



1 Propagation of Instrumental Self-Noise Through Causal Response Correction: Implications for Seismic-
2 Network Detectability and Completeness

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11 **Abstract**

12 Seismic detectability is commonly interpreted as a consequence of network geometry, station density, and
13 environmental noise. However, instrumental self-noise and causal instrument correction can substantially
14 modify the effective station noise floor and therefore influence earthquake detectability and completeness
15 magnitude. Despite their importance in instrumental seismology, these effects are rarely incorporated explicitly
16 into network-scale detectability analyses. This study develops a unified theoretical and computational
17 framework linking instrumental response, causal response correction, ground-referred self-noise,
18 environmental seismic noise, station-level signal-to-noise ratio, probabilistic detection, and network-scale
19 completeness magnitude. The framework propagates instrumental self-noise through the inverse instrumental
20 response into corrected ground-motion units and integrates this contribution into a probabilistic detectability
21 formalism. Global broadband station geometries derived from publicly available FDSN metadata are combined
22 with physically informed environmental-noise fields and parameterized instrumental self-noise models to
23 investigate large-scale detectability behavior. The results demonstrate that seismic detectability emerges from
24 the coupled interaction among network geometry, environmental noise, and propagated instrumental self-
25 noise. Three principal operational regimes arise naturally from the framework: geometry-limited, ambient-
26 noise-limited, and self-noise-limited detectability conditions. Sparse networks are dominated primarily by
27 geometric effects, whereas densely instrumented low-noise networks become increasingly sensitive to
28 instrumental self-noise after causal response correction. The results further show that the marginal detectability



29 gain associated with network densification progressively decreases as station density increases, while the
30 relative importance of instrumental quality correspondingly grows. The framework additionally demonstrates
31 that instrumental self-noise may be substantially amplified near the limits of the instrumental passband after
32 response correction, producing detectability degradation even when raw instrumental-noise levels appear
33 comparatively small. Consequently, seismic-network performance cannot be interpreted exclusively as a
34 geometrical property, but instead reflects a multiscale interaction among instrument physics, environmental
35 conditions, and network organization. The proposed methodology provides a computationally reproducible
36 and physically interpretable basis for analyzing detectability limits and regime-dependent network
37 optimization in modern seismic monitoring systems, including dense broadband arrays and emerging low-cost
38 MEMS-based networks. The proposed methodology is intended primarily as a physically interpretable
39 framework for investigating first-order detectability controls and regime-dependent network optimization
40 rather than as an operational estimator of real-world global completeness magnitude.

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42

43 **1. Introduction**

44 Modern seismic monitoring systems are increasingly evaluated not only in terms of instrumental sensitivity,
45 bandwidth, dynamic range, or installation quality, but also through their effective capability to detect, identify,
46 and characterize seismic signals across a broad range of magnitudes, distances, site conditions, and network
47 geometries. In operational seismology, this capability is commonly summarized through the concept of
48 detectability, namely the probability that an earthquake signal exceeds the effective observational threshold of
49 a seismic station or network and can therefore be reliably detected, associated, and processed. Detectability
50 directly controls the magnitude of completeness M_c , which in turn governs the reliability of earthquake
51 catalogues, seismicity-rate estimates, b-value calculations, earthquake forecasting models, and seismic-hazard
52 assessments.

53 Traditionally, detectability and catalogue completeness have been interpreted primarily as consequences of
54 network geometry, station spacing, azimuthal coverage, environmental noise, and phase-picking performance.
55 Simulation-based approaches have been developed to assess the location capability and magnitude thresholds
56 of real seismic networks, including the Seismic Network Evaluation through Simulation (SNES) framework



57 applied to national and regional monitoring systems (D'Alessandro et al., 2011; D'Alessandro and Ruppert,
58 2012; D'Alessandro and Stickney, 2012). These studies demonstrated that network performance is spatially
59 heterogeneous and strongly dependent on station geometry, phase availability, and regional coverage. Similar
60 analyses were later extended to the evolution and strengthening of regional seismic networks (D'Alessandro
61 et al., 2013a). However, these approaches primarily focused on geometrical and statistical aspects of network
62 performance rather than on the propagation of instrumental noise through response correction.

63 In parallel, ambient seismic-noise characterization has become a standard tool for evaluating station quality
64 and monitoring performance. The New Low Noise Model (NLNM) and New High Noise Model (NHNM)
65 introduced by Peterson (1993) remain foundational references for broadband seismic-noise assessment, while
66 probabilistic power spectral density approaches have enabled quantitative characterization of the temporal and
67 spatial variability of seismic background noise (McNamara and Buland, 2004). More recently, spatial analyses
68 of seismic-noise variability have shown that ambient-noise levels can significantly influence network
69 detectability and completeness magnitude. Detailed PSD/PDF analyses performed on the Italian Seismic
70 Network further demonstrated the strong spatial and temporal variability of background seismic noise and its
71 direct implications for seismic-network detectability and completeness magnitude (D'Alessandro et al., 2021).
72 In particular, Catania et al. (2025) demonstrated a strong spatial correspondence between background seismic-
73 noise levels and completeness-magnitude variability within the Italian seismic network, emphasizing that
74 detectability cannot be interpreted exclusively as a geometrical property of the network. Similarly, the
75 development of quasi-real-time seismic-noise monitoring systems, such as SEISMONOISY (Ruzza et al.,
76 2024), highlights the increasing importance of dynamic noise-field characterization for operational seismic-
77 network assessment and optimization.

78 Instrumental self-noise has long been recognized as a limiting factor in broadband seismology and sensor
79 characterization. Three-channel correlation techniques provide rigorous estimates of the intrinsic self-noise of
80 digitizers and seismic sensors (Sleeman et al., 2006), and subsequent studies demonstrated that instrumental
81 self-noise depends strongly on sensor type, frequency band, installation conditions, and sensor orientation
82 (Gerner and Bokelmann, 2013; Wang et al., 2023). Recent analyses comparing multiple seismometer classes
83 further confirmed that no single instrument minimizes self-noise across the entire seismic frequency spectrum



84 (Wang et al., 2023). Consequently, detectability depends not only on environmental noise and source
85 amplitude, but also on the interaction between instrumental response and intrinsic sensor noise.

86 These issues become even more relevant in the context of emerging dense and low-cost seismic networks.
87 MEMS accelerometers and compact low-cost sensors have enabled new monitoring strategies for urban
88 seismology, structural health monitoring, rapid post-earthquake assessment, and dense local earthquake
89 detection. D'Alessandro and D'Anna (2013) evaluated the suitability of low-cost MEMS accelerometers for
90 strong-motion seismology, while D'Alessandro et al. (2014) proposed an urban MEMS-based seismic network
91 for rapid post-earthquake disaster assessment. D'Alessandro et al. (2017) subsequently characterized MEMS
92 accelerometer self-noise using power spectral density and Allan variance analysis, demonstrating that self-
93 noise characterization is essential before low-cost sensors can be reliably integrated into seismic monitoring
94 systems. More recent studies and reviews further highlighted the rapid evolution of urban and distributed
95 seismic networks based on heterogeneous low-cost instrumentation (Anthony et al., 2019; Vitale et al., 2022;
96 Scudero et al., 2023).

97 These developments are particularly important because dense low-cost networks inherently modify the balance
98 between geometry-driven and instrumentation-driven detectability limitations. In sparse networks,
99 detectability is primarily controlled by inter-station spacing and azimuthal coverage. However, as network
100 density increases, instrumental self-noise progressively becomes a non-negligible contribution to the effective
101 detection threshold, especially after instrument-response correction into ground-motion units. Under these
102 conditions, detectability can no longer be interpreted solely as a consequence of network geometry or ambient
103 noise.

104 Despite these advances, a conceptual gap still exists between instrumental self-noise characterization and
105 network-scale detectability analysis. Existing studies commonly treat self-noise as a station-level instrumental
106 property, whereas detectability and M_c are usually considered network-level quantities controlled by geometry
107 and environmental noise. However, seismic waveforms are not interpreted directly in recorded digital units.
108 Instead, instrumental response removal or causal correction is applied to estimate the underlying ground
109 motion. This operation maps instrumental self-noise into ground-motion units through the inverse transfer
110 function of the sensor–digitizer system, potentially amplifying specific frequency bands near the instrumental



111 limits. As a consequence, the effective station noise floor depends simultaneously on environmental noise,
112 instrumental self-noise, and the spectral properties of the response correction.

