AC2

Response to Referee #1

We are very grateful to Referee #1 for taking the time to carefully evaluate our work and for providing valuable feedback, particularly the kind suggestion to write two separate papers.

Our motivation to write this manuscript (MS) is substantially owing to the consideration that we are facing a moment at which the physics-based NWP (Numerical Weather Prediction) methodology appears increasingly difficult in terms of further enhancing the NWP model's forecasting capabilities. In addition to the illustrations shown in Fig. 1 in our MS, it is found that, as an example, Macnally (2025) has mentioned a similar predicament even with a cartoon-style diagram (cf. the figures herein， Fig. 1). As a result, informing or reminding our worldwide colleagues of such a predicament is of great urgency.

Fig. 1 The screenshot from Macnally (2025).

By the way, we will revise our MS according to your comments one by one, from A to Z, integrating the other relevant comments, when the current phase of discussion is about to end. The following aspects such as those on the completeness of Literature Review coverage and on the issues needing to be supported by objective evidence are some among them:

1) On the completeness of Literature Review coverage.

As you have noticed, our present MS involves a subtitle showing its substantial theme being related to catastrophe theory and its applications. As far as this point is concerned, the coverage of the current MS is complete, which might be implied by the statement of "Over the scope covered by this article (since 1975), the results related to applications of CT to the atmospheric sciences might be briefly summarized as follows." (ll. 210-212, egusphere-2026-2981).

As for tipping points, though it is discussed as the supplementary* counterpart in contrast to catastrophes in catastrophe theory in the current MS, its completeness is certainly important as well. Therefore, we are going to include (say) these three articles (that are all concerned with tipping points) as mentioned in your comments into our coming revised MS.

*The wording "supplementary counterpart" here is somewhat influenced by the following line of reasoning that "commentators observe that the idea behind tipping points is not new; the editors of Nature, for example, suggest it is old wine in new bottles" (Russill and Nyssa, 2009).

As a matter of fact, traditionally, catastrophe theory is one of the so-called "new three theories" (also as the DSC-theories: Dissipative structures theory, Synergetics, and Catastrophe theory credited respectively by I. Prigogine, H. Haken, and R. Thom) that came out in 1970s, as compared relatively to the three theories (also as the SCI-theories: System theory, Cybernetics, and Information theory by L. Bertalanffy, N. Wiener, and C. Shannon, respectively) created around 1940s. All of these theories belong to the transdisciplinary science like mathematics (say, the differential equation theory) whose research object is not only a certain field or a certain substance, but is transversely embedded in many fields and even all fields.

However, on the other hand, there has been no theory called "tipping point theory" or something like that so far. As a result, in addition to the subtitle of our paper that stresses our substantial theme being related to catastrophe theory, we just mentioned in the abstract that "Studies of catastrophe have never stopped but the closely related terminology (tipping) changed subtly, and thus similarities and differences between catastrophe and tipping as well as the rhetoric course of change and development with tipping points have been ascertained in this article with the result showing that CT itself never declines and it gets a new replacer only."

For further details, please see also (e.g.), **3.2 On tipping points in the atmospheric/climatic system,** in the preprint egusphere-2026-2981 (ll. 342-362), as well as ll. 480-498.

2) On the issues needing to be supported by objective evidence

Thank you for having mentioned similar comments several times and, it indeed is a good question. Here we 'd like to take the issue on the need for more explicit use of the second law of thermodynamics as an example to respond "how what they advocate could be implemented in practice". As a matter of fact, as early as more than 15 years ago, a preliminary attempt at specifically using the second law of thermodynamics for improve NWP forecast accuracy had been done (see e.g. Liu and Liu, 2005; and, also a comprehensive review, Liu et al., 2011 with milestone of reaching 20 and 400 reads, respectively, as are informed by *ReseachGate* last July; as well as, a relevant Chapter Liu et al. 2010).

