



Correlated Geometric Scaling of Tornado Damage Paths in the SPC Database (1950–2024)

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Abstract.

We investigate the large-scale geometric organisation of tornado damage paths using the NOAA Storm Prediction Center (SPC) tornado database over the period 1950–2024. The analysis focuses primarily on the correlated geometry of tornado path length and width independently of any energetic weighting procedure.

5 A robust anisotropic scaling relation is identified between tornado path width W and path length L ,

$$W \sim L^\alpha, \quad \alpha \approx 0.43,$$

indicating that longer tornadoes undergo systematic sublinear lateral broadening during propagation. This correlated geometric growth naturally induces a hierarchy for the geometry-only observable

$$A = LW,$$

10 which obeys the scaling law

$$A \sim L^{1+\alpha} \approx L^{1.43}.$$

Complementary cumulative distribution functions reveal broad heavy-tailed statistics spanning several decades in scale. Finite-size scaling analysis further demonstrates that the geometric hierarchy remains remarkably stable across independent historical periods despite major observational transitions associated with Doppler-radar deployment and evolving survey practices.

15 Higher-order conditional moments remain predominantly affine and are shown to be fully consistent with the measured width–length scaling relation. Null-model comparisons demonstrate that the observed hierarchy cannot be explained solely by broad marginal distributions or simple algebraic construction effects, but instead reflects genuine correlations linking tornado length and width across scales.



20 Although the present analysis remains fundamentally statistical, the results suggest that tornado damage geometry may re-
flect the large-scale statistical imprint of persistent coherent vortex organisation embedded within strongly nonlinear convective
dynamics.

1 Introduction

Tornadoes constitute some of the most extreme and intermittent manifestations of atmospheric convection. They emerge
25 from strongly nonlinear interactions involving moist instability, vertical wind shear, mesocyclonic rotation, vortex stretch-
ing, boundary-layer turbulence, and dissipative processes acting simultaneously across a broad range of spatial and temporal
scales. Although major progress has been achieved in severe-weather forecasting and Doppler-radar observations during recent
decades, the collective statistical organisation of tornado damage paths remains only partially understood. In particular, while
tornado climatology has extensively characterised occurrence frequencies, seasonal variability, geographical distributions, and
30 intensity statistics, much less is known regarding the possible existence of robust scaling laws governing tornado morphology
itself.

Most classical tornado studies focused primarily on descriptive statistics associated with the Fujita and Enhanced Fujita
classifications. Brooks (2004) demonstrated that tornado path length and width increase systematically with tornado intensity
and that their distributions exhibit broad non-Gaussian behaviour extending over several orders of magnitude. Subsequent
35 studies confirmed that tornado geometry contains physically meaningful information beyond discrete intensity classification
alone. Elsner et al. (2014) showed that path dimensions may be used to construct continuous tornado intensity estimators,
suggesting that tornado damage geometry reflects underlying properties of vortex organisation and convective dynamics.

At the same time, the interpretation of long-term tornado statistics remains complicated by major observational inhomog-
eneities. Agee and Childs (2014) emphasized that tornado databases are strongly affected by evolving reporting practices,
40 increasing population density, improved surveying procedures, and modifications of tornado classification methodologies. Ed-
wards et al. (2021) further demonstrated that the deployment of Doppler-radar networks and the transition from the original
Fujita scale to the Enhanced Fujita scale substantially modified tornado detection capabilities, particularly for weak tornadoes.
Apparent scaling relations may therefore emerge partly from reporting biases rather than from intrinsic atmospheric organi-
sation. A central challenge is thus to determine whether statistical structures remain robust across observational eras despite
45 these substantial inhomogeneities.

Beyond descriptive climatology, several investigations suggested that severe convective systems may exhibit properties anal-
ogous to those encountered in complex nonlinear systems. Malamud and Turcotte (2012) analysed tornado outbreaks using
concepts inspired by statistical physics and self-organised criticality, reporting approximate scale-free behaviour in outbreak
statistics and tornado path distributions. Tippett and Cohen (2016) later showed that tornado outbreak variability approximately
50 follows Taylor scaling laws, where the variance grows as a power of the mean outbreak rate. Such behaviour is characteristic
of multiplicative amplification processes and suggests the possible existence of emergent multiscale organisation in severe
convective activity.



55 More generally, atmospheric science has progressively incorporated concepts originating from turbulence theory, multiplicative cascades, and multifractal analysis. Since the pioneering work of Lovejoy and Schertzer (1985), atmospheric variability has often been interpreted as the manifestation of scale-invariant cascade processes extending across broad ranges of scales. Schertzer and Lovejoy (1987) and Tessier et al. (1993) later developed universal multifractal models capable of reproducing intermittency corrections in rainfall and cloud-radiance fields. Subsequent studies demonstrated that precipitation, hydrological extremes, atmospheric radiative fields, and geophysical turbulence frequently exhibit hierarchical scaling laws and heavy-tailed statistics analogous to those observed in fully developed turbulence (Veneziano et al., 2006; Lovejoy and Schertzer, 2013).

60 Comparable scale-invariant organisation has also been identified in a variety of geophysical extreme-event systems beyond atmospheric turbulence itself. Earthquake statistics obey the Gutenberg–Richter law and exhibit scale-free clustering properties (Turcotte, 1997; Corral, 2004). Wildfire areas display broad heavy-tailed distributions and finite-size scaling behaviour (Malamud et al., 1998). Avalanches, fragmentation processes, and river-network statistics similarly reveal emergent organisation laws characteristic of strongly nonlinear nonequilibrium systems (Bak et al., 1987; Stanley et al., 1999; Sornette, 2006). These studies collectively suggest that extreme geophysical events often self-organise into robust statistical hierarchies despite strong local stochasticity.

70 Despite this extensive literature on atmospheric scaling and geophysical intermittency, tornadoes themselves have rarely been investigated from the perspective of correlated geometric organisation. Most previous studies treated tornado path length, path width, intensity distributions, or outbreak frequencies as largely independent climatological observables. Although broad heavy-tailed behaviour has occasionally been reported, the existence of robust scaling relations linking tornado length and width across scales remains largely unexplored.

75 An additional difficulty concerns the interpretation of energetic observables. Direct kinetic-energy measurements are unavailable at climatological scales, and tornado databases primarily contain geometrical descriptors together with discrete EF intensity classes. Consequently, energetic proxies constructed from path geometry and intensity weighting may introduce partial circularity into scaling analyses. If an observable is built directly from path length and width, then conditioning moment statistics on path length can artificially generate apparent scaling behaviour. Likewise, assigning discrete velocity amplitudes to EF classes may produce hierarchical structures dominated primarily by geometric correlations rather than by genuine energetic organisation.

80 The present work addresses these issues explicitly. Rather than beginning from an energy-weighted observable, we first investigate whether tornado damage geometry itself possesses intrinsic multiscale organisation independently of any EF-based weighting procedure. Using the NOAA Storm Prediction Center (SPC) tornado database covering the period 1950–2024, we analyse the scaling properties of tornado path length L and tornado path width W .

85 Our analysis combines heavy-tail statistics, finite-size scaling collapse, local scaling diagnostics, conditional moment hierarchies, and null-model comparisons inspired by statistical physics and nonlinear dynamics. Particular attention is devoted to determining whether the observed hierarchy reflects genuine correlations linking tornado length and width or merely trivial consequences of broad marginal distributions.

We show that tornado geometry obeys a robust anisotropic scaling relation of the form



$$W \sim L^\alpha, \quad \alpha \approx 0.43,$$

which remains stable across independent historical periods spanning more than seven decades. This correlated geometric
90 growth naturally induces a hierarchy for the geometry-only observable

$$A = LW,$$

leading to the effective scaling relation

$$A \sim L^{1+\alpha}.$$

Conditional moments additionally exhibit predominantly affine scaling behaviour consistent with the measured width–length
95 relation. Finite-size scaling analysis further reveals substantial collapse of the rescaled geometric distributions across historical periods despite major observational transitions between 1950 and 2024.

The analysis therefore suggests that tornado damage paths possess a previously underexplored level of correlated multiscale
geometric organisation that is not reducible to simple observational artifacts or trivial heavy-tailed randomness. At the same
time, the present work deliberately adopts a cautious interpretation. The observed hierarchy does not by itself demonstrate
100 the existence of a genuine turbulent cascade mechanism. Rather, the results indicate that tornado geometry exhibits robust emergent scale-invariant organisation analogous to that encountered in several complex geophysical systems.

The objective of the present study is therefore twofold. First, we aim to determine whether tornado damage geometry pos-
sesses statistically robust scale-invariant structure independent of EF-based energetic weighting. Second, we seek to establish
a quantitative bridge between tornado climatology and modern approaches originating from nonlinear dynamics, statistical
105 physics, and intermittency theory.

2 Methods

2.1 Tornado database and observational framework

The present study uses the NOAA Storm Prediction Center (SPC) tornado database covering the period 1950–2024. The SPC
archive constitutes the largest long-term tornado catalogue currently available for the United States and contains, for each
110 confirmed tornado event, the occurrence date, path geometry, geographical coordinates, and Fujita or Enhanced Fujita (EF)
intensity classification.

The database spans the complete tornado intensity range from EF0 to EF5 and includes several major observational tran-
sitions, notably the progressive deployment of Doppler-radar networks, improvements in storm-survey procedures, and the



transition from the original Fujita scale to the Enhanced Fujita scale in 2007. Previous studies emphasized that tornado statis-
115 tics are affected by evolving reporting practices, population-density effects, and increasing observational capabilities Agee and
Childs (2014); Edwards et al. (2021). Rather than attempting to remove all such effects explicitly, the present work investigates
whether robust geometric scaling structures persist despite these observational heterogeneities.

Only tornadoes satisfying

$$L > 0, \quad W > 0,$$

120 were retained, where L denotes tornado path length and W tornado path width. Path lengths originally reported in miles were
converted to kilometres, while widths reported in yards were converted to metres. Events containing missing or non-physical
geometrical values were excluded.

To investigate temporal robustness, the catalogue was partitioned into three independent historical periods,

$$1950\text{--}1974, \quad 1975\text{--}1999, \quad 2000\text{--}2024,$$

125 corresponding approximately to the pre-modern observational era, the transitional reporting period, and the modern Doppler-
radar era.

2.2 Geometric observables and anisotropic scaling

A central objective of the present work is to determine whether tornado damage paths exhibit intrinsic correlated geometric
organisation independently of any energetic weighting procedure.

130 Let L denote tornado path length and W the corresponding path width. We first investigate the empirical scaling relation

$$W \sim L^\alpha,$$

which quantifies how tornado width evolves statistically with propagation distance.

