of our manuscript. Thank you!

The remainder of the paper (which I must say I find uselessly lengthy and repetitive) bears on the present limits of Numerical Weather Prediction. The authors start from the observation, supported by Figure 1, that the quality of NWP has not improved significantly in recent years. They conclude that there is a 'development bottleneck' that needs to be broken. Their point is that this is to be achieved by resorting to catastrophe theory (CT). Well, diminishing returns are observed in many fields of science and technology (not to mention economics ...), and are not necessarily proof of a methodological 'bottleneck'. They may be due to ultimate limits (for instance, of energy), or lack, or non-use, of resources (after all, the spatial resolution of the model of the European Centre for Medium-Range Weather Forecasts, from which most diagnostics shown in Fig. 1 have been obtained, has apparently not increased since at least 2018).

Yes, your acumen impressed us so much. As a result, we revised our draft substantially; particularly, according to your comments, we suggest some future directions of research in the revised MS. On the other hand, indeed in many cases, only enhancing the spatial resolution of a model would not certainly improve its forecasting accuracy.

In order to make their point to support the use of catastrophe theory, the authors make a number of statements which are simply unfounded. In addition to the one I mention at the beginning of this review, I have noticed

Ll. 87-88, … weather forecast failure happens frequently in case catastrophe/tipping phenomena emerge in the atmosphere. What is the evidence ? Can you give precise examples of such forecast failures ?

The authors mention (ll. 99-103) a number of atmospheric phenomena, which they claim could be better described by catastrophe theory. That may be true, but these phenomena are simulated to an acceptable degree of accuracy by the present physical models (concerning for instance prediction of sudden stratospheric warmings, see Karpechko et al., 2018).

Ll. 631-632, Numerous catastrophe phenomena have been observed in the Earth atmosphere, but many of these discontinuous phenomena still defy analysis. Again, can you give precise examples to support the use of catastrophe theory in those situations ?

Similar comments could be made, for instance on the possible multiplicity of solutions of nonlinear equations (and on the use of ensemble forecasts in that context), or on the need for more explicit use of the second law of thermodynamics. But my main point is that the authors do not

give any example, even highly idealized, of how what they advocate could be implemented in practice.

Indeed, our argument is not rigorous enough and we sincerely apologize for that. Many thanks for you to identify these problems one by one. We revised them carefully in the revised MS and among them, however, for some (say, on the need for more explicit use of the second law of thermodynamics) we actually have already made a few efforts for what we advocate could really be implemented in practice. As a matter of fact, as early as more than 15 years ago, a preliminary attempt at specifically using the second law of thermodynamics for improve NWP forecast accuracy had been done (see e.g. Liu and Liu, 2005; and, also a comprehensive review, Liu et al., 2011 with milestone of reaching 20 and 400 reads, respectively, as are informed by ReseachGate last July; as well as, a relevant Chapter, see Liu et al. 2010). By the way, in fact, either the spectral models or grid-point ones can most likely be improved in virtue of the inclusion of the 2nd law of thermodynamics (Liu and Liu 2005)

I certainly do not want to imply that what the authors say about the possible usefulness of CT for NWP is erroneous. I simply say that, as interesting as that possibility may be, what the authors write is unjustified at this stage. They write (ll. 614-615) that they will organize a separate paper to specifically address the question of a novel CT-based forecasting methodology. That is certainly what they must do now.

The revised MS has been substantially revised on considering all the comments and suggestions including especially yours which really contribute to significant improvement to our MS and, we suppose, your assessment on it might become different this time. Thank you!

My recommendation is to reject the present paper, and to suggest to the authors to write a more complete and better structured review paper on the question of tipping points /catastrophes in the atmospheric and climatic system. And in addition, as they anticipate, to write another paper, with appropriate examples, on the application of catastrophe theory in Numerical Weather Prediction.

We are grateful very much you for your nice recommendation to write two separate articles. However, considering that most of our peers worldwide, we suppose, might realize that the current review article of this sort should be published as soon as possible. Your consent is crucially expected.

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Liu, C; Ma, L.; Liu, Y.; Luo, Z.; Lei, X.; Zhou, X.; Wang, D.; Xu, H.: Linking tropical cyclone number over the western north pacific with sea surface temperatures, Hurricanes and Climate Change (eds: Elsner, J.B., Hodges, R.E., Malmstadt, J.C., Scheitlin, K.N.), Springer: New York, NY, USA, Volume 2, doi: 10.1007/978-90-481-9510-712, 2010.

Chongjian Liu, Ying Liu, Zhexian Luo, Xiaotu Lei, Donghai Wang and Xiaogang Zhou: Studies of the Hurricane Evolution Based on Modern Thermodynamics，Recent Hurricane

Research: Climate, Dynamics, and Societal Impacts by A. Lupo (ed.), InTech, ISBN 978-953-307-238-8, 2011