113 The central conceptual contribution of this work is the explicit propagation of instrumental self-noise through
114 causal response correction into probabilistic network-detectability metrics, thereby establishing a direct
115 physical link between instrument-response physics and large-scale seismic monitoring performance.

116 The present study develops a unified theoretical and computational framework that links instrument physics,
117 causal response correction, ground-referred self-noise, station-level signal-to-noise ratio, probabilistic station
118 detection, and network-scale detectability into a single physically consistent formalism. The central hypothesis
119 of this work is that seismic detectability is an emergent multiscale property resulting from the coupled
120 interaction among: (i) source amplitude and spectral content, (ii) propagation effects, (iii) environmental noise,
121 (iv) instrumental self-noise, (v) causal instrument correction, and (vi) network aggregation geometry.

122 Within this framework, instrumental self-noise is propagated into corrected ground-motion units through the
123 inverse instrumental response, allowing the definition of an effective station noise floor that combines
124 environmental and instrumental contributions. This effective noise model is then integrated into a probabilistic
125 station-detection formalism and subsequently aggregated into network-scale detectability metrics and spatial
126 completeness fields.

127 The manuscript further introduces the concept of detectability regimes. Depending on network density, ambient
128 noise, and effective self-noise, a monitoring system may operate in a geometry-limited regime, an ambient-
129 noise-limited regime, or a self-noise-limited regime. These regimes imply fundamentally different
130 optimization strategies for seismic-network design. In sparse networks, densification may provide the
131 dominant gain in detectability. In dense networks, reducing instrumental self-noise may become more effective
132 than adding further stations. Intermediate configurations instead require joint optimization of station geometry
133 and sensor quality.

134 The objectives of this work are therefore fourfold: (1) to formalize the propagation of instrumental self-noise
135 through causal instrument correction; (2) to define an effective station noise floor combining environmental
136 and ground-referred instrumental noise; (3) to quantify the effect of this effective noise on station-level and
137 network-level detectability; and (4) to establish a regime-aware framework for seismic-network optimization
138 linking instrument physics to network performance.



139 Figure 1 summarizes the conceptual architecture of the proposed framework, illustrating the physical chain
140 connecting ground motion, instrumental response, self-noise propagation, station-level signal-to-noise ratio,
141 probabilistic detection, and network-scale detectability metrics. The purpose of the present study is therefore
142 not to provide an operational estimate of global earthquake detectability or catalogue completeness, but rather
143 to establish a physically consistent and computationally reproducible framework capable of isolating and
144 quantifying the first-order interactions between causal instrument correction, propagated instrumental self-
145 noise, environmental seismic noise, and network geometry. Within this perspective, the proposed methodology
146 should be interpreted primarily as a regime-oriented and physically interpretable detectability framework
147 designed to investigate how instrumental and geometrical limitations jointly control seismic monitoring
148 performance across different network configurations.

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150

151 **2. Theoretical Framework**

152 The detectability of seismic signals by a monitoring network is ultimately controlled by the coupled interaction
153 among source radiation, wave propagation, environmental noise, instrumental response, and network
154 geometry. In standard seismic processing, recorded waveforms are transformed from instrumental digital
155 counts into estimates of physical ground motion through response removal or causal instrument correction.
156 This operation fundamentally modifies the spectral structure of both signal and noise, implying that
157 detectability must be formulated in corrected ground-motion units rather than in raw instrumental counts. The
158 framework developed in this work explicitly propagates instrumental self-noise through causal response
159 correction and integrates it into a probabilistic network-detectability formalism. Figure 2 illustrates the spectral
160 propagation of environmental and instrumental noise contributions into the effective station noise floor.

161 Let $g(t)$ denote the true ground-motion signal and $h_i(t)$ the causal impulse response of the i -th seismic station,
162 including sensor, digitizer, and acquisition chain. The recorded waveform $d_i(t)$ can be written as

163

$$165 \quad d_i(t) = h_i(t) * g(t) + n_i^{\text{env}}(t) + n_i^{\text{self}}(t), \quad (1)$$

164



166 where $n_i^{\text{env}}(t)$ represents the environmental noise contribution, $n_i^{\text{self}}(t)$ is the intrinsic instrumental self-noise,
167 and $*$ denotes convolution.

168 In the frequency domain, Eq. (1) becomes

169

$$171 \quad D_i(\omega) = H_i(\omega)G(\omega) + N_i^{\text{env}}(\omega) + N_i^{\text{self}}(\omega). \quad (2)$$

170

172 To estimate the underlying ground motion, the instrumental response is removed through inverse filtering. In
173 practical applications, strictly acausal deconvolution is generally avoided because it may amplify unstable
174 spectral components and violate real-time processing constraints. The corrected signal is therefore obtained
175 through stabilized causal correction:

176

$$178 \quad \hat{G}_i(\omega) = \frac{D_i(\omega)}{H_i(\omega)} W_i(\omega), \quad (3)$$

177

179 where $W_i(\omega)$ is a stabilization operator accounting for regularization, tapering, and causal filtering constraints.
180 Such stabilization procedures are necessary to avoid numerical instabilities and non-physical amplification
181 near zeros of the instrumental response (Scherbaum, 2001; Wielandt, 2002).

182 Substituting Eq. (2) into Eq. (3) yields

183

$$185 \quad \hat{G}_i(\omega) = G(\omega)W_i(\omega) + \frac{N_i^{\text{env}}(\omega)}{H_i(\omega)} W_i(\omega) + \frac{N_i^{\text{self}}(\omega)}{H_i(\omega)} W_i(\omega). \quad (4)$$

184

186 Equation (4) highlights a key physical aspect of the problem: instrumental self-noise is transformed into
187 corrected ground-motion units through the inverse response of the instrument. Consequently, the detectability
188 threshold depends not only on environmental noise and source amplitude, but also on the spectral amplification
189 induced by response correction.

190 The power spectral density (PSD) of the ground-referred instrumental self-noise is defined as

191



193
$$S_{n,i}^{\text{self,gr}}(\omega) = \frac{S_{n,i}^{\text{self}}(\omega)}{|H_i(\omega)|^2}. \quad (5)$$

192

194 Here, $S_{n,i}^{\text{self}}(\omega)$ denotes the intrinsic instrumental self-noise PSD and $H_i(\omega)$ is the complex transfer function
195 of the instrument. Equivalent formulations are commonly used in instrumental self-noise characterization and
196 seismic sensor intercomparison studies (Sleeman et al., 2006; Ringler and Hutt, 2010). Equation (5) further
197 shows that response correction may significantly amplify instrumental noise near the limits of the instrumental
198 passband, especially at long periods and high frequencies where $|H_i(\omega)|$ becomes small. This effect is
199 illustrated conceptually in Figure 2.

200 The total effective station noise floor is modeled as the sum of environmental and ground-referred instrumental
201 noise:

202

204
$$S_{n,i}^{\text{eff}}(\omega) = S_{n,i}^{\text{env}}(\omega) + S_{n,i}^{\text{self,gr}}(\omega). \quad (6)$$

203

205 This formulation assumes statistical independence between environmental and instrumental noise sources.
206 Although this assumption may not hold strictly under all installation conditions, it provides a first-order
207 approximation suitable for network-scale detectability modeling. The formulation is conceptually consistent
208 with probabilistic PSD/PDF approaches commonly used in broadband seismic-noise characterization
209 (Peterson, 1993; McNamara and Buland, 2004). Figure 3 illustrates the transition from station-level
210 detectability kernels to network-scale completeness and spatial detectability fields obtained through
211 probabilistic aggregation of multiple stations.

212 Let $S_s(M, r, \omega)$ denote the source spectrum associated with an earthquake of magnitude M at hypocentral
213 distance r . The station-level signal-to-noise ratio is defined as

214

216
$$\text{SNR}_i(M, r, \omega) = \frac{S_s(M, r, \omega)}{S_{n,i}^{\text{eff}}(\omega)}. \quad (7)$$

215



217 The source spectrum may include attenuation, geometrical spreading, radiation-pattern effects, and site
218 amplification terms. In this work, the framework is intentionally formulated in a general form independent of
219 a specific attenuation law in order to preserve broad applicability.