This relation constitutes the primary non-trivial scaling result of the present study. In particular, the exponent α charac-
terises the anisotropic geometric growth of tornado damage paths and determines whether longer tornadoes preserve constant
135 transverse structure or instead broaden systematically during propagation.

Using these observables, we define the geometry-only quantity

$$A = LW,$$

which represents an effective geometric damage-path area.



140 Importantly, A is not introduced as an independent observable unrelated to the width-length scaling law. On the contrary,
once the empirical relation

$$W \sim L^\alpha$$

is established, one naturally expects

$$A = LW \sim L^{1+\alpha}.$$

145 The observable A therefore serves primarily as a consistency diagnostic allowing us to test whether the measured geometry
remains coherent across multiple statistical levels, including finite-size collapse and moment hierarchies.

The quantity A should not be interpreted as the exact physical damage area of a tornado, since actual damage strongly
depends on local exposure, land-use heterogeneity, and structural vulnerability. Instead, A constitutes a coarse-grained geo-
metrical descriptor designed to capture the large-scale correlated morphology of tornado damage paths.

2.3 Energy-weighted observable

150 Although the principal focus of the present study concerns geometric organisation, we additionally introduce an energy-
weighted observable in order to compare purely geometric scaling with coarse intensity-weighted statistics.

Direct measurements of tornado kinetic energy are unavailable at climatological scales. Tornado databases provide path
geometry and EF intensity classifications rather than fully resolved vortex velocity fields. Following previous studies relating
tornado intensity to damage characteristics Brooks (2004); Elsner et al. (2014), we associate a characteristic velocity amplitude
155 V with each EF category using midpoint estimates derived from NOAA operational wind-speed intervals.

An effective energy-weighted observable is then defined as

$$E \sim V^2 LW.$$

The quantity E should not be interpreted as the exact kinetic energy of a tornado vortex. Instead, it represents a phenom-
ological observable combining intensity classification with geometrical extent.

160 Because EF-scale velocities take only a finite number of discrete values, the quantity E necessarily contains a degree of
artificial discretisation inherited from the EF classification itself. Consequently, the observable E is not treated here as a
primary dynamical quantity, but rather as a secondary robustness test designed to determine whether the geometric hierarchy
identified in $A = LW$ persists under coarse energetic weighting.

The principal conclusions of the present work therefore rely primarily on the geometry-only observables rather than on the
165 EF-weighted quantity E .



2.4 Scaling-law estimation

Empirical scaling laws were estimated through logarithmic regressions.

For two observables X and Y , relations of the form

$$Y \sim X^\eta$$

170 were estimated through ordinary least-squares (OLS) regression in logarithmic coordinates,

$$\log_{10} Y = \eta \log_{10} X + c,$$

where c denotes the intercept.

The principal scaling relations investigated are

$$W \sim L^\alpha,$$

175 $A \sim L^\eta,$

and

$$E \sim L^\beta.$$

The coefficient of determination

$$R^2$$

180 was used to quantify the quality of the logarithmic fits.

We emphasize that the width-length relation exhibits substantial intrinsic scatter, reflecting the strong intermittency and heterogeneity of tornado morphology. Consequently, the exponent α should not be interpreted as defining a deterministic geometric law for individual tornadoes, but rather as an effective statistical scaling exponent characterising average correlated growth over several decades in scale.

185 Because ordinary least-squares regression in log-log space may be affected by broad dispersion and heteroscedasticity, the measured exponents are interpreted primarily as effective scaling diagnostics rather than exact asymptotic critical exponents. Additional robustness checks were performed through temporal subsampling and null-model comparisons.



The SPC tornado archive is known to contain larger observational uncertainties for tornado path widths than for path lengths, particularly during the early decades of the record prior to modern Doppler-radar coverage and standardized survey procedures. Consequently, the measured scaling exponents should be interpreted as effective statistical descriptors rather than exact asymptotic critical exponents.

Rather than attempting to reconstruct event-by-event uncertainty propagation, the present work evaluates robustness through temporal subsampling, finite-size collapse, and null-model comparisons across independent observational eras.

2.5 Heavy-tail statistics

The statistical distributions of tornado observables were investigated using complementary cumulative distribution functions (CCDFs). For a positive random variable X , the CCDF is defined as

$$\text{CCDF}(x) = \mathbb{P}(X \geq x).$$

CCDF representations are particularly well suited for intermittent systems because they reduce sensitivity to binning procedures and preserve information associated with rare extreme events.

The distributions of L , W , A , and E were analysed over multiple decades in logarithmic coordinates. Special attention was devoted to the existence of broad heavy tails, finite-size truncation effects, and temporal robustness across historical observational periods.

The present analysis does not assume the existence of exact asymptotic power laws. Instead, the objective is to determine whether tornado observables exhibit approximate scale-free organisation over an extended intermediate range.

2.6 Finite-size scaling analysis

To investigate the robustness of the geometric hierarchy across observational eras, we employed a finite-size scaling framework inspired by statistical physics and critical phenomena.

Assuming the empirical geometric relation

$$A \sim L^\eta,$$

we define the reduced variable

$$X = \frac{A}{L^\eta}.$$

If tornado geometry obeys an approximately scale-invariant organisation law, then distributions associated with distinct historical periods should collapse approximately toward a common scaling function when expressed in terms of the reduced variable X .



215 The quality of the collapse was examined visually through CCDF comparisons before and after renormalisation. This pro-
cedure allows us to test whether the geometric hierarchy remains statistically stable despite major observational transitions
between 1950 and 2024.

2.7 Moment hierarchy and affine consistency relations

To characterise the statistical hierarchy associated with tornado geometry, we investigated conditional moments of the geometry-
220 only observable $A = LW$.

For each moment order q , we define

$$M_q(L) = \langle A^q | L \rangle,$$

where angular brackets denote averaging over tornadoes belonging to the same logarithmic length bin.

Because

225 $A = LW,$

one has identically

$$A^q = L^q W^q,$$

and therefore

$$M_q(L) = L^q \langle W^q | L \rangle.$$

230 Consequently, if the width-length relation obeys the effective scaling law

$$W \sim L^\alpha,$$

one naturally expects the approximate affine hierarchy

$$M_q(L) \sim L^{(1+\alpha)q},$$

leading to the prediction

235 $\zeta(q) \approx (1 + \alpha)q.$



The moment analysis is therefore not interpreted here as an independent proof of multifractality. Instead, it constitutes a consistency test designed to determine whether the observed geometric hierarchy remains coherent across multiple statistical levels.

The exponents $\zeta(q)$ were estimated through logarithmic regressions,

240 $\log M_q(L) \sim \zeta(q) \log L,$

over the moment range

$$0.25 \leq q \leq 3.$$

Residuals relative to the best affine fit were additionally computed in order to quantify possible departures from purely affine behaviour.

245 The same methodology was subsequently applied to the energy-weighted observable E in order to compare geometric and intensity-weighted hierarchies.

2.8 Null-model construction

A major objective of the present work is to determine whether the observed hierarchy reflects genuine correlated geometry or merely trivial consequences of broad marginal distributions.

250 Two null models were therefore constructed.

The first null model consists of randomly shuffling tornado widths independently of path lengths. This procedure preserves the marginal distributions of both L and W while destroying their correlations.

The second null model consists of synthetic lognormal fields possessing broad heavy-tailed distributions with variance comparable to the observed data.

255 The same hierarchy analysis was then applied to both null models. Comparing their scaling spectra with those of the real tornado catalogue allows us to determine whether the observed organisation originates from genuine correlated multiscale geometry rather than from marginal heavy-tailed statistics alone.

2.9 Local scaling analysis

To determine whether the measured scaling laws correspond to genuine intermediate scale-invariant regimes rather than purely
260 global regression artifacts, we computed the local scaling exponent

$$\eta(L) = \frac{d \log A}{d \log L}.$$



The derivative was estimated numerically using logarithmically binned data. A relatively stable local exponent over an extended range of scales supports the existence of a genuine scaling regime, whereas systematic drift would instead suggest the absence of robust scale invariance.

265 2.10 Numerical implementation

All analyses were performed using Python scientific libraries including NumPy, Pandas, SciPy, and Matplotlib. Statistical regressions were carried out in logarithmic space using ordinary least-squares methods. Moment calculations employed logarithmically spaced length bins in order to preserve statistical robustness across several decades in scale.

All figures and diagnostics presented in the present work were generated directly from fully reproducible analysis scripts
270 specifically developed for the SPC tornado catalogue.

3 Results

3.1 Width–length geometric scaling

We first investigate the intrinsic geometric organisation of tornado damage paths independently of any energetic weighting procedure. Figure 1 shows the relationship between tornado path width W and path length L over the full SPC tornado
275 database.

Despite substantial dispersion, a coherent average scaling law emerges,

$$W \sim L^{0.43},$$

indicating that tornadoes do not simply grow isotropically with path length. Instead, longer tornado tracks become proportionally wider according to a sublinear anisotropic growth law.

280 The moderate coefficient of determination $R^2 \approx 0.39$ reflects the strong intermittency expected in highly nonlinear atmospheric phenomena. Consequently, the exponent should not be interpreted as a deterministic law governing individual tornadoes, but rather as an effective statistical scaling relation emerging over multiple decades in scale.

Most importantly, the persistence of the scaling relation across the full SPC record strongly suggests the existence of a robust correlated geometric organisation linking tornado propagation and lateral broadening.

285 3.2 Geometry-only scaling: $A = LW$

Using the measured width–length relation, we define the geometry-only observable

$$A = LW,$$

which represents an effective damage-path area independent of EF-scale velocity assignments.

