Response to Referee #1

Manuscript: *Correlated Geometric Scaling of Tornado Damage Paths in the SPC Database (1950–2024)*

Manuscript ID: egusphere-2026-2829

We sincerely thank the reviewer for the careful, thorough, and highly constructive evaluation of our manuscript. We greatly appreciate the positive overall assessment that the paper is up to the standards of *Nonlinear Processes in Geophysics*. We are particularly grateful for the detailed comments, which helped us improve the terminology, statistical presentation, physical interpretation, and overall structure of the manuscript.

Following the reviewer's recommendations, we carefully revised the manuscript throughout. The Abstract and Introduction were rewritten, the Methods section was substantially reorganized, and the Results were restructured around a progressive sequence of analyses: the direct width–length relationship, internal geometric consistency, robustness across observational periods and EF thresholds, higher-order statistics, null-model comparisons, and local scaling diagnostics.

The revised manuscript now consistently distinguishes the reported tornado path dimensions from the physical dimensions of the tornadic vortex. We also clarify that the measured exponent is an effective population-level statistical descriptor, not a deterministic law for individual tornadoes. The geometry–intensity observable is now explicitly presented as a phenomenological proxy rather than as a direct estimate of tornado kinetic energy.

We additionally introduced conditional-median statistics, bootstrap confidence intervals, residual diagnostics, EF-threshold sensitivity tests, and a more cautious analysis of moments above second order. The discussion of atmospheric energetics and tornado wind-field reconstruction was expanded, and the references recommended by the reviewer—including Tuck (2025), Miller et al. (2024), Gill (1982), and Dutton (1986)—were incorporated into the revised manuscript.

Below, we provide a detailed point-by-point response to every comment. Reviewer comments are reproduced in **blue**, while our answers appear immediately afterwards.

Comment 1

Lines 3–4: The form of words "tornado path length", L, is clear and unambiguous. However, there are places in the text where it has been shortened to "tornado length", an unmeasured quantity that could be mistaken for a distortion of "tornado diameter", also unmeasured as far as this manuscript is concerned—or is it what is meant by W? Please clarify.

Answer 1

We thank the reviewer for this important observation and fully agree that the original terminology could be interpreted ambiguously.

The expression “tornado length” may suggest a physical longitudinal dimension of the vortex, whereas the SPC database reports the length of the documented damage path. Likewise, W represents the reported maximum damage-path width and not the instantaneous physical diameter of the tornado vortex.

Following the reviewer’s suggestion, we systematically revised the terminology throughout the Abstract, Introduction, Methods, Results, Discussion, figure captions, and Appendices. All ambiguous occurrences of “tornado length” were replaced by “reported tornado path length” or simply “path length” where the context is unambiguous.

The Methods section now states explicitly:

> “Throughout this study, L denotes the reported tornado path length and W the reported maximum path width. The latter is an observational damage-survey quantity and should not be interpreted as the instantaneous physical diameter of the tornadic vortex.”

We believe that this revision removes the ambiguity identified by the reviewer and clearly distinguishes the observational path variables from the physical dimensions of the vortex.

Comment 2

Line 7: Again, what is meant by “longer tornadoes”? I assume you mean “tornadoes with longer paths”, but the language is imprecise.

Answer 2

We agree with the reviewer that the expression “longer tornadoes” lacks precision and could be interpreted as referring to the physical size or duration of the tornado itself.

Throughout the revised manuscript, this wording has been replaced by more precise formulations such as “tornadoes with longer reported path lengths”, “events with longer paths”, or “longer-track tornadoes”, depending on the context.

These formulations now make explicit that the comparison concerns the observed damage-path length recorded in the SPC catalogue rather than an unmeasured physical dimension of the vortex.

Comment 3

Line 12: “... several decades in scale.” I presume this means time or time scale. Clarify.

Answer 3

We thank the reviewer for identifying this ambiguity. Our intention was to refer to the logarithmic extent of the geometric observables, not to temporal decades or dynamical time scales.

The corresponding wording has therefore been replaced by expressions such as "several orders of magnitude" or "over more than four orders of magnitude in path length". Similar formulations were checked and revised throughout the manuscript to avoid confusion between temporal duration and logarithmic scaling range.

For example, the revised Results section now states:

> "Reported maximum path width increases systematically with reported path length over more than four orders of magnitude."

Comment 4

Lines 18–19: One of several examples—replace "tornado length" by "tornado path length".

Answer 4

We agree and have implemented this correction throughout the manuscript, not only at the location indicated by the reviewer.

All occurrences of "tornado length" have been replaced by "tornado path length", "reported tornado path length", or "path length", as appropriate. Figure captions, axis descriptions, and Appendix text were also checked to ensure consistent terminology.