220 For practical detectability estimates, a band-integrated SNR is considered:

221

$$223 \quad \text{SNR}_i^{\text{band}}(M, r) = \int_{\omega_1}^{\omega_2} W_b(\omega) \text{SNR}_i(M, r, \omega) d\omega, \quad (8)$$

222

224 where $W_b(\omega)$ is a frequency-weighting function defining the target detection band. Band-integrated SNR
225 metrics are widely used in seismic-station performance evaluation and automatic event-detection systems
226 (Trnkoczy, 2012). Figure 4 illustrates the propagation of effective station noise into frequency-dependent SNR
227 degradation and station-level detectability.

228 Detection is modeled probabilistically rather than deterministically. Let $P_i(M, x)$ denote the probability that
229 the i -th station detects an earthquake of magnitude M occurring at spatial position x . Following a generalized
230 logistic formulation,

231

$$233 \quad P_i(M, x) = \frac{1}{1 + \exp[-\alpha_i(\text{SNR}_i^{\text{band}}(M, x) - \theta_i)]}, \quad (9)$$

232

234 where θ_i is the effective detection threshold and α_i controls the steepness of the transition between non-
235 detection and detection. This probabilistic formulation accounts for the gradual transition between detectable
236 and non-detectable conditions resulting from noise variability and waveform uncertainty. Taken together, Eqs.
237 (1)–(9) define a unified detectability formalism in which instrumental self-noise becomes dynamically coupled
238 to seismic-network performance through causal response correction and probabilistic network aggregation.

239 The network-level probability of detection is defined as the complement of the probability that all stations fail
240 to detect the event:

241



243
$$P_{\text{detect}}(M, x) = 1 - \prod_{i=1}^N [1 - P_i(M, x)]. \quad (10)$$

242

244 Equation (10) represents a simplified independence model in which station detections are treated as statistically
245 independent conditional on the event parameters. Although this assumption neglects correlated noise, telemetry
246 losses, and phase-association complexity, it provides a tractable first-order approximation for large-scale
247 detectability analysis. Probabilistic formulations of seismic-network detection capability have long been used
248 in seismic monitoring and network optimization studies (Ringdal, 1975; Kværna and Ringdal, 1986).

249 The completeness magnitude $M_c(x)$ is then defined as the minimum magnitude satisfying

250

252
$$P_{\text{detect}}(M_c, x) \geq P_0, \quad (11)$$

251

253 where P_0 is a target detection probability, typically chosen between 0.9 and 0.95. Figure 5 illustrates the
254 resulting spatial detectability fields and the emergence of completeness patterns jointly controlled by geometry
255 and effective station noise.

256 The framework naturally predicts the existence of distinct detectability regimes depending on the relative
257 importance of geometry, environmental noise, and instrumental self-noise. The geometry-limited regime is
258 characterized by

259

261
$$\frac{\partial M_c}{\partial \log_{10} N} \gg \frac{\partial M_c}{\partial S_{\text{self}}}, \quad (12)$$

260

262 where detectability is controlled primarily by station density and spatial coverage.

263 The self-noise-limited regime is characterized by

264

266
$$\frac{\partial M_c}{\partial S_{\text{self}}} \gg \frac{\partial M_c}{\partial \log_{10} N}, \quad (13)$$

265

267 where instrumental self-noise dominates the effective detection threshold.



268 The ambient-noise-limited regime is characterized by

269

271
$$S_n^{\text{env}}(\omega) \gg S_n^{\text{self,gr}}(\omega), \quad (14)$$

270

272 where environmental noise overwhelms the instrumental contribution.

273 These regimes define distinct optimization strategies for seismic-network design and are investigated

274 quantitatively in Figures 6 and 7.

275 The complete framework developed in this work links instrumental response physics, causal correction,

276 ground-referred self-noise, station-level SNR, probabilistic detection, network aggregation, and spatial

277 completeness magnitude into a unified end-to-end detectability formalism. Figure 8 summarizes this

278 conceptual architecture and illustrates how network performance emerges from the coupled interaction

279 between instrumental properties, environmental noise, and network geometry.

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281

282

283 **3. Data**

284 The framework developed in this study is intentionally designed to remain general and transferable across

285 different classes of seismic monitoring systems. The adopted datasets are intentionally designed to support a

286 physically interpretable and computationally reproducible exploration of first-order detectability controls

287 rather than an operational reconstruction of real-world global catalogue completeness. Consequently, the

288 present framework should be interpreted as a surrogate but physically grounded representation of the coupled

289 interactions among network geometry, environmental noise, instrumental self-noise, and causal response

290 correction. Figure 3 summarizes the transition from station-level detectability kernels to network-scale

291 completeness and spatial detectability fields, while Figures 5–7 illustrate the resulting detectability patterns

292 and regime classifications.

293 The station geometry used throughout this work is derived from publicly accessible metadata distributed

294 through international FDSN web services, including EarthScope/IRIS and EIDA infrastructures (IRIS DMC,

295 2014; EIDA Consortium, 2023). Only broadband seismic stations equipped with high-gain channels (BH?,



296 HH?, LH?) were considered in order to maintain consistency with the broadband detectability framework
297 investigated here. The retrieved station metadata include latitude, longitude, network code, station code, and
298 operational status. After merging and removing duplicate entries, the resulting global inventory contains
299 several thousand broadband seismic stations with heterogeneous spatial coverage representative of current
300 large-scale monitoring systems.

301 The adopted station distribution exhibits dense coverage across North America, Europe, Japan, and selected
302 tectonically active regions, whereas large portions of oceanic domains, polar regions, continental interiors, and
303 developing regions remain sparsely instrumented. This spatial heterogeneity is a fundamental component of
304 the present analysis because it naturally generates geometry-controlled variations in detectability and
305 completeness magnitude. The resulting network geometry therefore provides a realistic first-order
306 approximation of the present-day global broadband monitoring infrastructure, despite unavoidable temporal
307 variations in station availability, instrumentation, and operational status.

308 The present framework does not explicitly simulate waveform propagation through three-dimensional Earth
309 structure. Instead, detectability is modeled through simplified geometry-dependent attenuation kernels
310 designed to isolate the first-order interaction between source amplitude, source-to-station distance, and
311 effective station noise. This choice intentionally prioritizes physical interpretability, numerical stability, and
312 computational reproducibility over regional propagation realism. Consequently, the resulting detectability
313 fields should not be interpreted as operational estimates of absolute completeness magnitude, but rather as
314 internally consistent representations of relative detectability variability.

315 Environmental seismic noise is represented through synthetic but physically informed global noise fields
316 constructed to mimic the large-scale spatial variability observed in broadband seismic-noise studies (Peterson,
317 1993; McNamara and Buland, 2004). The adopted environmental-noise model includes broad continental-
318 scale gradients and regionalized perturbations intended to approximate the contrast between low-noise
319 continental interiors, moderate-noise tectonically active regions, and high-noise coastal or urbanized
320 environments. The adopted representation therefore preserves realistic large-scale spatial structure without
321 attempting to reproduce site-specific noise spectra or temporal nonstationarity.

322 The environmental noise level associated with the i -th station is represented through the PSD field $S_{n,i}^{\text{env}}(\omega)$,
323 while instrumental self-noise is introduced through a family of parameterized spectral models representing



324 different instrumental quality classes. Rather than assigning manufacturer-specific self-noise curves to
325 individual stations, the analysis adopts representative self-noise scenarios spanning low-noise broadband
326 instruments, intermediate-quality sensors, and higher-noise compact or low-cost instrumentation. This
327 approach allows systematic exploration of the transition between geometry-limited and self-noise-limited
328 detectability regimes.

329 The self-noise representation is intentionally simplified and does not attempt to reproduce the detailed spectral
330 behavior of individual sensors. In particular, the present study neglects possible variations associated with
331 installation quality, vault conditions, digitizer characteristics, sensor orientation, aging effects, calibration
332 uncertainties, or site-specific coupling effects. Nevertheless, the adopted parameterization preserves the first-
333 order physical mechanism through which causal response correction propagates instrumental noise into
334 corrected ground-motion units.