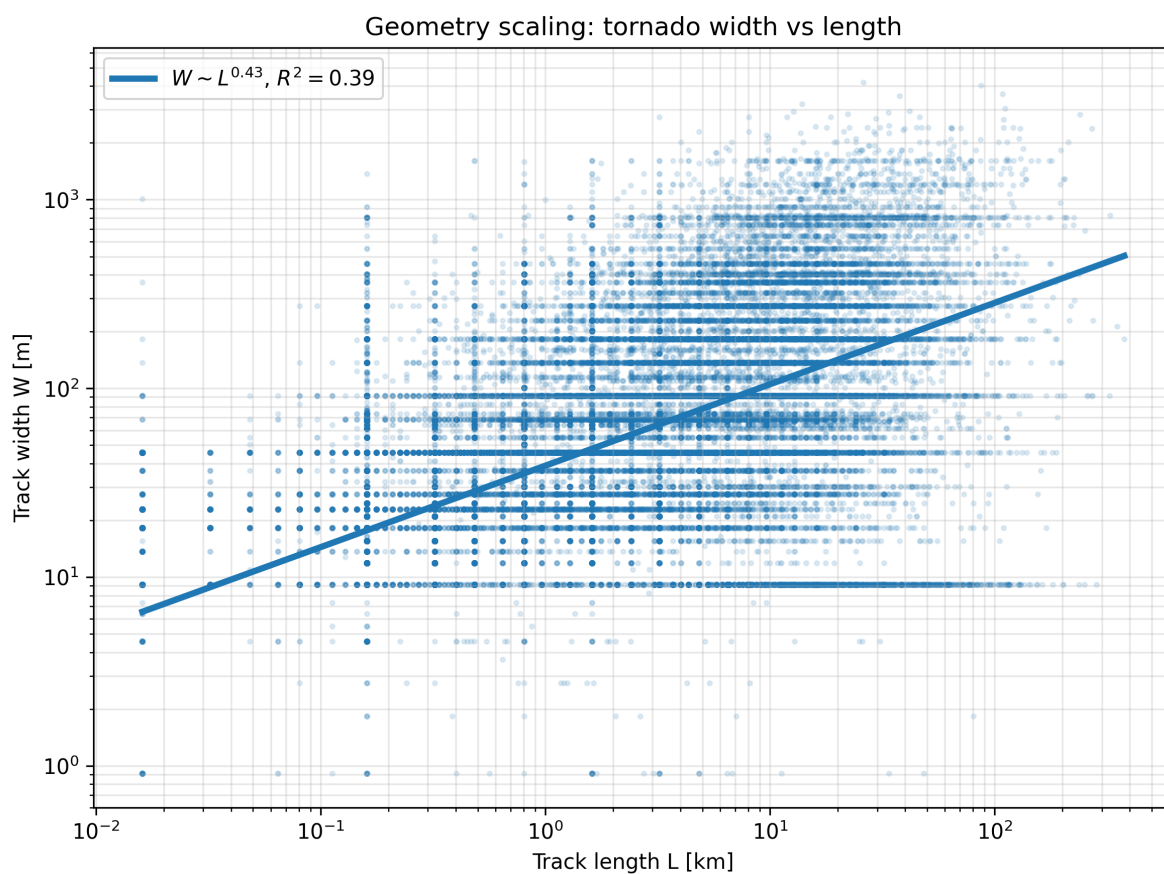


Figure 1. Scaling relation between tornado path width W and path length L . A statistically significant power-law trend is observed over several decades, with an effective scaling exponent $W \sim L^{0.43}$. The substantial scatter reflects the intrinsic intermittency and heterogeneity of tornado morphology, while the persistence of the average trend indicates a non-trivial correlated geometric organisation of tornado damage paths.



Figure 2 presents the scaling of A as a function of tornado length.

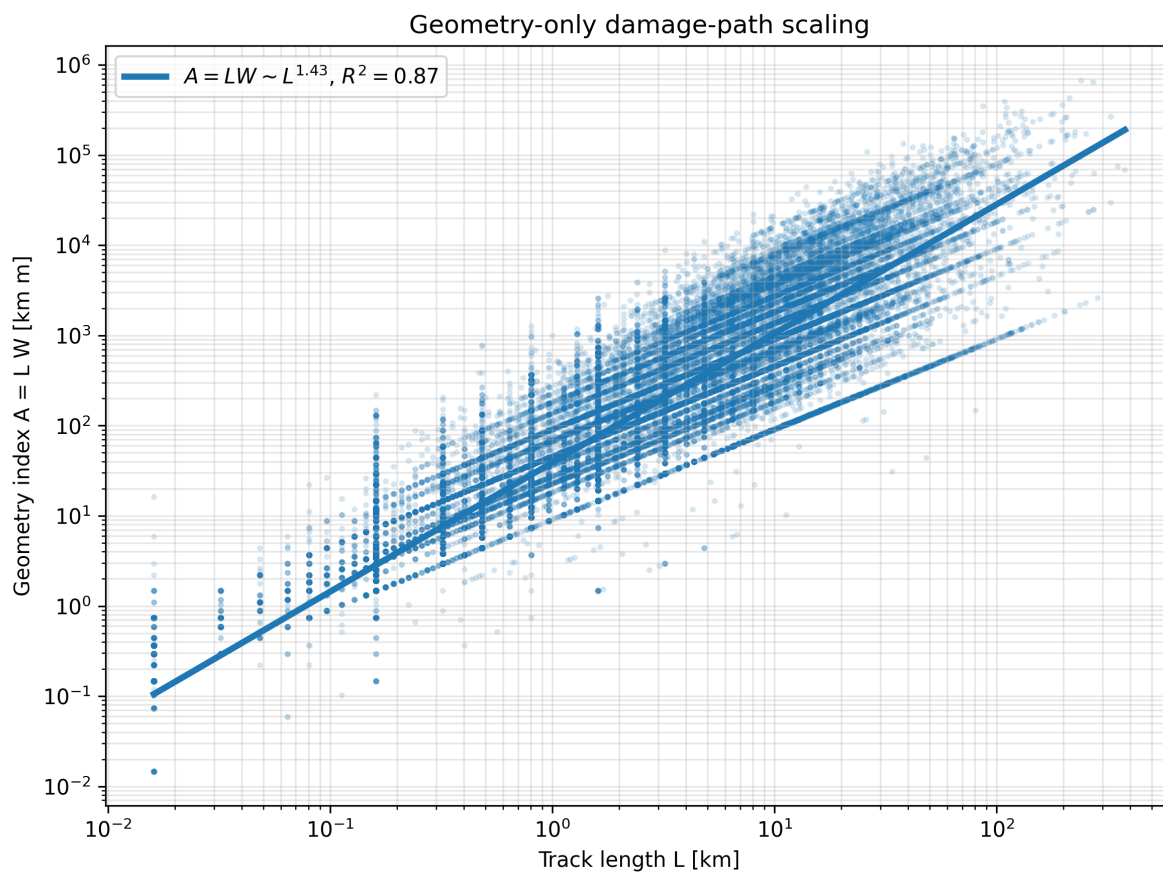


Figure 2. Scaling of the geometry-only damage-path observable $A = LW$ as a function of tornado path length L . A coherent scaling law $A \sim L^{1.43}$ emerges over several decades.

290 A robust effective scaling relation is obtained,

$$A \sim L^{1.43}.$$

Importantly, this result is not interpreted as an independent scaling law unrelated to the width–length relation. Since

$$A = LW,$$

and



295 $W \sim L^\alpha,$

one naturally expects

$$A \sim L^{1+\alpha}.$$

Using the independently measured exponent

$$\alpha \approx 0.43,$$

300 the predicted scaling exponent becomes

$$\eta = 1 + \alpha \approx 1.43,$$

in excellent agreement with the directly measured value.

The significance of this result therefore lies not in the existence of an independent area scaling law, but rather in the strong internal consistency of the correlated geometric hierarchy linking tornado length and width across scales.

305 Most importantly, the dominant multiscale organisation already exists at the purely geometric level, independently of any energetic weighting procedure involving EF-scale velocities.

3.3 Heavy-tailed geometry distributions

We next examine the statistical distributions of the geometric observables. Figure 3 presents the complementary cumulative distribution functions (CCDFs) of L , W , and A .

310 All distributions display pronounced heavy tails extending over several orders of magnitude. The absence of a characteristic scale is consistent with the behaviour of many geophysical extreme-event systems exhibiting approximate scale-free organisation.

However, unlike systems characterised primarily by a single size-distribution observable, tornado geometry exhibits a strong correlated scaling relation linking path length and width. The geometry observable A therefore inherits a coherent multiscale
315 organisation associated with correlated anisotropic growth.

3.4 Finite-size collapse across historical periods

To investigate temporal robustness, the SPC catalogue was divided into three independent historical periods: 1950–1974, 1975–1999, and 2000–2024.

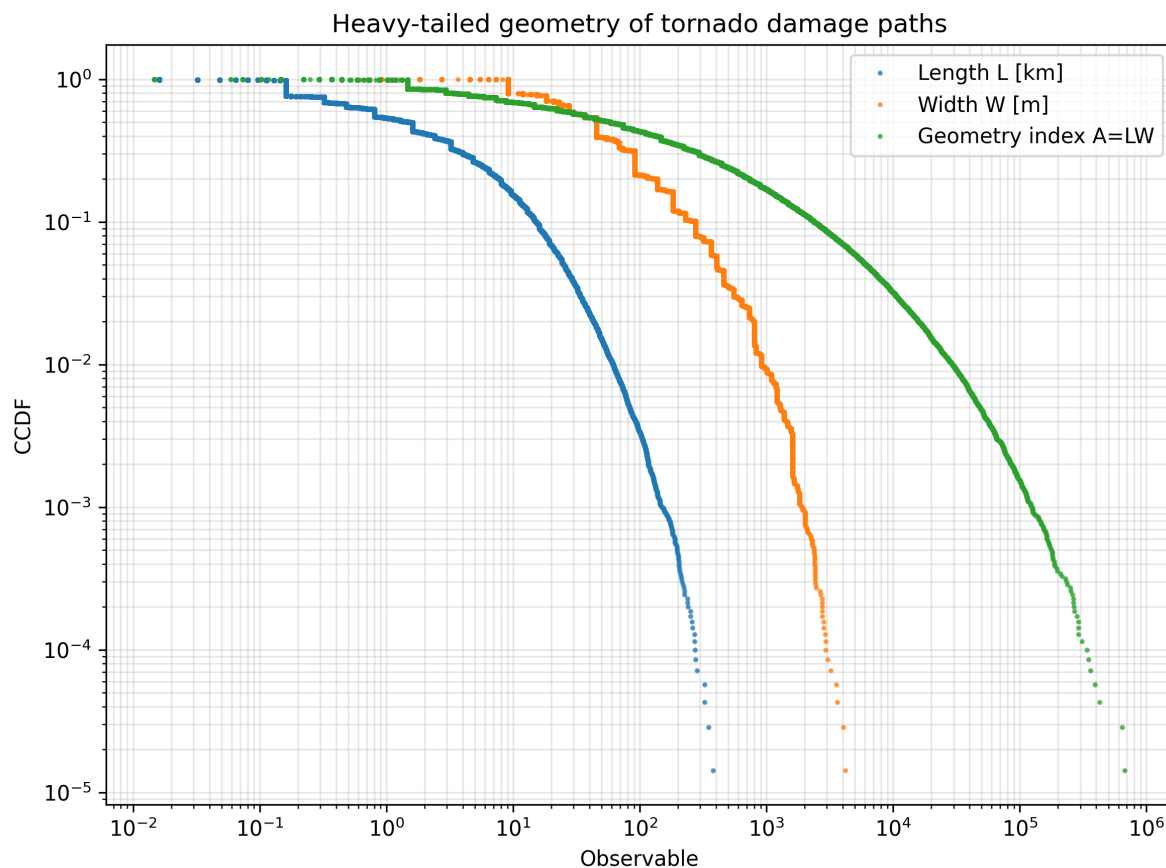


Figure 3. Complementary cumulative distribution functions (CCDFs) of tornado path length L , width W , and geometry observable $A = LW$. All observables exhibit broad heavy-tailed distributions spanning multiple decades, indicating the absence of a characteristic tornado size scale.