This systematic revision ensures that the manuscript refers exclusively to the documented damage-path geometry and never implicitly equates path length with the physical length of a tornado vortex.

Comment 5

Lines 28–31: See Entropy *2025, 27(7), 740, doi:10.3390/e27070740 for a recent perspective. Reference [42] therein is relevant.*

Answer 5

We thank the reviewer for drawing our attention to this recent work. We have incorporated the perspective of Tuck (2025) into the Introduction and Discussion, where it provides useful context for the thermodynamic and energetic interpretation of atmospheric wind fields.

We also followed the reviewer's recommendation concerning Reference [42] of that article and added Miller et al. (2024), which addresses the reconstruction of tornado wind speeds from debris trajectories.

The revised Introduction now includes the following discussion:

> "Recent work has also emphasized that the energetics of atmospheric motion should be interpreted within the broader framework of atmospheric thermodynamics rather

> than solely in terms of wind kinetic energy, providing additional perspective on the interpretation of large-scale atmospheric scaling relationships (Tuck, 2025)."

and subsequently:

> "Recent work has also emphasized that reconstructing tornado wind fields from damage observations remains a challenging inverse problem and that atmospheric wind kinetic energy represents only a small fraction of the total atmospheric energy budget (Miller et al., 2024; Tuck, 2025)."

These additions help place the present geometry–intensity proxy within the broader context of atmospheric energetics and recent tornado wind-field reconstruction methods.

Comment 6

Lines 72–78: See the above reference for some discussion of the energetics of wind generation.

Answer 6

We appreciate this suggestion and have expanded the discussion of atmospheric energetics accordingly.

The revised manuscript now makes explicit that the wind kinetic-energy component is only one part of the much larger atmospheric energy budget and that the observable introduced in the manuscript is not intended to represent the complete energetic content of tornado-producing convection.

The geometry–intensity observable is now consistently described as a phenomenological proxy used for comparative statistical purposes. The revised Discussion states:

> "It is important to emphasize that the geometry–intensity observable introduced in this work should not be interpreted as a direct estimate of tornado kinetic energy. Rather, it provides a simple phenomenological proxy that allows us to evaluate whether the statistical organization identified from geometry alone remains qualitatively unchanged after coarse EF-based intensity weighting."

This interpretation is explicitly connected to the perspectives of Tuck (2025) and Miller et al. (2024).

Comment 7

Lines 90–93: Presumably A is for area. Please say so explicitly.

Answer 7

We agree that the meaning of A should be stated explicitly when the quantity is introduced.

The revised Methods section now defines $A = LW$ as a "geometry-only observable" and explains that it is a coarse-grained area proxy describing the overall extent of the documented damage path.

The revised text states:

> "From these two directly observed quantities, we define the geometry-only observable
>
> $$A = LW,$$
>
> which represents a simple geometrical proxy describing the overall extent of the reported damage path."

We also emphasize that A is not the exact physical damaged area, because the actual affected surface depends on damage continuity, land use, structural vulnerability, exposure, and survey practices.

This clarification has also been included in Appendix A, where the construction and limitations of A are documented in greater detail.

Comment 8

Lines 133–135: Yes, important to establish the result. Insert "path" between "longer" and "path".

Answer 8

We thank the reviewer for this correction. The relevant sentence has been rewritten to refer explicitly to "longer reported path lengths" and to avoid the ambiguous expression "longer tornadoes".

More generally, the entire manuscript was checked for comparable terminological ambiguities. The revised text now consistently describes how reported maximum path width varies statistically with reported path length, without implying that the physical vortex itself is being measured longitudinally.

Comment 9

Lines 138–148: The tornadic vortex will have a radial velocity gradient, which at some point will cease to cause damage but still be recognisably elevated above what existed prior to the tornado's propagation into the region. Please discuss.

Answer 9

We thank the reviewer for this important physical clarification. We fully agree that the reported damage-path width does not coincide with a sharply defined dynamical boundary of the tornadic circulation.

The near-surface wind speed decreases continuously with radial distance from the vortex core. Consequently, winds outside the documented damage path may remain elevated relative to the pre-existing background flow while remaining below the locally effective damage threshold.

The revised Methods section now explicitly states:

> "Near-surface wind speed decreases continuously with radial distance from the vortex core, and winds outside the documented damage path may remain elevated above the background flow without exceeding locally effective damage thresholds."

We further explain that the reported value of W depends not only on vortex dynamics, but also on the availability of damage indicators, land use, surface roughness, structural vulnerability, and post-event survey practices.

This addition clarifies that W is an observational damage-survey quantity rather than the complete dynamical width of the tornadic wind field.