335 To evaluate network-scale detectability, the Earth's surface is discretized onto a regular latitude–longitude grid.
336 For each grid node, the source-to-station distance is computed relative to all available stations, allowing
337 construction of station-level detectability kernels and subsequent probabilistic aggregation into network
338 detection probability $P_{\text{detect}}(M, x)$. Spatial smoothing is then applied to reduce discretization artifacts and
339 emphasize large-scale detectability patterns rather than grid-dependent local fluctuations. Figures 5 and 6 show
340 the resulting global detectability fields and regime classifications.

341 The detectability calculations are intentionally performed within a generic frequency-domain framework rather
342 than for a specific earthquake source model. The source spectrum $S_s(M, r, \omega)$ is therefore treated
343 parametrically, allowing the framework to remain independent of any single attenuation relation, stress-drop
344 model, or regional propagation law. This choice reflects the primary objective of the manuscript, namely the
345 investigation of how instrumental self-noise and network geometry jointly control detectability after causal
346 instrument correction.

347 Several assumptions and limitations must therefore be emphasized. First, the present framework does not
348 incorporate realistic phase picking, event association, waveform correlation, or machine-learning-based
349 detection algorithms. Second, station detections are treated as conditionally independent in the probabilistic
350 aggregation model. Third, the environmental-noise field is synthetic and spatially smoothed rather than derived
351 directly from continuous waveform archives. Fourth, the source model neglects source complexity, focal



352 mechanism variability, frequency-dependent scattering, and three-dimensional propagation effects. Finally, the
353 detectability fields presented in this work should be interpreted as physically informed sensitivity maps rather
354 than operational completeness estimates.

355 Despite these limitations, the adopted dataset configuration provides a robust and reproducible framework for
356 isolating the first-order coupling among causal instrument correction, ground-referred self-noise, network
357 geometry, and seismic detectability. The resulting framework is therefore particularly suitable for investigating
358 regime-dependent network optimization strategies and the transition between geometry-driven and
359 instrumentation-driven detectability limits.

360

361

362 4. Methods

363 The methodological framework developed in this study is designed to quantify how instrumental self-noise,
364 environmental noise, and network geometry jointly control seismic detectability after causal instrument
365 correction. The workflow combines physically motivated station-level signal modeling with probabilistic
366 network aggregation, allowing the construction of spatial detectability fields, completeness-magnitude maps,
367 and regime-classification diagnostics. The complete workflow is summarized conceptually in Figure 8, while
368 Figures 3–7 illustrate the successive stages of the computational framework.

369 The first step of the methodology consists of constructing an effective station noise model in corrected ground-
370 motion units. For each station i , the environmental-noise PSD $S_{n,i}^{env}(\omega)$ is combined with the ground-referred
371 instrumental self-noise $S_{n,i}^{self,gr}(\omega)$ defined in Eq. (5). The resulting effective station noise floor is therefore

372

$$374 S_{n,i}^{eff}(\omega) = S_{n,i}^{env}(\omega) + S_{n,i}^{self,gr}(\omega). \quad (15)$$

373

375 The effective noise field is evaluated over a broad frequency range representative of standard broadband
376 seismic observations. The adopted spectral representation is intentionally simplified and is not intended to
377 reproduce instrument-specific transfer functions in detail. Instead, the objective is to isolate the first-order
378 contribution of response-corrected self-noise to detectability degradation. The adopted PSD-based



379 representation is conceptually consistent with probabilistic seismic-noise characterization methods widely
380 used in broadband station monitoring (McNamara and Buland, 2004).

381 The source spectrum associated with an earthquake of magnitude M at hypocentral distance r is represented
382 through a generalized attenuation-based formulation:

383

$$385 \quad S_s(M, r, \omega) = A(M, \omega) G(r) Q(r, \omega) R(\omega), \quad (16)$$

384

386 where $A(M, \omega)$ represents source scaling, $G(r)$ accounts for geometrical spreading, $Q(r, \omega)$ represents
387 attenuation effects, and $R(\omega)$ includes site and radiation-pattern contributions. The present framework
388 intentionally avoids adopting a region-specific attenuation law in order to preserve generality and
389 computational reproducibility.

390 For each station and grid node, the frequency-dependent signal-to-noise ratio is computed according to Eq.
391 (7). A band-integrated station-level SNR is then evaluated using

392

$$394 \quad \text{SNR}_i^{\text{band}}(M, r) = \int_{\omega_1}^{\omega_2} W_b(\omega) \frac{S_s(M, r, \omega)}{S_{n,i}^{\text{eff}}(\omega)} d\omega, \quad (17)$$

393

395 where $W_b(\omega)$ defines the frequency weighting within the target detection band. The adopted weighting
396 function is smooth and broadband in order to avoid introducing artificial detectability discontinuities
397 associated with narrow-band amplification. Band-integrated SNR metrics are widely used in seismic-station
398 performance evaluation and automatic event-detection systems (Trnkoczy, 2012). Figure 4 illustrates the
399 propagation of effective station noise into frequency-dependent SNR degradation and station-level
400 detectability.

401 Detection is modeled probabilistically rather than deterministically. Let $P_i(M, x)$ denote the probability that
402 the i -th station detects an earthquake of magnitude M occurring at spatial position x . Following a generalized
403 logistic formulation,

404



406
$$P_i(M, x) = \frac{1}{1 + \exp[-\alpha_i(\text{SNR}_i^{\text{band}}(M, x) - \theta_i)]}, \quad (18)$$

405

407 where θ_i represents the effective station detection threshold and α_i controls the steepness of the detectability
408 transition. This probabilistic representation accounts for the fact that detection capability does not exhibit a
409 sharp binary threshold, but rather transitions gradually as a function of signal amplitude and noise conditions.
410 Similar probabilistic formulations are commonly adopted in seismic-network performance and automatic-
411 detection studies (Ringdal, 1975; Allen, 1978; Withers et al., 1996).

412 The spatial detectability kernel associated with each station is computed by evaluating Eq. (18) over the full
413 latitude–longitude grid. Figure 3 illustrates the resulting station-level kernels and their aggregation into
414 network-scale detectability fields. The kernel approach allows efficient evaluation of large-scale detectability
415 variability while preserving explicit dependence on station geometry and effective station noise. Kernel-based
416 representations of seismic-network performance are conceptually related to simulation-based network
417 evaluation frameworks such as SNES (D’Alessandro et al., 2011; D’Alessandro and Ruppert, 2012).

418 Network-level detectability is then computed using the probabilistic aggregation model introduced in Eq. (10):

419

421
$$P_{\text{detect}}(M, x) = 1 - \prod_{i=1}^N [1 - P_i(M, x)]. \quad (19)$$

420

422 This formulation assumes conditional independence among station detections. Although this assumption
423 neglects correlated environmental noise, telemetry failures, and event-association complexity, it provides a
424 computationally tractable first-order approximation for large-scale detectability analysis. Probabilistic
425 formulations of seismic-network detection capability have long been used in seismic monitoring and network
426 optimization studies (Ringdal, 1975; Kväerna and Ringdal, 1986).

427 The completeness magnitude $M_c(x)$ is estimated by solving

428

430
$$P_{\text{detect}}(M_c, x) = P_0, \quad (20)$$

429



431 where P_0 is the target detection probability threshold. In this work, P_0 is fixed at 0.95 unless otherwise stated.

432 Figure 5 illustrates the resulting spatial detectability maps and completeness fields.

433 To reduce discretization artifacts associated with the finite spatial grid and heterogeneous station spacing,

434 spatial smoothing is applied to the detectability fields using isotropic Gaussian filtering. The smoothing scale

435 is intentionally chosen to suppress grid-dependent fluctuations while preserving first-order continental-scale

436 detectability gradients. Spatial smoothing approaches are commonly used in completeness and seismic-

437 network capability studies to stabilize large-scale detectability patterns and suppress artificial discretization

438 effects (Woessner and Wiemer, 2005). No attempt is made to infer local operational completeness values at

439 subregional scales.