Figure 4 shows the CCDFs of the geometry observable A before rescaling.

320 Moderate differences appear between periods, particularly in the tails, likely reflecting changes in tornado detection and reporting practices over time. Nevertheless, the overall structure of the distributions remains remarkably similar.

We then perform a finite-size collapse using the global scaling exponent

$$\eta \approx 1.43.$$

The rescaled distributions are shown in Figure 5.

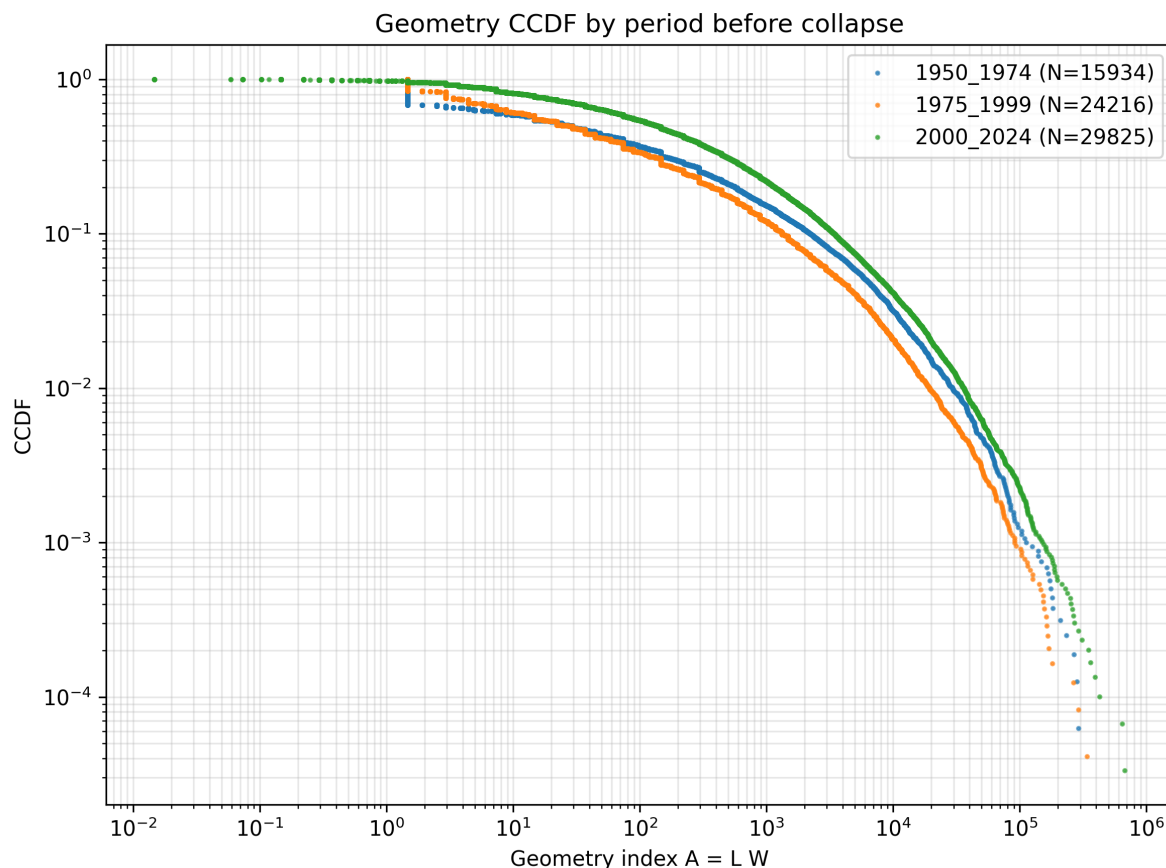


Figure 4. CCDFs of the geometry observable $A = LW$ for three independent historical periods before finite-size rescaling. Moderate differences are visible due to observational evolution and reporting completeness, but the global distributional structure remains stable across decades.

325 After rescaling, the distributions collapse approximately onto a common master curve over a broad statistical range. This result strongly suggests that the underlying geometric organisation remains approximately stationary despite major observational transitions between 1950 and 2024.

The observed hierarchy therefore appears to reflect an intrinsic statistical property of tornado morphology rather than a purely observational artifact.

330 3.5 Temporal robustness of the scaling exponents

To quantify temporal robustness more directly, we independently estimate the scaling exponents within each historical period. Figure 6 compares the directly measured exponent

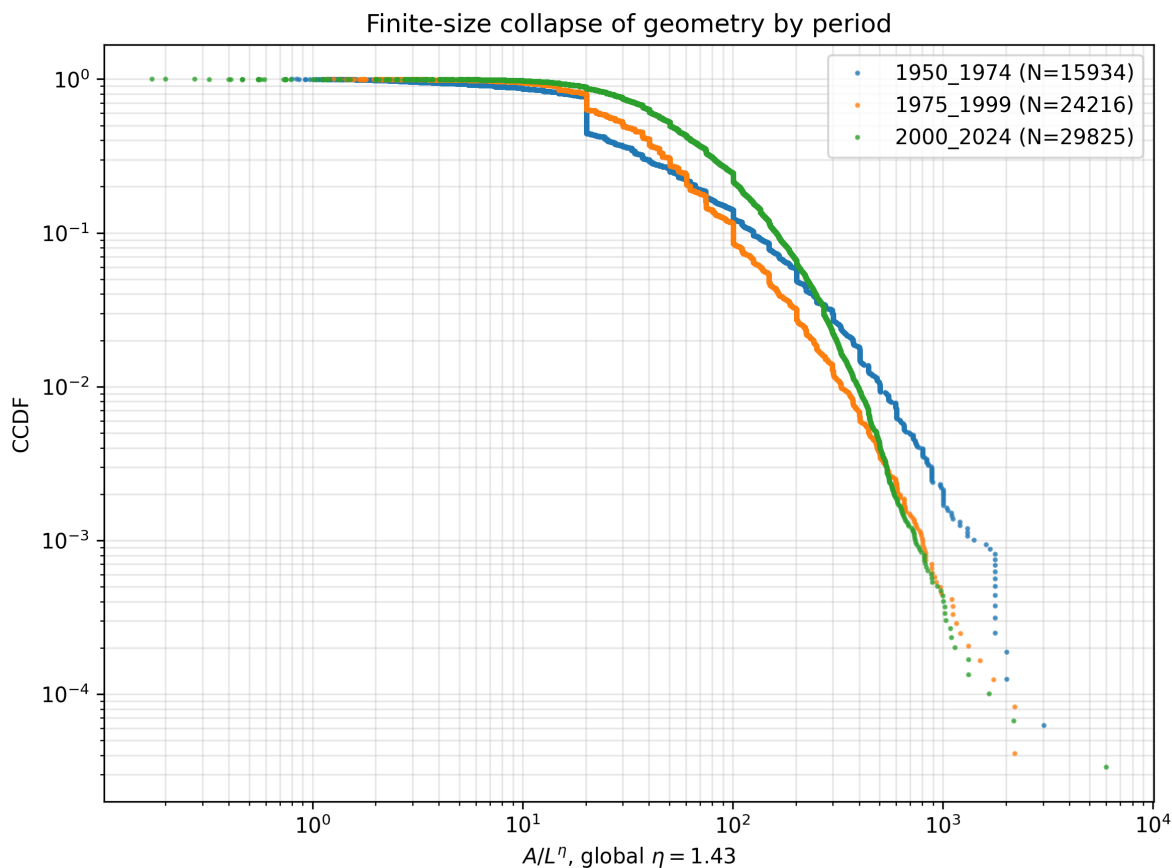


Figure 5. Finite-size collapse of the geometry-observable distributions using the global exponent $\eta \approx 1.43$. The collapse demonstrates that the geometric statistics remain approximately invariant across historical periods despite major observational evolution.

$$A \sim L^\eta,$$

with the geometrically predicted value

335 $\eta = 1 + \alpha,$

deduced independently from the width–length relation

$$W \sim L^\alpha.$$

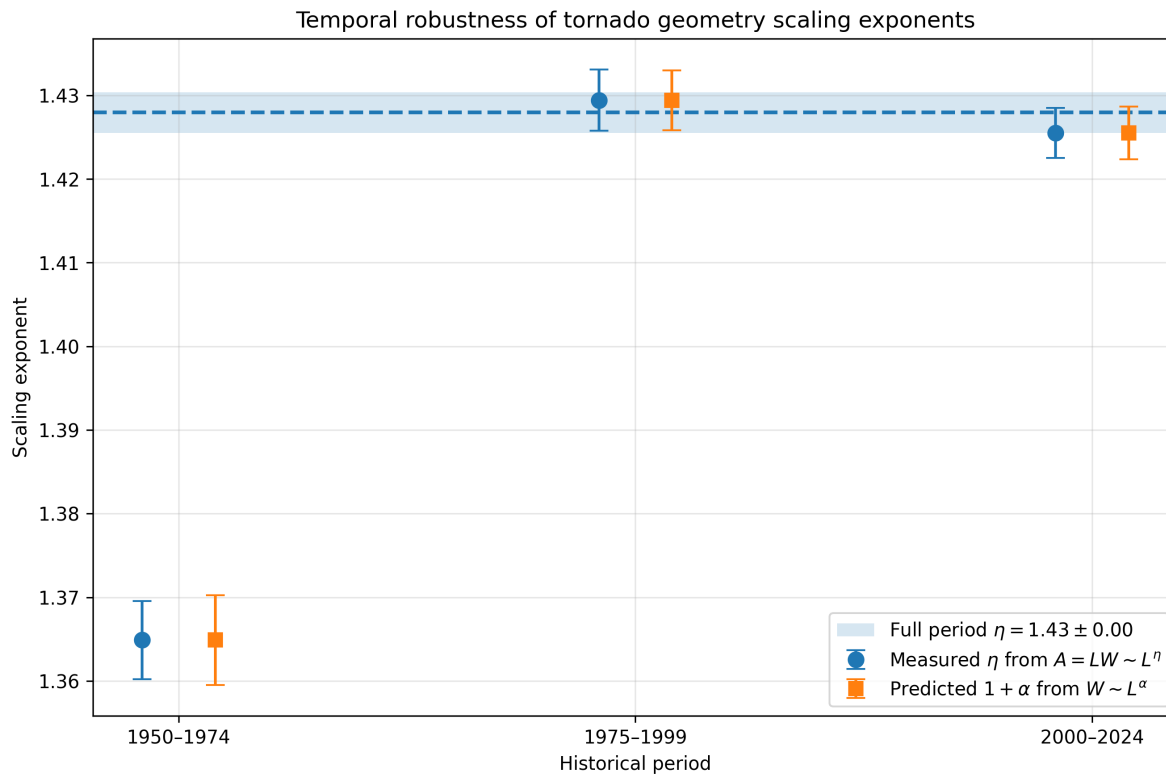


Figure 6. Temporal robustness of the geometric scaling exponents across independent historical periods. Blue circles show the directly measured exponent η from the scaling relation $A \sim L^\eta$, while orange squares show the predicted value $1 + \alpha$ deduced independently from the width–length relation $W \sim L^\alpha$. The close agreement across all historical periods demonstrates both the temporal stability and the internal consistency of the geometric hierarchy. Error bars correspond to bootstrap uncertainties.