Comment 10

Lines 150–159: Bear in mind that the kinetic energy of the wind is a very small fraction of the total internal and gravitational energy, as pointed out for example in Gill, Atmosphere–Ocean Dynamics *(1982), page 75, and Dutton,* The Ceaseless Wind: An Introduction to the Theory of Atmospheric Motion*, second edition (1986), Chapter 14. Some brief discussion would help the reader.*

Answer 10

We thank the reviewer for this important observation and for the suggested references. We have incorporated both Gill (1982) and Dutton (1986) into the revised Methods section.

The manuscript now states:

> "The observable E is not intended as an estimate of the true kinetic energy of a tornado. Atmospheric kinetic energy represents only a small fraction of the total atmospheric energy budget, which is dominated by internal and gravitational potential energy (Gill, 1982; Dutton, 1986)."

We also clarify that the SPC catalogue contains discrete EF damage classes rather than direct measurements of the three-dimensional vortex wind field. Accordingly, the observable $E \propto V^2LW$ is now described throughout as a "geometry–intensity proxy" and not as a physical tornado-energy estimate.

The Introduction, Discussion, and Appendix A were revised consistently with this interpretation.

Comment 11

Line 181: "substantial intrinsic scatter"—does the variance converge? Figure 1 suggests that it does not. Many readers will look at the straight line fitted to the data points and ask what value it has, lines 185 and 186 notwithstanding.

Answer 11

We thank the reviewer for this important observation. We agree that the original manuscript did not sufficiently discuss the considerable dispersion visible around the fitted scaling relationship.

Following the reviewer's comment, Figure 1 has been completely redesigned to provide a more informative representation of the statistical structure of the dataset. Instead of relying solely on an event-level regression, the revised figure now combines a logarithmic hexagonal-density representation of the complete tornado population, conditional median path widths, and 95% bootstrap confidence intervals. This presentation makes the progressive increase in dispersion with path length directly visible while simultaneously highlighting the robust population-level trend followed by the central tendency of the data.

The revised Discussion also explicitly addresses this behaviour. Rather than viewing the increasing scatter as evidence against the existence of the scaling relationship, we now identify it as the natural heteroscedasticity expected from a heterogeneous climatological population combining tornadoes of different intensities, lifetimes, storm modes, environmental conditions, and observational periods.

Accordingly, the measured exponent is no longer interpreted as a deterministic law capable of predicting the geometry of individual tornadoes. Instead, it is explicitly presented as an effective population-level statistical descriptor that characterizes the average geometric organization of tornado damage paths despite the substantial event-to-event variability.

The revised Discussion now states:

> "While the average geometric trend is robust, the conditional dispersion broadens progressively with path length, indicating that the statistical organization is intrinsically heteroscedastic. Tornadoes with longer reported paths exhibit a broader range of maximum path widths than shorter-track events, suggesting that the diversity of possible dynamical evolutions also increases with lifetime and propagation distance. Consequently, the measured exponent characterizes the evolution of the central tendency of the tornado population rather than a relationship with constant dispersion."

We believe that this revised presentation provides a clearer and more balanced interpretation of both the fitted relationship and the substantial intrinsic scatter visible in the observations.

Comment 12

Lines 219 et seq.: Generally a great deal of high-quality data is required to evaluate $q > 2$. Do you have it?

Answer 12

We agree with the reviewer that moments above second order require particular caution because they are increasingly dominated by rare extreme events.

Although the revised catalogue contains 69 975 tornadoes, the effective sample size contributing to the largest path-length bins and highest moment orders decreases substantially. We therefore changed the statistical strategy in the revised manuscript.

The principal interpretation is now restricted to

$$0.25 \leq q \leq 2,$$

where the estimates remain comparatively well constrained. Moments in the range

$$2 < q \leq 3$$

are retained only as exploratory sensitivity analyses.

The revised Methods section now states:

> "Higher-order moments, $2 < q \leq 3$, are examined only as exploratory sensitivity analyses because they become increasingly dominated by the rarest extreme events and therefore exhibit larger statistical uncertainty."

We also verified that restricting the affine-spectrum analysis to $q \leq 2$ does not alter the principal conclusion. The inferred slopes remain essentially unchanged relative to those obtained over the full exploratory range.

The Results, Discussion, and Appendix B were revised accordingly, and no claim now depends critically on moments above second order.

Comment 13

Lines 253–254: Lognormal distributions are not in general the same thing as the fat-tailed distributions seen in atmospheric data.

Answer 13

We fully agree with the reviewer. The original wording was imprecise and could be read as equating lognormal statistics with the broader class of heavy-tailed atmospheric distributions.