440 The framework additionally evaluates the relative sensitivity of detectability to network geometry and

441 instrumental self-noise. The geometry sensitivity is quantified through

442

$$444 \quad \Gamma_N = -\frac{\partial M_c}{\partial \log_{10} N}, \quad (21)$$

443

445 while the self-noise sensitivity is quantified as

446

$$448 \quad \Gamma_{\text{self}} = \frac{\partial M_c}{\partial S_{\text{self}}}, \quad (22)$$

447

449 These two quantities are used to classify the dominant detectability regime at each grid node. Figure 6 shows

450 the resulting regime-classification maps, while Figure 7 illustrates the trade-off between geometry-driven and

451 self-noise-driven detectability limitations.

452 The regime-classification framework distinguishes among geometry-limited, self-noise-limited, and ambient-

453 noise-limited conditions according to the relative magnitude of Γ_N , Γ_{self} , and the environmental-noise

454 contribution. This classification provides a physically interpretable representation of how detectability controls

455 change across different network configurations and instrumental conditions.

456 Several methodological simplifications must nevertheless be emphasized. The framework does not include

457 realistic phase picking, waveform correlation, template matching, machine-learning-based detectors, or event-



458 association algorithms. Source radiation is represented through simplified isotropic spectral scaling, while
459 three-dimensional Earth structure and path-dependent scattering are neglected. Furthermore, the
460 environmental-noise model is synthetic and temporally stationary. Consequently, the present methodology
461 should be interpreted as a first-order physical framework for network detectability analysis rather than as an
462 operational earthquake-detection simulator.

463 Despite these limitations, the adopted methodology provides a computationally reproducible and physically
464 transparent framework for quantifying how causal instrument correction propagates instrumental self-noise
465 into seismic detectability and completeness magnitude. The resulting approach is particularly suitable for
466 investigating the transition between geometry-controlled and instrumentation-controlled monitoring regimes
467 in modern seismic networks.

468

469

470 **5. Results**

471 The proposed framework reveals that seismic detectability emerges from the coupled interaction among
472 network geometry, environmental noise, and instrumental self-noise after causal instrument correction. The
473 resulting detectability patterns exhibit strong spatial variability and naturally organize into distinct operational
474 regimes characterized by different dominant limiting factors. Figures 3–7 summarize the principal results
475 obtained from the station-level detectability kernels, network aggregation framework, and regime-
476 classification analysis.

477 Figure 3 illustrates the transition from individual station detectability kernels to network-scale completeness
478 fields. The station-level kernels exhibit the expected radial attenuation structure around individual stations,
479 with detectability decreasing progressively as source-to-station distance increases. However, the aggregated
480 network response is highly heterogeneous at the global scale due to the strongly nonuniform spatial distribution
481 of broadband seismic stations. Dense continental regions characterized by high station density exhibit
482 extensive overlapping detectability kernels, producing elevated network detection probabilities and reduced
483 completeness magnitude. In contrast, oceanic domains, polar regions, and sparsely instrumented continental
484 interiors display fragmented detectability structures associated with limited network redundancy.



485 The resulting global completeness patterns shown in Figure 5 demonstrate that network geometry alone
486 produces strong first-order spatial variability in detectability. Regions characterized by dense station coverage
487 exhibit systematically lower M_c values, whereas sparsely instrumented regions require substantially larger
488 event magnitudes to achieve the same network-level detection probability. Similar geometry-controlled
489 completeness variations have previously been observed in simulation-based seismic-network performance
490 analyses (D'Alessandro et al., 2011; D'Alessandro and Ruppert, 2012). These geometry-controlled gradients
491 emerge naturally from the probabilistic aggregation framework and remain stable under moderate variations
492 in smoothing scale and detection threshold.

493 The introduction of environmental noise significantly modifies the spatial detectability structure. Figure 4
494 shows that elevated environmental-noise levels reduce the effective station-level signal-to-noise ratio and
495 broaden the transition between detectable and non-detectable conditions. This behavior is consistent with
496 previous studies linking ambient seismic-noise levels to station performance and completeness variability
497 (McNamara and Buland, 2004; Catania et al., 2025). The degradation of detectability is nonlinear because the
498 logistic detection model amplifies the effect of small SNR perturbations near the effective detection threshold.

499 The impact of instrumental self-noise becomes evident after causal response correction. Figure 2 demonstrates
500 that response correction propagates instrumental self-noise into corrected ground-motion units through the
501 inverse instrumental transfer function. Such amplification effects are consistent with established instrumental
502 self-noise analyses and sensor intercomparison studies (Sleeman et al., 2006; Ringler and Hutt, 2010).
503 Although environmental noise remains the dominant contribution across most of the broadband frequency
504 range, instrumental self-noise becomes increasingly important at long periods and in low-noise monitoring
505 conditions. This effect is particularly pronounced for higher-noise instrumental classes and low-cost sensing
506 configurations.

507 Figure 4 further demonstrates that the contribution of instrumental self-noise is strongly frequency dependent.
508 In the low-frequency domain, response correction may substantially increase the effective station noise floor
509 even when the intrinsic instrumental noise remains relatively small in raw instrumental units. Consequently,
510 detectability degradation cannot be interpreted solely as a function of environmental noise amplitude, but must
511 also account for the spectral behavior of the corrected instrumental response.



512 The global detectability fields shown in Figure 5 reveal that the interaction between environmental noise and
513 instrumental self-noise produces spatially variable detectability degradation. In regions already characterized
514 by sparse station geometry, self-noise amplification produces relatively modest additional effects because
515 geometry remains the dominant limiting factor. In contrast, densely instrumented low-noise regions exhibit
516 significantly greater sensitivity to instrumental self-noise because geometry-related limitations are already
517 minimized. This result demonstrates that the relative importance of instrumental quality increases as network
518 density improves. This geometry-to-self-noise transition represents one of the principal conceptual outcomes
519 of the framework because it demonstrates that the dominant control on detectability evolves systematically as
520 seismic networks approach increasingly dense monitoring configurations. This transition is particularly
521 relevant for emerging dense and low-cost seismic networks, where instrumentation-related limitations may
522 progressively dominate over purely geometrical constraints (Anthony et al., 2019; Scudero et al., 2023).

523 The regime-classification analysis shown in Figure 6 provides a quantitative representation of these transitions.
524 Three principal detectability regimes emerge naturally from the framework. Comparable trade-offs between
525 network geometry, environmental noise, and monitoring capability have been discussed in previous seismic-
526 network optimization studies (Ringdal, 1975; D'Alessandro and Stickney, 2012). The geometry-limited regime
527 dominates oceanic domains, remote continental interiors, and polar regions characterized by sparse station
528 coverage. In these regions, detectability improvements are controlled primarily by increasing station density
529 and reducing inter-station spacing. The ambient-noise-limited regime is concentrated in regions characterized
530 by elevated environmental-noise levels, including coastal zones, tectonically active regions, and urbanized
531 environments. In these areas, reductions in ambient noise or improvements in site conditions produce larger
532 detectability gains than additional network densification.

533 The self-noise-limited regime appears primarily within dense low-noise networks where environmental noise
534 and geometry-related limitations are already relatively weak. In these conditions, the effective detection
535 threshold becomes increasingly sensitive to the spectral properties of the instrumental response and the
536 propagated ground-referred self-noise. This transition is particularly important because it implies that further
537 improvements in detectability may require reductions in instrumental self-noise rather than additional stations.
538 Figure 7 quantifies the trade-off between geometry-driven and instrumentation-driven detectability
539 improvements. The sensitivity analysis demonstrates that the relative gain associated with network



540 densification decreases progressively as station density increases. At low station density, detectability
541 improvements are dominated by geometric effects because additional stations substantially increase spatial
542 redundancy and reduce hypocentral distance to the nearest sensor. However, beyond a critical network density,
543 the marginal detectability gain associated with additional stations becomes progressively smaller.
544 In contrast, the relative importance of instrumental self-noise increases systematically with network density.
545 The sensitivity curves shown in Figure 7 indicate that low-noise instrumental configurations produce only
546 limited improvements in sparse networks, but become increasingly effective within already dense monitoring
547 systems. This nonlinear transition represents one of the principal results of the present study because it
548 demonstrates that optimal network design strategies depend strongly on the operational detectability regime.
549 The framework additionally shows that detectability degradation associated with instrumental self-noise is
550 strongly scale dependent. Small increases in self-noise may produce negligible effects under geometry-limited
551 conditions while generating substantial detectability losses in low-noise dense networks. Consequently, the
552 same instrumental improvement may have dramatically different operational value depending on the
553 surrounding network configuration.
554 The global detectability fields remain qualitatively stable under moderate variations in smoothing scale,
555 detection threshold P_0 , and parameterized self-noise amplitude. Although the absolute completeness values
556 vary as expected, the large-scale spatial organization of geometry-limited, ambient-noise-limited, and self-
557 noise-limited regions remains robust. This stability indicates that the regime-classification framework reflects
558 intrinsic properties of the coupled detectability system rather than numerical artifacts associated with a
559 particular parameter choice.
560 Overall, the results demonstrate that seismic detectability cannot be interpreted exclusively as a geometrical
561 property of seismic networks. Instead, detectability emerges from a multiscale interaction among station
562 geometry, environmental noise, instrumental response, and propagated self-noise after causal correction. The
563 resulting framework provides a physically interpretable basis for analyzing the transition between geometry-
564 controlled and instrumentation-controlled monitoring regimes in modern seismic networks.

