Supplementary material

Supplement S1: L^1 normalization

The formulations in the main text are derived using the standard L^2 normalization of the CWT, for which

$$||\psi_{a,b}(t)||^2 = ||\psi(t)||^2, \qquad \psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right).$$

which preserves energy across scales. Alternatively, the L^1 normalization, for which

$$||\psi_{a,b}(t)|| = ||\psi(t)||, \qquad \psi_{a,b}(t) = \frac{1}{a}\psi\left(\frac{t-b}{a}\right),$$

preserves amplitude across scales.

Due to the different scaling exponent, the variance reconstruction formula becomes (proof analog to Appendix A2):

$$\langle f, f\rangle = \frac{1}{C_\psi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left|W_f(a,b)\right|^2 db\,\frac{da}{a}.$$

This leads to the following discrete formulations, using the modified $C_\psi = \int_0^{+\infty}\frac{|\hat{\psi}(\omega)|^2}{\omega}d\omega$:

Real-valued wavelets

$$\sigma_x^2 = \frac{\delta j \ln(2)}{NC_\psi}\sum_{n=1}^{N}\sum_{j=1}^{J}\left|W_x\left(\frac{a_j}{\delta t}, n\right)\right|^2,$$

$$cov(x,y) = \frac{\delta j \ln(2)}{NC_\psi}\sum_{n=1}^{N}\sum_{j=1}^{J}W_x\left(\frac{a_j}{\delta t}, n\right)\overline{W_y\left(\frac{a_j}{\delta t}, n\right)}.$$

Analytic wavelets and real-valued signals

$$\sigma_x^2 = 2\frac{\delta j \ln(2)}{NC_\psi}\sum_{n=1}^{N}\sum_{j=1}^{J}\left|W_x\left(\frac{a_j}{\delta t}, n\right)\right|^2,$$

$$cov(x,y) = 2\frac{\delta j \ln(2)}{NC_\psi}\sum_{n=1}^{N}\sum_{j=1}^{J}Re\left[W_x\left(\frac{a_j}{\delta t}, n\right)\overline{W_y\left(\frac{a_j}{\delta t}, n\right)}\right].$$

Under this normalization, variance and covariance contributions are therefore directly comparable across scales.

Supplement S2: Linear scaling

For linearly spaced scales, i.e. scales separated by a constant increment δj, the discrete formulations are obtained by replacing da with $\frac{\delta j}{\delta t}$ in the continuous formulations. Expressions for L^2 normalization follow directly from this substitution:

Real-valued wavelets

$$\sigma_x^2 = \frac{\delta t}{N}\frac{\delta j}{C_\psi}\sum_{n=1}^{N}\sum_{j=1}^{J}\frac{\left|W_x\left(\frac{a_j}{\delta t},n\right)\right|^2}{a_j^2},$$

$$cov(x,y) = \frac{\delta t}{N}\frac{\delta j}{C_\psi}\sum_{n=1}^{N}\sum_{j=1}^{J}\frac{Re\left[W_x\left(\frac{a_j}{\delta t},n\right)\overline{W_y\left(\frac{a_j}{\delta t},n\right)}\right]}{a_j^2}.$$

Analytic wavelets and real-valued signals

$$\sigma_x^2 = 2\frac{\delta t}{N}\frac{\delta j}{C_\psi}\sum_{n=1}^{N}\sum_{j=1}^{J}\frac{\left|W_x\left(\frac{a_j}{\delta t},n\right)\right|^2}{a_j^2},$$

$$cov(x,y) = 2\frac{\delta t}{N}\frac{\delta j}{C_\psi}\sum_{n=1}^{N}\sum_{j=1}^{J}\frac{Re\left[W_x\left(\frac{a_j}{\delta t},n\right)\overline{W_y\left(\frac{a_j}{\delta t},n\right)}\right]}{a_j^2}.$$

Formulations for L^1 normalization are, for each case:

Real-valued wavelets

$$\sigma_x^2 = \frac{\delta j}{NC_\psi}\sum_{n=1}^{N}\sum_{j=1}^{J}\frac{\left|W_x\left(\frac{a_j}{\delta t},n\right)\right|^2}{a_j},$$

$$cov(x,y) = \frac{\delta j}{NC_\psi}\sum_{n=1}^{N}\sum_{j=1}^{J}\frac{Re\left[W_x\left(\frac{a_j}{\delta t},n\right)\overline{W_y\left(\frac{a_j}{\delta t},n\right)}\right]}{a_j}.$$