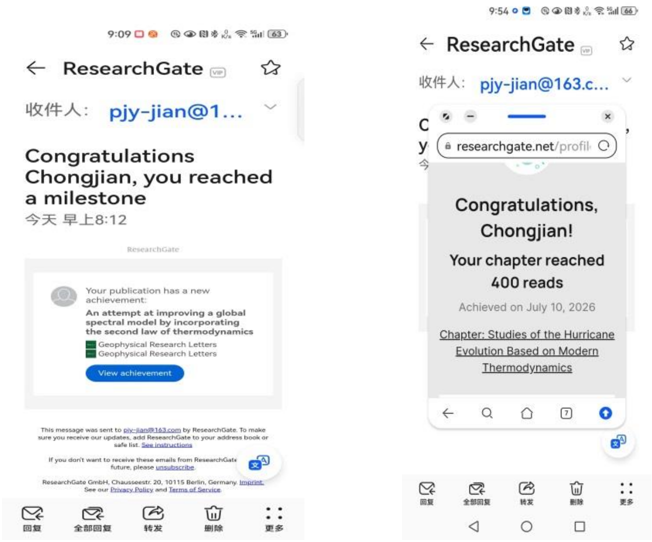

Fig. 2 The screenshot from the email-message by *ReseachGate*.

All of the papers above are listed in the References of the preprint egusphere-2026-2981.

3) In addition, an idealized case (PHASE TRANSITION OF MOISTURE) about how what we advocate could be implemented in practice would be given here to help introduce some basic concepts and methods of catastrophe theory as well as show how to construct a catastrophe model for understanding a specific catastrophic phenomenon.

The involvement of catastrophe theory with thermodynamics, like most thermodynamic processes, has produced more heat than light. However, just as for caustics (Poston and Stewart 1978) catastrophe theory is a necessary component in any physical treatment of phase transitions short of exact solution of Schrödinger equation for bulk matter. As we know, the Boyle's law description $PV = RT$ of the 'ideal gas' is generally accepted in the meteorological sciences although, as we shall see, the description of phase transitions by such an equation of state is not always adequate. At least it is unable to be used for describing the nature of catastrophe (sudden phase transition) for a substance such as moisture at the critical point (say, the triple point). Van der Waals made the first attempt (in 1873) beyond the state equation for an ideal gas and proposed the following modified equation that could distinctly exhibit its ability of describing catastrophic behavior:

$$(P + \alpha/V^2)\,(V - \beta) = RT, \qquad (1)$$

where P, V and T are the pressure, volume and temperature of the sample of fluid concerned, respectively, with R being a constant. The constants α and β are chosen, nowadays, by experimental best fit, and are no longer given a physical meaning (Callen 1960).

In the form above, van der Waals' equation is usually represented graphically as in Fig. 3a below, by drawing the graph of P against V for various T. A three-dimensional version might be the surface of Fig. 3b, showing the points in (P, V, T)-space that satisfy the equation.

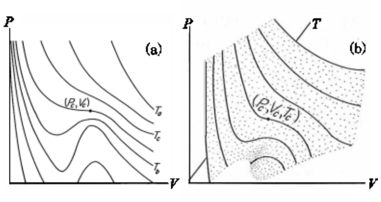

Fig. 3 (a) A graphical representation for van der Waals' equation with pressure (P) against volume (V) for various temperature (T)； and (b) a three-dimensional version for its representative surface, showing the points in (P, V, T)-space that satisfy van der Waals' equation. Pc, Vc and Tc are the critical values of P, V and T, respectively, and Ta, Tb and Tc mark their respective isotherms.

It may be seen that as the substance described experiences decreasing pressure at constant temperature, there are several possibilities:

(i) If the temperature is T_a (> T_c, the critical temperature), the volume can smoothly increase;

(ii) If the temperature is T_c, then V is a continuous but not a differentiable function of P;

(iii) If the temperature is T_b (< T_c), there are pressures for which several volumes are possible, and the equation does not give volume as a function of pressure. At some point, it would appear that matter attempting to follow the curve must jump. Above the jump, the matter resists a small reduction in volume with a much greater increase in pressure than below; the hard-to-compress liquid suddenly becomes an easy-to-compress gas.

How does the matter decide at which point to jump? Maxwell gave in 1875 an argument from the equivalence of heat and work, whose conclusion may be geometrically expressed as the equal area rule (Fig. 4).

