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On the impact of a misestimation of the lidar ratio in the retrieval of the overlap function of an elastic/Raman lidar and a two-step explicit formulation of the overlap function retrieval

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Abstract. In a relatively recent paper by the authors, an explicit (i.e. non iterative) formula was provided for recovering the overlap factor in an aerosol lidar equipped with a N2/O2 Raman channel close to an elastic one to retrieve the aerosol extinction coefficient. One of the advantages of that formula is that it allows for the estimation of the impact of an erroneous lidar ratio in the retrieved overlap function. In the cited paper, the equation that relates the retrieved overlap function to the true one and the error on the assumed lidar ratio was given with almost no proof, on the grounds that its derivation was "boring and cumbersome, but otherwise straightforward". After having discussed that equation with some interested researchers, it turns out that its derivation is not perhaps that straightforward. Therefore, we provide here its derivation. As a subproduct of that derivation, we propose a streamlined, two-step explicit formulation to retrieve the overlap function.

1 Introduction

Comerón et al. derived an explicit formulation for the retrieval of the overlap function in an elastic and Raman aerosol lidar (Comerón et al., 2023), which is equivalent to the iterative method derived by Wandinger and Ansmann (Wandinger & Ansmann, 2002). The accuracy of the retrieval in these two methods is affected by how good is the estimation of the lidar ratio of the aerosol in presence. The explicit formulation permits to assess clearly the impact of an erroneous assumption of the lidar ratio on the retrieved overlap function. The retrieved overlap function, $O'(R)$, where R is the range from the lidar, relates to the "true" overlap function, $O(R)$, through (Comerón et al., 2023)

$$O'(R) = O(R)\exp\left[-2\int_R^{R_m} \Delta S_{a0}(x)\beta_{a0}(x)dx\right], \tag{1}$$

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where $\beta_{a0}(R)$ is the aerosol backscatter coefficient at range R, R_m is a reference range where $\beta_{a0}(R_m)=0$ and $\Delta S_{a0}(R)$ is the difference at range R between the assumed lidar ratio at the emitted wavelength, $S'_{a0}(R)$, and the "true" lidar ratio at the same wavelength $S_{a0}(R)$, i. e.

$$\Delta S_{a0}(R) = S'_{a0}(R) - S_{a0}(R). \tag{2}$$

From Eq. (1) it is clear that $O'(R)=O(R)$ if the aerosol lidar ratio is estimated correctly, i.e., $\Delta S_{a0}=0$, or if there is no aerosol .i.e. $\beta_{a0}(R)=0$, in which case there is no aerosol lidar ratio to be considered.

In section 2, we derive Eq. (1). In section 3 we propose a streamlined, explicit method to retrieve the overlap function, alternative to that derived in (Comerón et al., 2023). Section 4 contains the conclusions.

2 Derivation of the relationship between the true overlap function and the estimated one

We start, as indicated in (Comerón et al., 2023), by noting that, under the assumption that the overlap function is the same for the elastic and the Raman signals (Wandinger & Ansmann, 2002), the range-corrected elastic and Raman signals can be expressed respectively as

$$X(R) = A\,O(R)\left[\beta_{m0}(R)+\beta_{ao}(R)\right]\exp\left\{-2\int_0^R\left[\alpha_{a0}(x)+S_{m0}\beta_{m0}(x)\right]dx\right\}, \tag{3a}$$

$$X_R(R) = B\,O(R)\beta_{mR}(R)\exp\left\{-\int_0^R\left[\alpha_{a0}(x)+\alpha_{aR}(x)+S_{m0}\beta_{m0}(x)+S_{mR}\beta_{mR}(x)\right]dx\right\}, \tag{3b}$$

where A and B are instrument constants, $\alpha_{a0}(R)$ and $\alpha_{aR}(R)$ are the extinction coefficients at the excitation and Raman wavelengths respectively, $\beta_{m0}(R)$ and $\beta_{mR}(R)$ are the molecular backscatter coefficients at the excitation and Raman wavelengths respectively, and S_{m0} and S_{mR} are the molecular lidar ratios at the excitation and Raman wavelengths respectively (D'Amico et al., 2016), (Bucholtz, 1995).

We turn now to the explicit expression of the overlap function given in Eq. (15) of (Comerón et al., 2023), where, by the way, we note that two typos slipped into. One affects the argument of the fourth exponential function, which should be raised to -1. The other typo is found in the initial factor in that equation, $\dfrac{\beta_{m0}(R_m)X_R(R)}{\beta_m(R)X_R(R_m)}$, which should be $\dfrac{\beta_{m0}(R_m)X_R(R)}{\beta_{m0}(R)X_R(R_m)}$, R_m being an aerosol-free, sufficiently far reference range, where the aerosol backscatter coefficient can be assumed to be 0 and the overlap function has stabilized to a constant value that we make arbitrarily 1, i.e. $O(R_m)=1$. We repeat here that expression for convenience and, because it uses an aerosol lidar ratio at the excitation wavelength that might not be the true one, we denote that lidar ratio as $S'_{a0}(R)$ (see Eq. (2)) and the overlap function as $O(R)$:

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$$
\begin{aligned}
O'(R) &= \frac{\beta_{m0}(R_m)X_R(R)}{\beta_{m0}(R)X_R(R_m)}\exp\left\{2\int_R^{R_m}\left[S'_{a0}(x)-S_{m0}\right]\beta_{m0}(x)dx\right\} \\
&\exp\left\{\int_R^{R_m}\left[S_{m0}\beta_{mo}(x)-S_{mR}\beta_{mR}(x)\right]dx\right\}\exp\left\{\int_R^{R_m}\left[\alpha_{a0}(x)-\alpha_{aR}(x)\right]dx\right\} \\
&\exp\left\{2\frac{X_R(R_m)}{X(R_m)}\int_R^{R_m}\frac{S'_{a0}(x)\beta_{m0}X(x)}{X_R(x)}\exp\left(-\int_x^{R_m}\left[S_{m0}\beta_{m0}(x')-S_{mR}\beta_{mR}(x')\right]dx'\right)\right. \\
&\left.\exp\left(-\int_x^{R_m}\left[\alpha_{a0}(x')-\alpha_{aR}(x')\right]dx'\right)dx\right\}^{-1}.
\end{aligned}
\tag{4}
$$

By substituting the expressions (3) for $X(R)$ and $X_R(R)$ in the argument of the fourth exponential function of Eq. (4), and taking into account that the ratio $\frac{\beta_{mR}(R)}{\beta_{m0}(R)}$ is independent of the range, hence $\frac{\beta_{mR}(R_m)\beta_{m0}(x)}{\beta_{m0}(R_m)\beta_{mR}(x)}=1$, one obtains after some algebra

$$
\begin{aligned}
O'(R) &= \frac{\beta_{m0}(R_m)X_R(R)}{\beta_{m0}(R)X_R(R_m)}\exp\left[-2\int_R^{R_m}S'_{a0}(x)\beta_{a0}(x)dx\right] \\
&\exp\left\{-\int_R^{R_m}\left[S_{m0}\beta_{m0}(x)+S_{mR}\beta_{mR}(x)\right]dx\right\}\exp\left\{\int_R^{R_m}\left[\alpha_{a0}(x)-\alpha_{aR}(x)\right]dx\right\}.
\end{aligned}
\tag{5}
$$

Likewise, by substituting Eq. (3b) in the factor $\frac{\beta_{m0}(R_m)X_R(R)}{\beta_{m0}(R)X_R(R_m)}$ appearing in Eq. (5), and, taking into account again the independence with range of the ratio $\frac{\beta_{mR}(R)}{\beta_{m0}(R)}$, we obtain

$$
\frac{\beta_{m0}(R_m)X_R(R)}{\beta_{m0}(R)X_R(R_m)} = O(R)\exp\left\{\int_R^{R_m}\left[\alpha_{a0}(x)+\alpha_{aR}(x)+S_{m0}\beta_{m0}(x)+S_{mR}\beta_{mR}(x)\right]dx\right\}. \tag{6}
$$

Note that in Eq. (6) there is no assumption on the aerosol lidar ratio and, therefore, the overlap function appearing is the "true" one.

The substitution of Eq. (6) in Eq. (5) yields therefore

$$
O'(R) = O(R)\exp\left\{\int_R^{R_m}2\left[\alpha_{a0}(x)-S'_{a0}(x)\beta_{a0}(x)\right]dx\right\}, \tag{7}
$$

which, knowing that, by definition, $\alpha_{a0}(x)=S_{a0}(x)\beta_{a0}(x)$, and substituting, according to Eq. (2), $S'_{a0}(x)=S_{a0}(x)+\Delta S_{a0}(x)$, results finally in Eq. (1).

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3 Alternative retrieval of the overlap function

In our way to deriving Eq. (1), we have found a way to compute explicitly (i.e. non iteratively) the overlap function, other than the daunting Eq. (4) (Eq. (15) of (Comerón et al., 2023) corrected for typos), namely Eq. (5) of the present paper. All the variables intervening in it are found in Eq. (4), except the aerosol backscatter coefficient at the emitted wavelength β_{a0}. But this coefficient can be retrieved, assuming, as we have done since the beginning, that the overlap functions of the elastic and the Raman channels are identical (Eqs. (3)), as (Ansmann et al., 1992)

$$\beta_{a0}(R)=\frac{X(R)X_R(R_m)\beta_{m0}(R)}{X(R_m)X_R(R)}\frac{\exp\left\{-\int_R^{R_m}\left[\alpha_{a0}(x)+S_{m0}\beta_{m0}(x)\right]dx\right\}}{\exp\left\{-\int_R^{R_m}\left[\alpha_{aR}(x)+S_{mR}\beta_{mR}(x)\right]dx\right\}}-\beta_{m0}(R)\,. \tag{8}$$

Substituting the result of Eq. (8) for $\beta_{a0}(x)$ in Eq. (5) yields the same result as Eq. (4).

4 Conclusion

We have shown the details of the derivation of the ratio between the retrieved overlap function – which needs the assumption of an aerosol lidar ratio that may in principle differ from the actual, unknown one – and the true one in terms of the difference between the assumed lidar ratio and the true one. Obviously, this difference is also unknown, but helps in detecting too high or too low assumptions and the expression clearly shows the interest of doing the retrieval of the overlap function in clear atmospheres, with $\beta_{a0}(R)$ as low as possible to minimize the error.

At the same time, we propose a two-step explicit formulation for the retrieval of the overlap function, easier to apply than that proposed in (Comerón et al., 2023): 1st step: compute $\beta_{a0}(R)$ according to Eq. (8); 2nd step: compute the overlap function after Eq. (5).

Author contributions

AC developed the new formulation, wrote the text. CMP and ARG contributed with data analysis. The contributions of MS, FD, CGD, DCFDSO and FR have been invaluable in the inception, development and consolidation of our lidar system, besides the contribution in measurements and data analysis.

Competing interests

The contact author has declared that none of the authors has any competing interests.

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