Analytic wavelets and real-valued signals

$$\sigma_x^2 = 2\frac{\delta j}{NC_\psi}\sum_{n=1}^{N}\sum_{j=1}^{J}\frac{\left|W_x\left(\frac{a_j}{\delta t},n\right)\right|^2}{a_j},$$

$$cov(x,y) = 2\frac{\delta j}{NC_\psi}\sum_{n=1}^{N}\sum_{j=1}^{J}\frac{Re\left[W_x\left(\frac{a_j}{\delta t},n\right)\overline{W_y\left(\frac{a_j}{\delta t},n\right)}\right]}{a_j}.$$

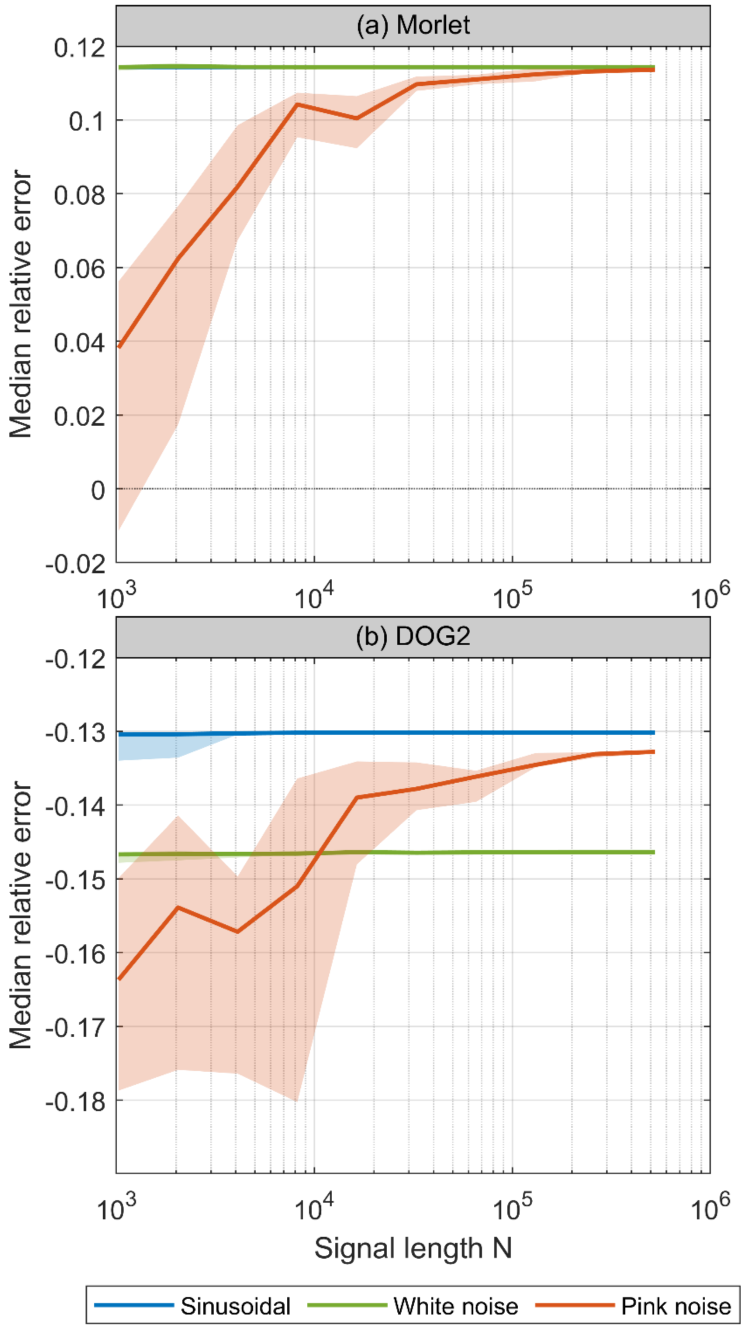

Figure S1: Convergence of the TC-based variance estimator as a function of signal length for sinusoidal (blue), white-noise (green), and pink-noise (red) signals using (a) Morlet and (b) DOG2 wavelets. Solid lines denote median relative errors (σ^^2/σ^2-1) and shaded areas indicate interquartile ranges.