The independently measured exponents remain highly consistent across all periods and agree closely with the predicted values $1 + \alpha$.

340 This agreement constitutes an important internal consistency test of the scaling framework. Although small variations exist between eras, all measured exponents remain compatible within uncertainty bars with the global value

$$\eta \approx 1.43.$$

The correlated geometric hierarchy therefore appears statistically robust despite substantial historical changes in observational technology and reporting completeness.



345 3.6 Consistency of the moment hierarchy with geometric scaling

We next investigate conditional moment statistics of both the geometry-only observable A and the energy-weighted quantity

$$E = V^2 LW,$$

where V is assigned from EF-scale velocity intervals.

Because the geometry-only observable is defined as

$$350 \quad A = LW,$$

the previously measured width–length relation

$$W \sim L^\alpha$$

naturally implies

$$A \sim L^{1+\alpha}.$$

355 More generally, one expects the conditional moments

$$M_q(L) = \langle A^q | L \rangle$$

to scale approximately as

$$M_q(L) \sim L^{(1+\alpha)q},$$

provided that the width statistics remain sufficiently self-similar across scales.

360 The purpose of the present subsection is therefore not to interpret the affine hierarchy as an independent multifractal discovery, but rather to test whether the observed geometric organisation remains statistically self-consistent across higher-order moments.

Figure 7 compares the corresponding scaling spectra.

Both observables exhibit nearly affine spectra of the form

$$365 \quad \zeta(q) \simeq Hq,$$

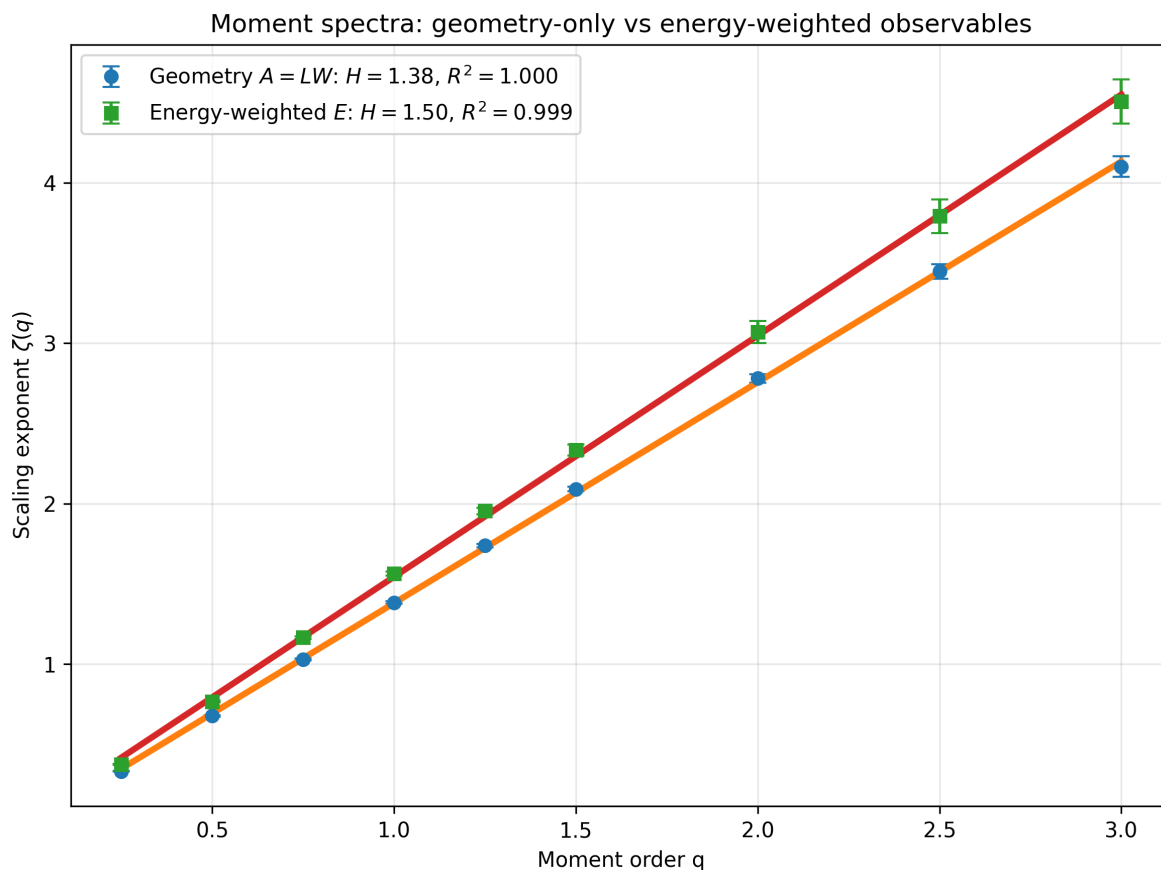


Figure 7. Moment-scaling spectra $\zeta(q)$ for the geometry-only observable $A = LW$ and the energy-weighted observable $E = V^2LW$. Both observables exhibit predominantly affine scaling behaviour over the investigated moment range.

with effective exponents

$$H_A \approx 1.38, \quad H_E \approx 1.50.$$

The geometry-only hierarchy is therefore fully consistent with the previously measured anisotropic geometric scaling relation linking tornado width and length.

370 The energy-weighted quantity produces a slightly steeper hierarchy, but the overall affine structure remains dominated by the underlying geometry. The EF-based velocity weighting therefore modifies the global scaling amplitude without fundamentally changing the dominant organisation law.

To quantify possible departures from purely affine behaviour, we analyse the residuals relative to the best linear fits. The results are shown in Figure 8.

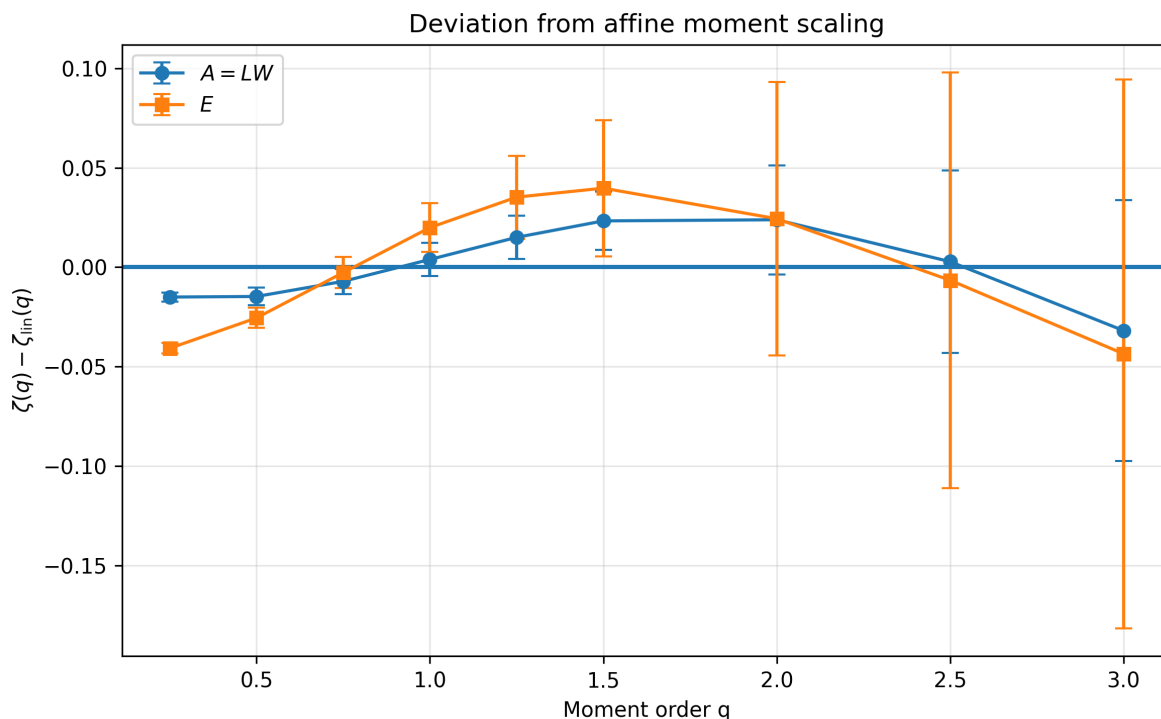


Figure 8. Residual deviations from affine moment scaling for both geometry-only and energy-weighted observables. Within uncertainty bars, the spectra remain broadly compatible with predominantly affine behaviour over the investigated moment range.

375 The deviations remain weak and largely compatible with statistical uncertainties. No strong multifractal curvature is observed over the accessible moment range.

Unlike strongly multifractal atmospheric rainfall fields, whose moment spectra often exhibit pronounced nonlinear curvature, tornado geometry appears considerably closer to a predominantly affine hierarchy with only weak departures from purely affine scaling.

380 3.7 Comparison with null models

A central question is whether the observed hierarchy could arise trivially from the construction of the observables themselves. To address this issue, we compare the real tornado statistics with two null models:

1. a shuffled-width model, where width values are randomly permuted independently of length;
2. a synthetic lognormal model with comparable variance.