565

566

567 **6. Discussion**



568 The results presented in this study demonstrate that seismic detectability is not controlled exclusively by
569 network geometry, but instead emerges from the coupled interaction among station distribution, environmental
570 noise, instrumental response, and propagated self-noise after causal instrument correction. The proposed
571 framework therefore extends traditional interpretations of network performance by explicitly linking
572 instrument physics to large-scale detectability behavior.

573 Conventional seismic-network assessments have generally focused on geometrical quantities such as inter-
574 station spacing, azimuthal coverage, phase availability, and travel-time residuals (Ringdal, 1975; D'Alessandro
575 et al., 2011; D'Alessandro and Ruppert, 2012). Within this perspective, detectability is typically interpreted as
576 a predominantly geometric property that improves monotonically with increasing station density. The present
577 results only partially support this interpretation. While geometry clearly dominates detectability in sparse
578 monitoring systems, the framework shows that the relative importance of instrumental self-noise progressively
579 increases as network density improves and environmental noise decreases.

580 This transition constitutes one of the central conceptual results of the study. In geometry-limited conditions,
581 detectability is controlled primarily by source-to-station distance and network redundancy. Under these
582 conditions, the addition of new stations produces substantial reductions in completeness magnitude because it
583 improves both spatial coverage and probabilistic aggregation efficiency. However, once network geometry
584 reaches sufficiently high density, the marginal benefit associated with additional stations decreases
585 significantly. At this stage, propagated instrumental self-noise becomes increasingly important because the
586 effective station noise floor approaches the ambient-noise level.

587 This behavior has important implications for modern seismic-network design. Dense broadband arrays, urban
588 monitoring systems, distributed MEMS deployments, and emerging low-cost seismic networks are
589 increasingly characterized by high station density but heterogeneous instrumental quality (Anthony et al.,
590 2019; Scudero et al., 2023). In these contexts, further densification alone may no longer provide optimal
591 detectability gains. Instead, improvements in instrumental self-noise, response stability, vault conditions, or
592 environmental-noise mitigation may become equally or more effective than adding additional stations.

593 More fundamentally, the present framework demonstrates that instrumental self-noise cannot be treated as an
594 isolated sensor property once causal response correction is applied, because the corrected self-noise becomes
595 an emergent component of the probabilistic detectability structure of the entire seismic network.



596 The results further emphasize the importance of causal response correction in detectability analysis.
597 Instrumental self-noise is commonly characterized in raw instrumental units, where it may appear negligible
598 relative to environmental noise across much of the broadband frequency range (Sleeman et al., 2006; Ringler
599 and Hutt, 2010). However, the present framework shows that response correction can substantially amplify
600 self-noise contributions near the limits of the instrumental passband, particularly at long periods and high
601 frequencies. Consequently, the detectability impact of self-noise cannot be evaluated independently of the
602 response-removal procedure.

603 This aspect is especially relevant for low-cost instrumentation and MEMS-based seismic networks. Although
604 low-cost sensors may provide acceptable performance in strong-motion applications or dense urban
605 deployments (D'Alessandro and D'Anna, 2013; D'Alessandro et al., 2014), their effective detectability after
606 response correction depends critically on the spectral behavior of the corrected self-noise floor. The present
607 framework therefore provides a physically consistent basis for evaluating the operational trade-offs between
608 station density and instrumental quality in dense low-cost monitoring systems.

609 The regime-classification framework introduced in this work additionally provides a physically interpretable
610 representation of how detectability controls vary spatially. The geometry-limited regime dominates sparsely
611 instrumented regions, where additional stations remain the primary mechanism for improving completeness
612 magnitude. In contrast, ambient-noise-limited regions are controlled predominantly by environmental
613 conditions and site characteristics. Finally, self-noise-limited regimes emerge preferentially within dense low-
614 noise networks where geometry-related limitations have already been minimized. This classification highlights
615 that optimal network-design strategies are intrinsically regime dependent.

616 The framework also helps explain why improvements in detectability may saturate in mature broadband
617 seismic networks. Once station spacing becomes sufficiently small, the effective detection threshold no longer
618 decreases proportionally with network density because the station-level noise floor imposes a lower bound on
619 usable signal amplitude. This saturation effect naturally emerges from the probabilistic aggregation model and
620 may partially explain the diminishing returns often observed in highly developed monitoring infrastructures.

621 Importantly, the primary objective of the present study is not the operational prediction of absolute
622 completeness magnitude at regional or global scales, but rather the development of a physically interpretable
623 framework capable of identifying the dominant controls governing seismic detectability under different



624 monitoring configurations. From this perspective, the simplified surrogate representations adopted here are
625 intentional because they allow isolation of the first-order coupling mechanisms linking instrumental response
626 physics, propagated self-noise, environmental noise, and network geometry. Several limitations nevertheless
627 constrain the interpretation of the present results. First, the environmental-noise field adopted in this study is
628 synthetic and temporally stationary. Real seismic-noise environments exhibit strong seasonal, meteorological,
629 anthropogenic, and diurnal variability that may substantially modify short-term detectability conditions
630 (McNamara and Buland, 2004; Ruzza et al., 2024). These effects have been extensively documented through
631 PSD/PDF analyses of the Italian broadband seismic network (D'Alessandro et al., 2021). Second, the
632 framework does not explicitly include realistic phase picking, waveform correlation, machine-learning
633 detectors, or event-association procedures. Modern AI-based detection systems may partially compensate for
634 elevated noise levels through improved signal extraction and pattern recognition.

635 Third, the present analysis neglects three-dimensional Earth structure, path-dependent attenuation variability,
636 anisotropic propagation, and scattering effects. The adopted attenuation formulation is intentionally simplified
637 in order to isolate first-order interactions between geometry and instrumental noise. Consequently, the derived
638 detectability fields should be interpreted as physically informed sensitivity patterns rather than operational
639 estimates of catalogue completeness.

640 The assumption of conditional independence among station detections also represents an important
641 simplification. In real seismic networks, environmental noise may exhibit spatial correlation, telemetry failures
642 may affect multiple stations simultaneously, and phase-association procedures introduce additional nonlinear
643 dependencies. Although the adopted aggregation framework provides a tractable first-order approximation,
644 future extensions could incorporate correlated detection statistics and time-dependent network reliability.

645 Several additional developments appear particularly promising. The framework could be extended to include
646 empirical PSD databases derived from continuous waveform archives, allowing direct integration of observed
647 station-noise statistics into detectability calculations. Similarly, instrument-specific transfer functions and
648 measured self-noise curves could replace the simplified parameterized instrumental classes adopted here.

649 Another important extension would involve coupling the framework with realistic earthquake source
650 populations and three-dimensional propagation simulations to generate operational detectability forecasts.



651 The methodology may also prove useful for evaluating future dense nodal arrays and distributed MEMS-based
652 monitoring systems. In such networks, station density may become sufficiently large that geometry-related
653 limitations are almost entirely suppressed, shifting detectability control toward instrumental stability and
654 environmental noise. Under these conditions, the optimization problem changes fundamentally from “where
655 should new stations be installed?” to “what level of instrumental quality is required to achieve further
656 detectability gains?”