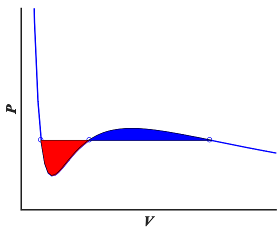

Fig. 4 Schematic diagram for Maxwell's equal area rule: part of the original curve be replaced by a horizontal straight line shown, such that the two shaded areas are equal.

Now we shall go into our substantial context to show its relationship with catastrophe theory by way of a standard reduction with a change of variables from Fowler (1972), which could be viewed to a great extent as a kind of classical prototype for constructing one of the elementary catastrophe models (the cusp catastrophe model in the present case). It is easily found that the point (P_c, V_c, T_c)

which lies at the heart of what is happening is $(\frac{\alpha}{27\beta^2}, 3\beta, \frac{8\alpha}{27\beta R})$. Normalizing this point to (1, 1, 1) by setting

$$P^{'} = \frac{P}{P_c}, V^{'} = V_c, \text{ and } T^{'} = T_c \, , \tag{2}$$

we get the reduced van der Waals' equation

$$(P^{'} + \frac{3}{V^{'2}})(V^{'} - \frac{1}{3}) = \frac{8}{3}T^{'} \, . \tag{3}$$

Replacing volume as variable by density *X*, that is setting $V^{'} = 1/X$, we get in some ways a more intuitive description of the 'state' of the matter. Finally, we transfer the origin to the point (1, 1, 1) around which everything is happening, by setting

$$p = P^{'} - 1, x = X - 1, \text{ and } t = T^{'} - 1 \, . \tag{4}$$

Thus, the dramatic form of equation emerges like that

$$x^3 + \frac{(8t + p)}{3}x + \frac{(8t - 2p)}{3} = 0 \, , \tag{5}$$

which is simply

$$x^3 + ax + b = 0 \, , \tag{6}$$

where $a = (8t + p)/3$ and $b = (8t - 2p)/3$. This is exactly the cusp catastrophe surface, and it can be drawn as Fig. 5 below in which the negative half has been cut off owing to pressure and temperature be non-negative.

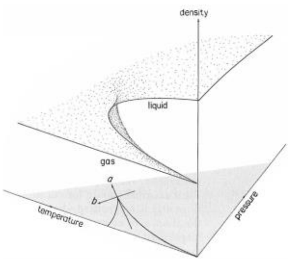

Fig. 5 The cusp catastrophe manifold given by van der Waals' equation, in which the negative half has been cut off owing to pressure and temperature be non-negative. The set of points at which jumps take place is a curve in (a, b)-space.

Application of the Maxwell equal area rule assigns a unique volume (and hence density) to the fluid for each temperature and pressure, unique that is except at the jump points. Carrying this over to our new coordinates, Fig. 5 becomes Fig. 6. The set of points at which jumps take place is a curve in (*a*, *b*)-space, shown by the heavy line in Fig. 6, tangent to the a-axis at (0, 0), but not coincident with it.

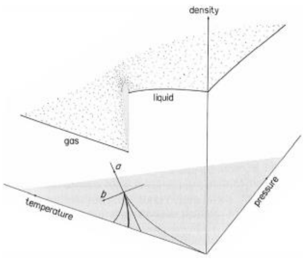

Fig. 6 The catastrophe manifold after the change of coordinates made with the Maxwell equal area rule used. The set of points at which jumps take place is a curve in (a, b)-space, shown by the heavy line.

The fact that van der Waals' equation transforms into the standard equation for the cusp surface is no coincidence, but the algebraic simplicity of the transformation is. The full relation with the 'van der Waals equation plus Maxswell's rule' model of liquid/gas phase transition should be more subtle.

To sum up, you may have noticed, the subtopic of the present paper is (it is indeed so) "a critical review on applications of catastrophe theory to the atmospheric science", and, as far as such a theme above is concerned, you would eventually justify our assertion that the current MS of this sort should be published as soon as possible.

Your kind consent is crucially expected.

Thank you in advance for your time and nice assistance.

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