385 Figure 9 presents the corresponding moment spectra.

The shuffled-width model produces a substantially weaker hierarchy,

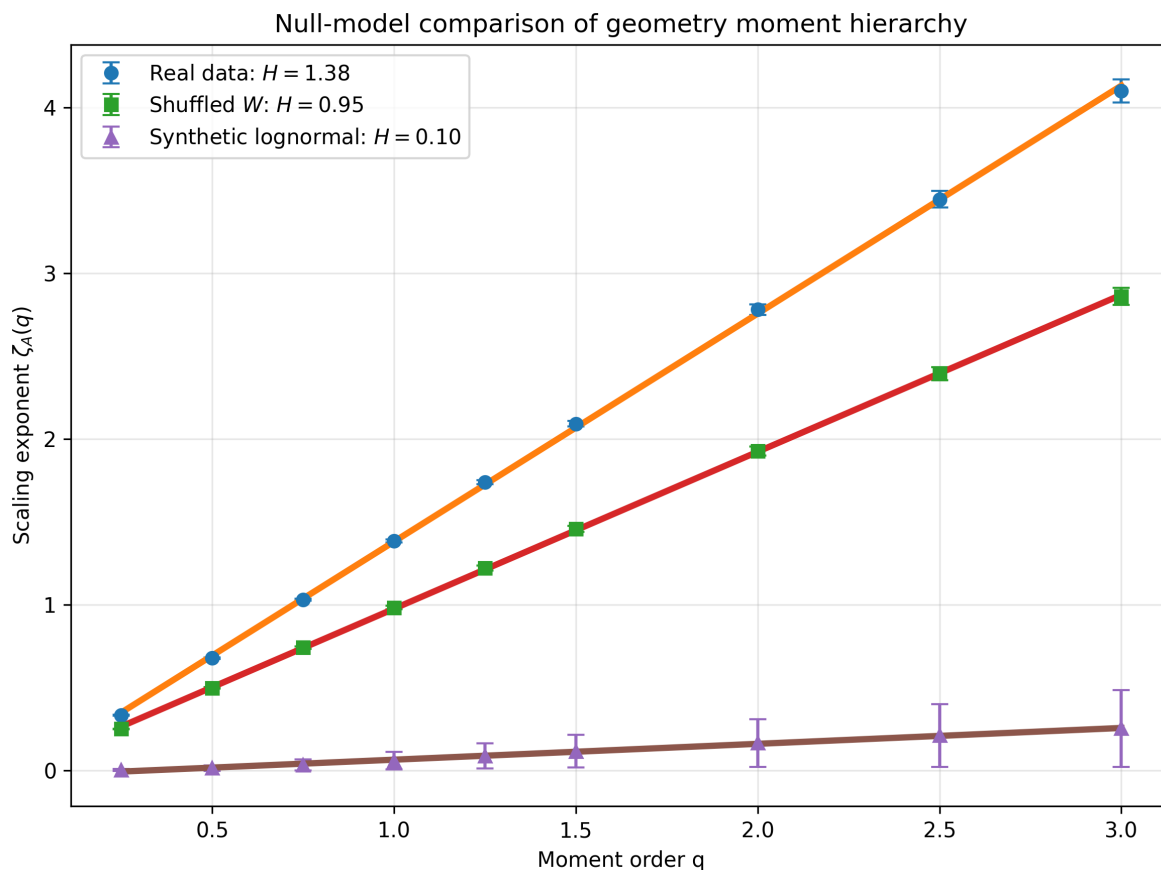


Figure 9. Comparison of moment-scaling spectra between real tornado geometry, shuffled-width data, and synthetic lognormal fields. The strong reduction of the scaling exponent in the null models demonstrates that the observed hierarchy is not a trivial consequence of broad marginal distributions alone.

$$H_{\text{shuffle}} \approx 0.95,$$

while the synthetic lognormal field yields only a very weak scaling behaviour,

$$H_{\text{lognormal}} \approx 0.10.$$

390 For a perfectly decorrelated geometry, one asymptotically expects

$$H = 1.$$



The small deviation of the measured shuffled exponent from unity most likely reflects finite-size effects, logarithmic binning, and residual sampling fluctuations.

395 Importantly, however, the shuffled hierarchy remains dramatically weaker than the hierarchy observed in the real tornado dataset.

These results demonstrate that the observed organisation cannot be explained solely by broad heavy-tailed marginal distributions. Instead, the hierarchy strongly depends on genuine correlations linking tornado length and width across scales.

This conclusion is reinforced by the CCDF comparison shown in Figure 10.

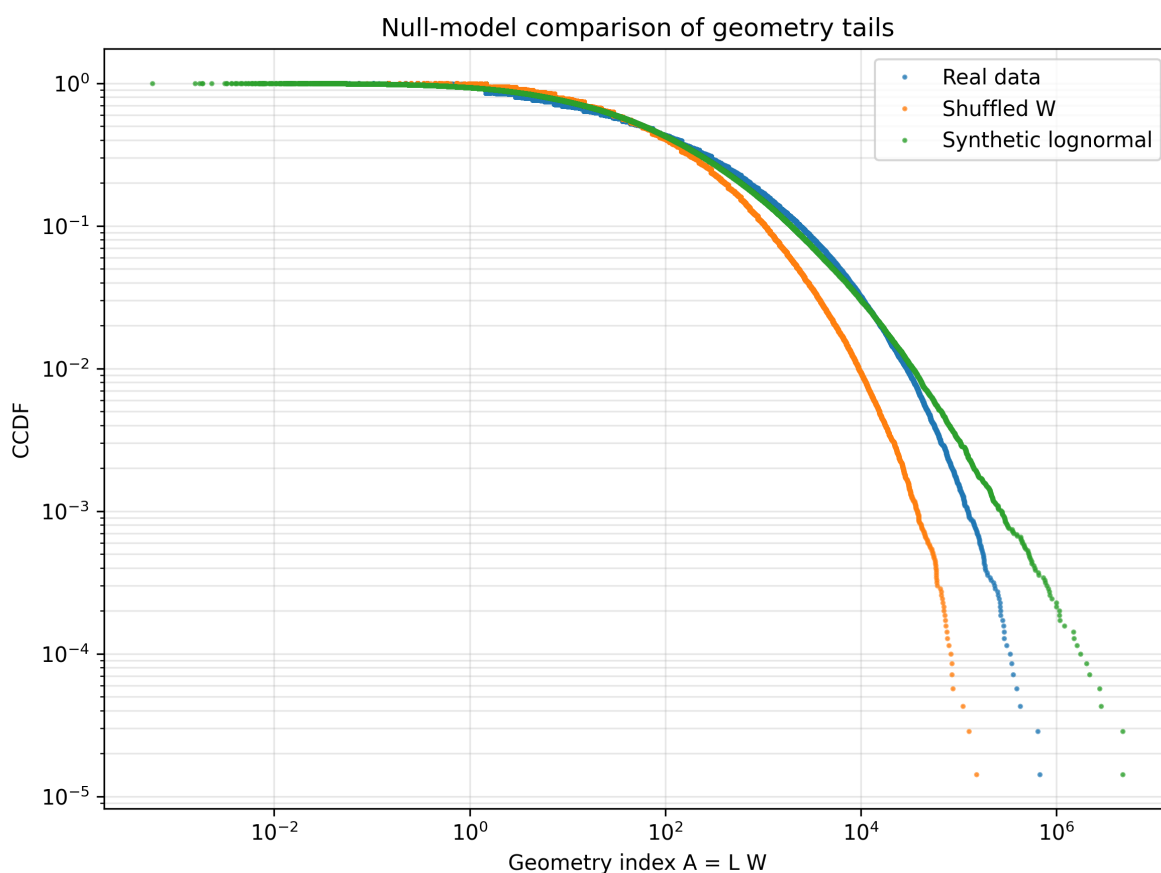


Figure 10. CCDF comparison between real tornado geometry and null models. Although synthetic and shuffled models reproduce broad-tailed distributions, they fail to reproduce the correlated scaling organisation observed in the real data.

400 Although the null models reproduce broad-tailed distributions, they fail to reproduce the coherent correlated hierarchy observed in the real tornado catalogue.



3.8 Local scaling exponent

Finally, we investigate the local scaling exponent defined by

$$\eta(L) = \frac{d \log A}{d \log L}.$$

Figure 11 presents the evolution of this local exponent as a function of scale.

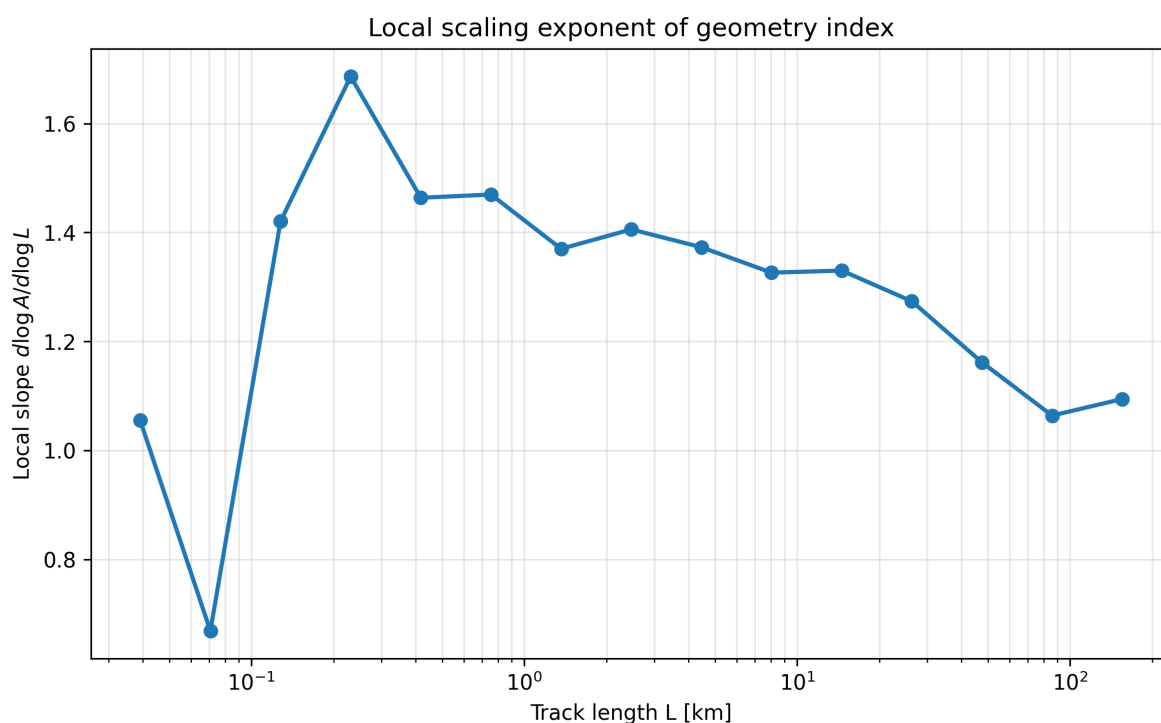


Figure 11. Local scaling exponent $\eta(L) = d \log A / d \log L$ of the geometry observable. After short-scale fluctuations, the exponent develops a relatively stable intermediate-scale regime, supporting the existence of a genuine scale-invariant geometric organisation rather than a purely global regression artifact.