657 More broadly, the results highlight the need to reinterpret seismic-network performance as a multiscale
658 emergent property linking instrument physics, environmental conditions, and spatial network organization.
659 Detectability is therefore not solely a geometrical characteristic of monitoring systems, but rather the outcome
660 of coupled interactions operating across multiple physical and observational scales.

661 The framework developed here provides a computationally reproducible and physically interpretable basis for
662 investigating these interactions. By explicitly propagating instrumental self-noise through causal response
663 correction into network-scale detectability metrics, the present study establishes a direct conceptual bridge
664 between instrumental seismology and probabilistic seismic-network performance analysis.

665

666

667 **7. Conclusions**

668 This study developed a unified theoretical and computational framework linking causal instrument correction,
669 instrumental self-noise, environmental seismic noise, and network geometry to seismic detectability and
670 completeness magnitude. The proposed approach explicitly propagates instrumental self-noise into corrected
671 ground-motion units through the inverse instrumental response and integrates this contribution into a
672 probabilistic network-detectability formalism.

673 The results demonstrate that seismic detectability is not controlled exclusively by network geometry. Instead,
674 detectability emerges from the coupled interaction among source amplitude, propagation effects,
675 environmental noise, instrumental response, and probabilistic network aggregation. This interaction naturally
676 generates strong spatial variability in completeness magnitude and produces distinct detectability regimes
677 characterized by different dominant limiting factors.



678 The framework identifies three principal operational regimes. Geometry-limited conditions dominate sparse
679 monitoring systems where detectability is controlled primarily by inter-station spacing and spatial coverage.
680 Ambient-noise-limited conditions occur where elevated environmental noise suppresses station-level signal-
681 to-noise ratios independently of network density. Self-noise-limited conditions emerge preferentially within
682 dense low-noise networks, where geometry-related limitations are already minimized and the effective station
683 noise floor becomes increasingly controlled by propagated instrumental self-noise.

684 One of the principal results of the study is the demonstration that the relative importance of instrumental quality
685 increases systematically with network density. In sparse networks, additional stations produce the largest
686 detectability improvements because geometric redundancy dominates network performance. However, as
687 station density increases, the marginal benefit associated with further densification progressively decreases,
688 while the contribution of instrumental self-noise becomes increasingly significant. This transition implies that
689 optimal network-design strategies are intrinsically regime dependent.

690 The results additionally demonstrate that causal response correction plays a fundamental role in detectability
691 analysis. Instrumental self-noise that appears negligible in raw instrumental units may become substantially
692 amplified after response correction near the limits of the instrumental passband. Consequently, the detectability
693 impact of instrumental noise cannot be evaluated independently of the response-removal procedure.

694 These findings are particularly relevant for modern dense monitoring systems, including urban seismic
695 networks, distributed broadband arrays, and emerging low-cost MEMS-based deployments. In such
696 configurations, improvements in instrumental self-noise, response stability, and environmental-noise
697 mitigation may become as important as — or more important than — further network densification.

698 Several limitations nevertheless constrain the present framework. The environmental-noise model is synthetic
699 and temporally stationary, realistic phase-picking and event-association procedures are not explicitly modeled,
700 and three-dimensional propagation effects are neglected. Consequently, the resulting detectability fields should
701 be interpreted as physically informed sensitivity patterns rather than operational estimates of catalogue
702 completeness.

703 Despite these limitations, the framework provides a computationally reproducible and physically transparent
704 basis for investigating the transition between geometry-controlled and instrumentation-controlled monitoring
705 regimes. The methodology further establishes a direct conceptual bridge between instrumental seismology and



706 probabilistic seismic-network performance analysis by explicitly incorporating response-corrected
707 instrumental self-noise into network-scale detectability metrics.

708 By explicitly propagating response-corrected instrumental self-noise into network-scale detectability metrics,
709 the present study reframes seismic-network performance as an emergent multiscale property linking instrument
710 physics, environmental noise, and spatial network organization within a unified probabilistic framework.

711 Future developments should incorporate empirical PSD archives, instrument-specific self-noise
712 measurements, realistic propagation simulations, and machine-learning-based detection frameworks. Such
713 extensions would allow the present methodology to evolve toward fully operational detectability forecasting
714 systems capable of supporting next-generation seismic-network optimization and performance assessment.

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717 **Competing interests:**

718 The authors declare that they have no conflict of interest.

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721 **8. References**

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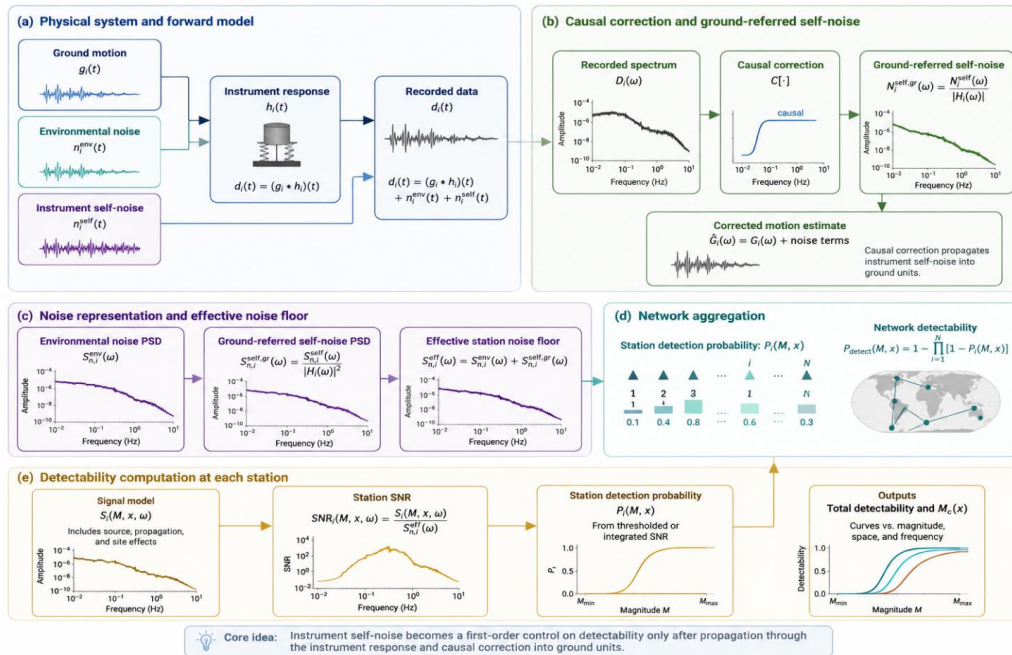
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825 **Figure 1.** Conceptual framework linking causal instrument correction to network-scale seismic detectability.

826 Panel (a) summarizes the observational model at station i , where recorded data result from the convolution of

827 ground motion with the instrumental response in the presence of environmental and instrumental noise

828 contributions. Panel (b) illustrates causal response correction and the propagation of instrumental self-noise

829 into ground-referred units after inverse filtering. Panel (c) defines the effective station noise floor resulting