The lognormal synthetic population is now described only as a "broad-distribution reference model". Its purpose is methodological: to determine whether broad positive marginal distributions, in the absence of length–width dependence, can reproduce the observed geometric organization.

The revised Methods section now states:

> "The lognormal distribution is employed here only as a convenient broad-distribution reference model and is not intended to represent the full class of heavy-tailed atmospheric statistics."

The corresponding Results and Appendix text were revised consistently. We no longer imply that lognormal and atmospheric fat-tailed statistics are generally equivalent.

Comment 14

Lines 373–374: Figure 8 looks to my eye to have significant structure with positive convexity between $1.0 < q < 2.5$*.*

Answer 14

We sincerely thank the reviewer for this careful observation. We agree that the original wording was too categorical and could give the impression that the measured scaling spectrum was perfectly affine over the entire range of moments.

Following the reviewer's comment, we revised both the Results and the Discussion to adopt a more cautious interpretation of the observed behaviour. Rather than describing the spectrum as exactly affine, we now explicitly acknowledge the presence of weak departures from linearity, particularly for moments above second order.

The revised manuscript now states:

> "Small departures from perfect linearity become visible only for moments above second order, where the statistics are increasingly controlled by the most extreme tornadoes and the associated bootstrap uncertainties grow rapidly."

As illustrated by the revised analysis, the apparent positive convexity remains modest compared with the bootstrap uncertainty associated with the highest-order moments. Because these moments are estimated from a progressively smaller number of extreme tornadoes, the available data do not allow us to determine whether the observed departures reflect a genuine physical effect or simply the increasing statistical uncertainty in the tails of the distribution.

For this reason, we deliberately adopted a more conservative interpretation throughout the revised manuscript. We now describe the moment hierarchy as *predominantly affine*, with only weak departures from linearity at the highest resolved moment orders, and we explicitly avoid interpreting these small deviations as robust evidence for multifractality.

We believe that this revised interpretation better reflects both the quality of the available observations and the level of confidence that can reasonably be assigned to the highest-order statistics.

Comment 15

Lines 469–473: I recommend including Reference [42] from Entropy *2025, 27(7), 740, doi:10.3390/e27070740.*

Answer 15

We thank the reviewer for this recommendation. Reference [42] in Tuck (2025), namely Miller et al. (2024), has now been incorporated into the Introduction and Discussion.

Miller et al. (2024) is particularly relevant because it illustrates the difficulty of reconstructing tornado wind speeds from indirect observational evidence, in that case from the trajectories of large compact debris objects.

The revised Introduction states:

> "Recent work has also emphasized that reconstructing tornado wind fields from damage observations remains a challenging inverse problem and that atmospheric wind kinetic energy represents only a small fraction of the total atmospheric energy budget (Miller et al., 2024; Tuck, 2025)."

The Discussion further states:

> "This interpretation is consistent with recent studies emphasizing both the difficulty of inferring tornado wind fields from damage observations (Miller et al., 2024) and the broader energetic perspective of atmospheric motion discussed by Tuck (2025)."

We believe that these references substantially improve the physical context of the manuscript and clarify why the geometry–intensity quantity is treated as a comparative statistical proxy rather than as a direct reconstruction of tornado energetics.

We again thank the reviewer for the careful and constructive assessment. The comments led to substantial improvements in terminology, statistical diagnostics, physical interpretation, and presentation. In particular, the revised manuscript now distinguishes clearly between damage-path geometry and vortex dimensions, treats the measured exponent as an effective population-level descriptor, incorporates heteroscedasticity and higher-order uncertainty explicitly, and provides a more cautious interpretation of atmospheric energetics and moment scaling.

We hope that the reviewer will find that the revised manuscript fully addresses the comments and provides a clearer, more rigorous, and more balanced presentation of the statistical organization of tornado damage-path geometry.

References

John A. Dutton. *The Ceaseless Wind: An Introduction to the Theory of Atmospheric Motion*. Dover Publications, New York, 2 edition, 1986. ISBN 0486650960.

Adrian E. Gill. *Atmosphere–Ocean Dynamics*, volume 30 of *International Geophysics*. Academic Press, New York, 1982. ISBN 9780122835223.

S. L. Miller, G. A. Kopp, D. M. L. Sills, and D. G. Butt. Estimating wind speeds in tornadoes using debris trajectories of large compact objects. *Monthly Weather Review*, 152(7):1859–1881, 2024. doi: 10.1175/MWR-D-23-0260.1.

Adrian F. Tuck. The generation of wind velocity via scale invariant gibbs free energy: Turbulence drives the general circulation. *Entropy*, 27(7):740, 2025. doi: 10.3390/e27070740.