405 After significant fluctuations at the smallest scales, the local exponent develops an extended intermediate regime approximately spanning

$$\eta(L) \approx 1.3 - 1.5,$$

over nearly two decades in scale. At the largest scales, stronger fluctuations reappear due to finite-size effects and sparse sampling of extreme tornado events.



410 The existence of this broad intermediate regime nevertheless demonstrates that the observed hierarchy is not merely an artifact of global regression fitting. Instead, tornado damage geometry exhibits a genuine correlated scale-invariant organisation over a broad range of spatial scales.

Overall, the results consistently indicate that tornado damage paths possess a robust correlated multiscale geometry characterised by heavy tails, anisotropic scaling, and approximate finite-size invariance across historical periods.

415 4 Discussion

The present study reveals that tornado damage paths exhibit a robust correlated geometric organisation across more than seven decades of observations in the SPC database. Despite the intrinsic intermittency, heterogeneity, and observational complexity of severe convective systems, the statistical relations linking tornado path length and width remain remarkably stable over a broad range of scales. The coexistence of heavy-tailed distributions, anisotropic geometric scaling, temporal robustness, finite-size
420 collapse, and coherent higher-order statistics strongly suggests that tornado morphology is governed by non-trivial multiscale geometric constraints rather than by purely random spatial organisation.

A central result of the present work is the existence of a robust anisotropic scaling relation between tornado path width and path length,

$$W \sim L^\alpha, \quad \alpha \approx 0.43.$$

425 This relation constitutes the primary non-trivial scaling law identified in the SPC tornado record. The exponent α quantifies the correlated growth of tornado geometry and indicates that longer tornadoes undergo systematic but sublinear lateral broadening during propagation.

The geometry-only observable

$$A = LW$$

430 naturally inherits this correlated organisation. Since

$$A = LW \quad \text{and} \quad W \sim L^\alpha,$$

one directly expects

$$A \sim L^{1+\alpha}.$$

The measured exponent



435 $\eta \approx 1.43$

is therefore fully consistent with the independently measured width–length relation. The significance of this agreement does not lie in the existence of an independent area scaling law, but rather in the strong internal consistency of the correlated geometric hierarchy across multiple statistical diagnostics.

This clarification is important for interpreting the higher-order moment hierarchy. The approximately affine spectra

440 $\zeta(q) \approx Hq$

should not be interpreted as evidence for a fully developed multifractal cascade analogous to those encountered in atmospheric turbulence or rainfall intermittency. Once the anisotropic geometric relation $W \sim L^\alpha$ is established, an approximately affine hierarchy for the conditional moments of $A = LW$ becomes a natural consequence of the correlated geometry itself. The moment analysis therefore acts primarily as a consistency test probing whether the geometric organisation remains coherent
445 across higher-order statistics.

The comparison with null models further reinforces this interpretation. When tornado widths are randomly shuffled independently of path length, the resulting hierarchy weakens substantially and approaches the trivial affine behaviour expected for decorrelated variables. Likewise, synthetic lognormal fields possessing broad heavy-tailed marginals fail to reproduce the observed hierarchy. These results demonstrate that the measured organisation cannot be explained solely by broad marginal
450 distributions or simple algebraic construction effects. Instead, the hierarchy depends critically on genuine correlations linking tornado length and width across scales.

Interestingly, the shuffled-width experiment produces an exponent close to

$$H_{\text{shuffle}} \approx 0.95,$$

rather than the exact value $H = 1$ expected for perfectly decorrelated variables. This small discrepancy most likely originates
455 from finite-size effects, logarithmic binning, and residual sampling fluctuations. The deviation nevertheless remains weak compared with the substantial separation between the real-data hierarchy and the null models, confirming that the dominant geometric organisation remains genuinely non-trivial.

The local scaling analysis provides additional evidence that the observed hierarchy is not merely an artifact of global regression fitting. The local exponent

460 $\eta(L) = \frac{d \log A}{d \log L}$



develops a relatively stable intermediate-scale regime extending over nearly two decades in scale. Although fluctuations remain visible at the smallest and largest scales, where finite-size effects become important, the existence of this extended plateau strongly supports the presence of a genuine scale-invariant geometric regime.

465 The finite-size scaling analysis additionally demonstrates that the observed hierarchy remains remarkably robust across independent historical periods. After appropriate renormalisation, the distributions associated with the periods 1950–1974, 1975–1999, and 2000–2024 collapse approximately onto a common master curve despite major changes in observational technology, Doppler-radar deployment, storm-survey procedures, and reporting practices. The temporal robustness of the independently measured scaling exponents further strengthens this conclusion.

470 This result is particularly important because tornado databases are known to suffer from substantial temporal inhomogeneities Agee and Childs (2014); Edwards et al. (2021). Weak tornadoes were likely underreported during the early decades of the SPC record, whereas modern Doppler-radar coverage significantly improved the detection of short-lived and low-intensity events. The persistence of nearly identical geometric scaling structures across historical periods therefore strongly suggests that the hierarchy reflects intrinsic atmospheric organisation rather than purely observational biases.

475 Although the present work remains fundamentally statistical, the observed scaling relations nevertheless suggest several plausible physical mechanisms capable of producing the correlated geometric hierarchy identified in tornado damage paths.

One possible interpretation involves the competition between vortex stretching, turbulent broadening, and finite tornado lifetime. If tornado path length is interpreted as an effective propagation distance associated with vortex survival time,

$$L \sim UT,$$

where U denotes a characteristic translation velocity and T an effective lifetime, then the measured relation

480 $W \sim L^{0.43}$

suggests that tornado width evolves sublinearly during propagation. Such behaviour is qualitatively compatible with progressive lateral broadening of coherent vortices under the combined effects of turbulent entrainment, environmental shear, and vortex stretching.

The measured exponent also remains relatively close to diffusive growth,

485 $W \sim L^{1/2},$

which may indicate that lateral expansion behaves approximately as a stochastic growth process constrained by finite vortex coherence. Within this interpretation, longer-lived tornadoes would not simply extend their trajectories while preserving constant transverse structure. Instead, their geometry would evolve through correlated anisotropic growth where longitudinal propagation and lateral broadening remain dynamically coupled.



490 The observable

$$A = LW$$

then naturally inherits a multiplicative amplification structure,

$$A \sim L^{1+\alpha},$$

consistent with the measured exponent

495 $\eta \approx 1.43$.

Such multiplicative geometric amplification is reminiscent of correlated growth processes encountered in several intermittent nonequilibrium systems. From a fluid-dynamical perspective, the observed hierarchy may therefore reflect the statistical imprint of persistent mesocyclonic organisation capable of sustaining coherent vortex structures over extended spatial and temporal scales.

500 At the same time, the present analysis does not demonstrate the existence of a true turbulent energy cascade in the strict fluid-mechanical sense. Several mechanisms may potentially contribute simultaneously to the observed scaling behaviour, including correlated geometric growth, constrained stochastic amplification, vortex stretching, and self-organised convective dynamics. The analogy with turbulence and multiplicative cascades should therefore be interpreted cautiously. What the data robustly demonstrate is the existence of correlated multiscale geometry and approximate scale invariance, not the direct existence of
505 inter-scale energy transfer processes.

The present approach also possesses several limitations. First, the energy-weighted observable

$$E \sim V^2 LW$$

remains a coarse proxy constructed from path geometry and discrete EF-scale velocity intervals rather than from fully resolved dynamical wind-field measurements. Because the EF scale contains only a small number of discrete categories, the
510 quantity E necessarily inherits a degree of artificial discretisation. The principal conclusions of the present work therefore rely primarily on the geometry-only observables rather than on the energetic proxy itself.

Second, although the hierarchy remains statistically stable across historical periods, residual reporting biases likely still influence the precise values of some exponents. In particular, weak tornadoes are more strongly affected by historical reporting incompleteness. Future work should therefore investigate the stability of the measured exponents under intensity-threshold
515 subsampling (e.g. EF1+, EF2+, or EF3+ events only) in order to determine whether the correlated geometric hierarchy remains universal across tornado intensity regimes or instead reflects distinct classes of convective vortex organisation.



Future work should also aim to connect the statistical results obtained here with explicit atmospheric dynamics. Coupling the present geometric scaling analysis with environmental variables derived from reanalysis datasets such as ERA5 may help determine how CAPE, vertical wind shear, storm-relative helicity, and storm organisation influence the measured exponents. 520 Similarly, high-resolution Doppler-radar observations and reduced coherent-vortex models could help establish more direct links between tornado morphology, vortex stretching, and coherent-structure persistence.

More broadly, the present results suggest that tornado climatology may benefit substantially from approaches originating from statistical physics, nonlinear dynamics, and complex-systems theory. Rather than treating tornadoes solely as isolated meteorological events, the observed scaling laws indicate that their collective organisation reflects emergent multiscale constraints associated with strongly nonlinear atmospheric dynamics. 525

The present results therefore suggest that tornado climatology may contain a previously underexplored level of correlated geometric organisation whose statistical structure remains remarkably stable despite the intrinsic intermittency and observational complexity of severe convective systems.

5 Conclusion

530 This work investigated the multiscale statistical organisation of tornado damage-path geometry using the long-term NOAA Storm Prediction Center tornado database covering the period 1950–2024. Rather than focusing exclusively on tornado occurrence frequencies or EF-scale statistics, we analysed the geometrical structure of tornado damage paths using methods inspired by statistical physics, turbulence theory, and nonlinear dynamics.

Several robust scaling laws were identified. Tornado path width scales with path length according to

535 $W \sim L^{0.43}$,

while the geometry-only damage-path observable

$$A = LW$$

obeys the relation

$$A \sim L^{1.43}.$$

540 These scaling laws remain stable across historical observational periods and persist over several decades in scale, indicating the existence of robust correlated geometric organisation.

The statistical distributions of tornado length, width, and geometry index exhibit broad heavy-tailed behaviour without a characteristic intrinsic scale. After finite-size renormalisation, the distributions associated with different historical periods



collapse approximately onto common master curves, suggesting that the underlying geometric hierarchy is remarkably stable
545 despite substantial observational evolution between 1950 and 2024.

A central result of the present study concerns the hierarchy of conditional moments. Both the geometry-only observable A and the energy-weighted quantity

$$E = V^2 LW$$

display nearly affine scaling spectra of the form

550 $\zeta(q) \approx Hq,$

with weak deviations from linearity over the accessible range of moments. Importantly, the geometry-only observable already possesses a strong scaling hierarchy prior to any energetic weighting. The dominant multiscale organisation therefore emerges fundamentally from correlated tornado geometry itself.

Comparison with shuffled and synthetic null models demonstrates that the observed hierarchy cannot be explained solely by
555 heavy-tailed marginal distributions or trivial construction effects. The scaling depends critically on the correlated organisation linking tornado length and width across scales. The local scaling analysis further confirms the existence of a genuine scale-invariant regime extending over nearly two decades.

The present results suggest that tornado damage paths constitute a statistically organised multiscale field characterized by correlated geometry, approximate scale invariance, and weak intermittency corrections. The observed hierarchy is compatible
560 with the interpretation of tornadoes as evolving coherent vortices embedded within strongly nonlinear convective systems. However, the present work does not directly demonstrate the existence of a turbulent energy cascade in the strict dynamical sense. Rather, it establishes the existence of robust geometric scaling organisation at climatological scales.

More broadly, this study illustrates how concepts originating from statistical physics and nonlinear dynamics can provide useful insight into severe-weather climatology. Tornadoes appear not merely as isolated extreme events, but as manifestations
565 of an emergent correlated multiscale atmospheric process governed by approximate scale invariance and coherent geometric organisation.

The present results therefore suggest that tornado climatology may contain a previously underexplored level of correlated geometric organisation whose statistical structure remains remarkably stable despite the intrinsic intermittency and observational complexity of severe convective systems.



570 Appendix A: Geometrical observables and energy-weighted quantities

A1 Geometrical observables

The principal objective of the present work is not to infer the exact kinetic energy of tornado vortices, but rather to determine whether tornado damage geometry itself possesses statistically robust multiscale organisation.

575 Direct dynamical measurements of tornado energetics are unavailable at climatological scales. Operational tornado catalogues primarily provide:

- tornado path length,
- tornado path width,
- EF intensity classification,
- and geographical information.

580 The present analysis therefore begins from purely geometrical observables independent of EF-scale velocity assignments. Let L denote tornado path length and W tornado path width. We define the geometry-only observable

$$A = LW,$$

which represents an effective damage-path area index.

585 The quantity A should not be interpreted as the exact physical damaged area produced by a tornado, since actual damage depends strongly on land-surface heterogeneity, structural vulnerability, and local exposure conditions. Instead, A constitutes a coarse-grained geometrical descriptor designed to characterize the large-scale statistical organisation of tornado morphology.

The observable $A = LW$ was deliberately chosen as the minimal physically interpretable geometrical descriptor of tornado damage-path extent. We did not introduce generalized combinations of the form

$$L^a W^b,$$

590 since additional free exponents could artificially improve scaling collapse through over-parameterisation. The purpose of the analysis is instead to determine whether the simplest geometrical combination already exhibits robust correlated scaling.

Importantly, the principal scaling hierarchy identified in the present work already emerges at the level of the geometry-only observable A , independently of any energetic weighting procedure.

A2 Construction of the energy-weighted observable

595 Although the main focus of the present study concerns geometric organisation, we additionally introduced an energy-weighted observable in order to compare geometry-only scaling with intensity-weighted statistics.



The energy-weighted quantity was constructed as

$$E \sim V^2 LW,$$

where:

- 600 – L denotes tornado path length,
- W tornado path width,
- V a representative velocity amplitude associated with the EF intensity class.

This construction is motivated by dimensional arguments. Assuming that tornado damage scales approximately with the action of a rotating vortex over an effective geometrical footprint,

605 $A \sim LW,$

one may expect an intensity-weighted observable of the form

$$E \sim \rho V^2 A,$$

up to multiplicative constants absorbed into the definition.

610 Importantly, the quantity E should not be interpreted as the exact kinetic energy of a tornado vortex. Instead, it represents a phenomenological observable combining geometric extent and EF-scale intensity information.

A major objective of the present work is precisely to determine whether the scaling hierarchy observed in E already exists within the purely geometric observable $A = LW$, or whether it emerges only after energetic weighting.

A3 EF velocity assignment

615 Representative velocity amplitudes were assigned to each EF category using midpoint values derived from NOAA operational intervals associated with the Enhanced Fujita scale.

The resulting values are summarized in Table A1.

Alternative assignments using lower-bound and upper-bound velocities were also tested. The resulting scaling exponents varied only weakly, indicating that the dominant statistical hierarchy is controlled primarily by correlated geometry rather than by the precise calibration of EF velocities.

620 A4 Robustness to observational periods

Because tornado databases are affected by substantial observational evolution Agee and Childs (2014); Edwards et al. (2021), robustness tests were additionally performed over several restricted subsets.



Table A1. Representative velocity amplitudes associated with EF classes.

EF class	Representative velocity (m/s)
EF0	35.8
EF1	49.2
EF2	61.7
EF3	74.8
EF4	89.4
EF5	100.6

The scaling analysis was repeated for: - the Doppler-radar era (1995–2024), - the Enhanced Fujita era (2007–2024), - and EF1+ or EF2+ subsets designed to reduce weak-event underreporting.

625 The principal geometric scaling relations remained stable throughout all subsets. In particular, the affine structure of the moment hierarchy persisted despite moderate fluctuations of the effective scaling exponent.

These results strongly suggest that the observed geometric organisation does not arise solely from evolving reporting practices or observational artifacts.

Appendix B: Robustness tests and null-model analysis

630 B1 Sensitivity to logarithmic binning

Moment-hierarchy estimation requires logarithmic binning of tornado path length. Since scaling exponents may be sensitive to binning procedures in intermittent systems, several robustness tests were performed using different logarithmic partitions.

The analysis was repeated using: - 10 bins, - 12 bins, - 15 bins, - and 20 bins.

635 In all cases, the measured moment spectrum retained a remarkably stable affine structure. Variations in the effective scaling exponent remained small compared with the global scaling amplitude.

This robustness indicates that the observed hierarchy is not an artifact of a particular binning choice.

B2 Robustness of the moment hierarchy

The moment analysis was additionally repeated over different moment ranges. In particular: - low-order moments, - high-order moments, - and truncated ranges excluding extreme events

640 were investigated separately.

The quasi-affine structure of

$\zeta(q)$



persisted throughout all tested ranges, although weak deviations from linearity became slightly more visible at large moment orders.

645 This behaviour is characteristic of weakly intermittent multiscale systems in which rare extreme events contribute disproportionately to high-order statistics.

Importantly, the near-affine structure of the moment spectrum should not be interpreted solely through the large coefficient of determination associated with linear fitting. The crucial result instead lies in the robustness of the hierarchy under null-model comparison, temporal subdivision, and local-scaling diagnostics.

650 **B3 Null-model comparison**

A major objective of the present study is to determine whether the observed scaling hierarchy reflects genuine correlated multiscale geometry or merely trivial consequences of broad marginal distributions combined with algebraic observable construction.

Two null models were therefore introduced.

655 The first null model consists of randomly shuffling tornado widths independently of path lengths. This procedure preserves the marginal distributions of L and W while destroying their correlations.

The second null model consists of synthetic lognormal fields possessing broad heavy-tailed distributions with variance comparable to the observed data.

The same moment-hierarchy analysis was then applied to both null models.

660 If the observed hierarchy resulted solely from the construction

$$A = LW,$$

then shuffled or synthetic fields possessing comparable marginal statistics would be expected to reproduce similar moment spectra.

This is not observed.

665 The shuffled-width model exhibits substantially weaker scaling, while the synthetic lognormal field produces only weak hierarchical organisation. These results demonstrate that the measured hierarchy depends critically on the existence of genuine correlations linking tornado path length and width across scales.

B4 Statistical uncertainty

Uncertainties on scaling exponents were estimated using ordinary least-squares regression errors in logarithmic coordinates together with bootstrap resampling procedures.

Although tornado geometry exhibits substantial scatter and intermittency, the very large size of the SPC catalogue ensures strong statistical convergence of the principal scaling exponents.

Bootstrap uncertainties remained small compared with the global scaling amplitudes throughout all robustness tests.



Appendix C: Tail statistics and possible physical interpretations

675 C1 Alternative tail statistics

The heavy-tailed distributions identified in the present work were compared with several alternative statistical models.

Pure algebraic fits of the form

$$\text{CCDF}(x) \sim x^{-\mu}$$

680 provide reasonable approximations over intermediate scales but systematically overestimate the probability of the largest events.

Lognormal distributions reproduce intermediate fluctuations more accurately but fail to reproduce the broadest tails associated with the geometric hierarchy.

The best overall agreement was obtained using stretched-exponential truncation,

$$\text{CCDF}(x) \sim \exp \left[- \left(\frac{x}{x_0} \right)^k \right],$$

685 with stretching exponents $k < 1$.

Such behaviour is frequently encountered in intermittent geophysical systems possessing finite-size constraints Lovejoy and Schertzer (1985); Veneziano et al. (2006).

C2 Possible physical interpretations

690 Although the present work remains fundamentally statistical, the measured scaling relations suggest several possible physical interpretations.

The observed relation

$$W \sim L^{0.43}$$

lies relatively close to diffusive scaling,

$$W \sim L^{1/2}.$$

695 If tornado path length is interpreted as an effective propagation distance proportional to vortex lifetime, such behaviour may indicate progressive broadening of coherent vortex structures during advection and stretching.

Similarly, the relation



$$A \sim L^{1.43}$$

suggests that tornado morphology obeys a robust correlated geometric organisation extending over several decades in scale.

700 These observations may be qualitatively consistent with simplified coherent-vortex broadening scenarios, although the present statistical analysis does not directly identify the underlying dynamical mechanisms.

Importantly, the present results establish statistical organisation laws rather than direct fluid-dynamical causality.

The approximately affine hierarchy identified here may emerge from several possible mechanisms, including: - correlated geometric amplification, - constrained stochastic growth, - vortex stretching, - or self-organised convective dynamics.

705 The present analysis alone cannot discriminate uniquely between these interpretations.

C3 Relation to other geophysical scaling systems

The scaling organisation identified in tornado geometry presents qualitative similarities with several other geophysical extreme-event systems.

710 Broad heavy-tailed distributions and approximate scale invariance are frequently observed in: earthquakes Turcotte (1997), wildfires Malamud et al. (1998), rainfall intermittency Lovejoy and Schertzer (1985); Tessier et al. (1993), avalanche systems Bak et al. (1987), and nonequilibrium multiscale processes more generally Stanley et al. (1999).

In many such systems, large-scale organisation emerges despite strong local stochasticity and intermittency.

The present results suggest that tornado damage paths may similarly constitute an emergent correlated multiscale field characterized by approximate scale invariance and weak intermittency corrections.

715 Future work combining tornado statistics with environmental atmospheric parameters and reduced vortex models may help establish more direct connections between the observed statistical hierarchy and the underlying severe-convective dynamics.



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