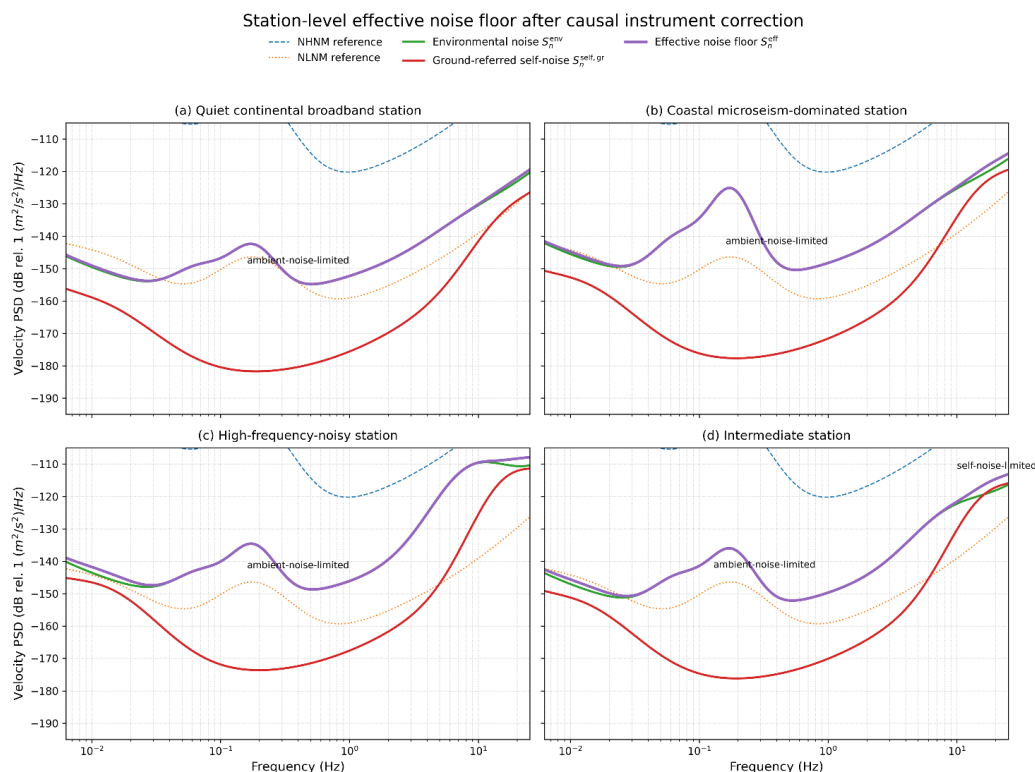
830 from the combined contribution of environmental noise and propagated self-noise. Panel (d) shows the station-

831 level detectability concept based on signal-to-noise ratio and probabilistic detection. Panel (e) extends the

832 formulation to the network scale through probabilistic aggregation of station detections into spatial

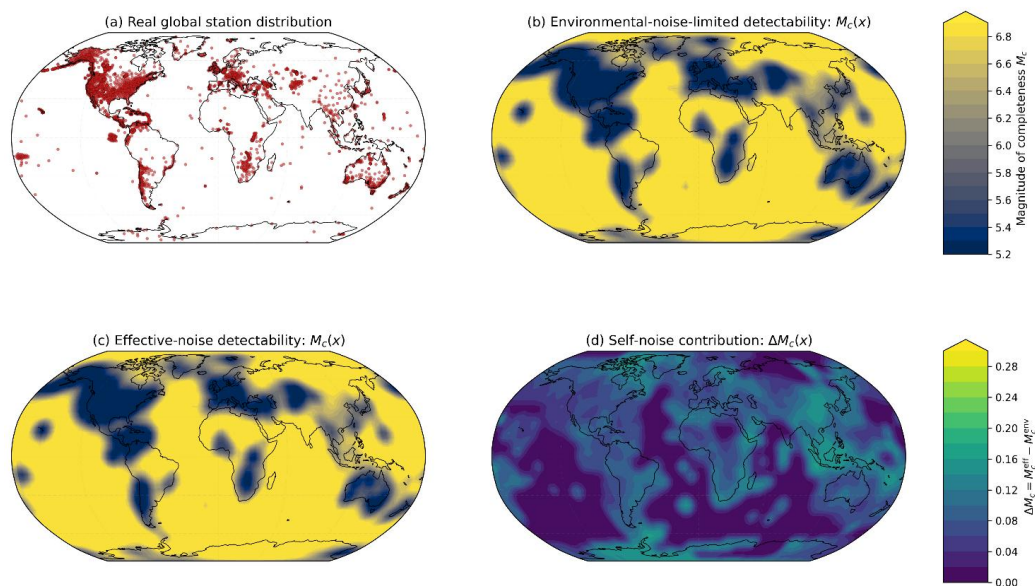
833 detectability and completeness-magnitude fields. The figure summarizes the physical and computational chain

834 linking instrument-response physics to large-scale seismic-network performance.



835

836 **Figure 2.** Station-level effective noise floor after causal instrument correction. Panels (a–d) show
 837 representative power spectral density (PSD) models for four broadband seismic stations characterized by
 838 different environmental conditions: (a) quiet continental site, (b) coastal microseism-dominated site, (c) high-
 839 frequency noisy site, and (d) intermediate conditions. Environmental noise spectra (green) and ground-referred
 840 instrumental self-noise (red), obtained after causal response correction, are combined into the effective station
 841 noise floor (purple). Dashed and dotted curves denote the NHNM and NLNM reference models of Peterson
 842 (1993). The results illustrate how instrumental self-noise becomes frequency dependent after response
 843 correction and may contribute significantly to the effective station noise floor under low-noise conditions or
 844 near the limits of the instrumental passband. Although environmental noise dominates most frequency bands,
 845 propagated self-noise establishes a lower bound on achievable signal-to-noise ratio and therefore on seismic
 846 detectability.



847

848 **Figure 3.** Global detectability and completeness under environmental and effective station noise.

849 Panel (a) shows the spatial distribution of broadband seismic stations derived from open FDSN Station

850 services (EarthScope and EIDA federator), highlighting the strong heterogeneity of the present-day global

851 monitoring infrastructure. Panel (b) presents the spatial distribution of completeness magnitude

852 $M_c(x)$ computed using environmental noise only within the probabilistic detectability framework described in

853 the text. Low M_c values coincide with densely instrumented regions, whereas sparsely monitored areas exhibit

854 substantially reduced detectability. Panel (c) shows the corresponding completeness field after inclusion of

855 propagated instrumental self-noise within the effective station noise model. The resulting detectability is

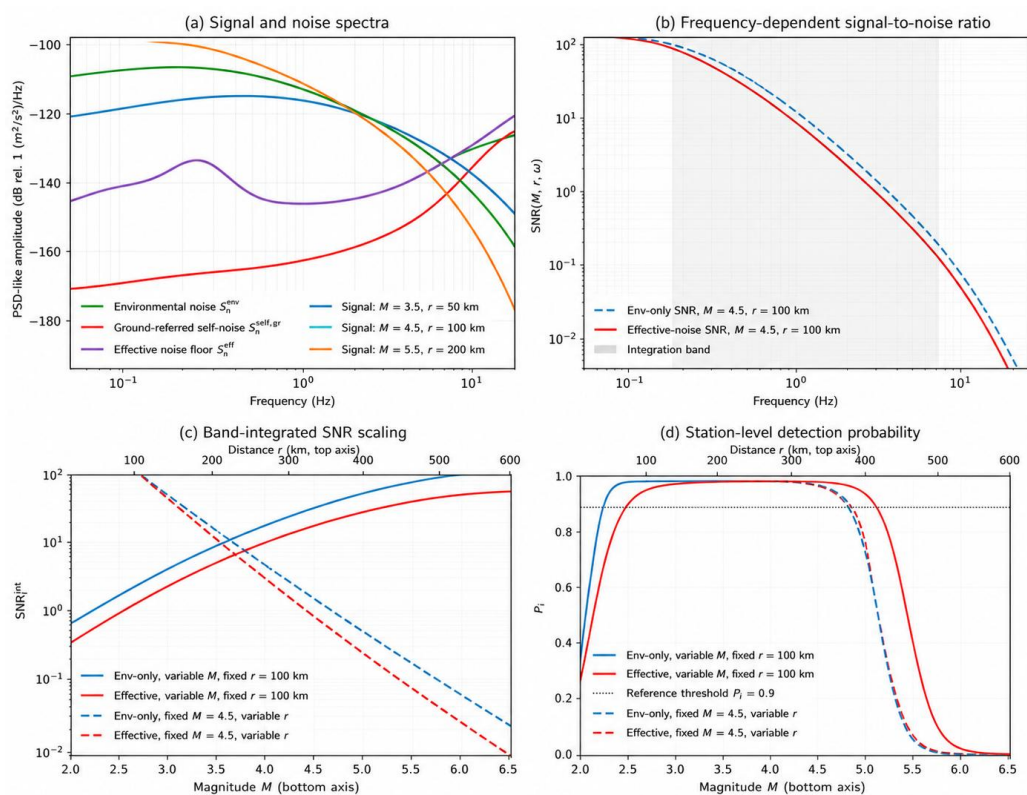
856 systematically degraded, leading to higher completeness magnitude values. Panel (d) illustrates the spatial

857 increment in completeness magnitude associated with instrumental self-noise. Although generally moderate,

858 the effect is spatially heterogeneous and becomes more pronounced in regions characterized by sparse

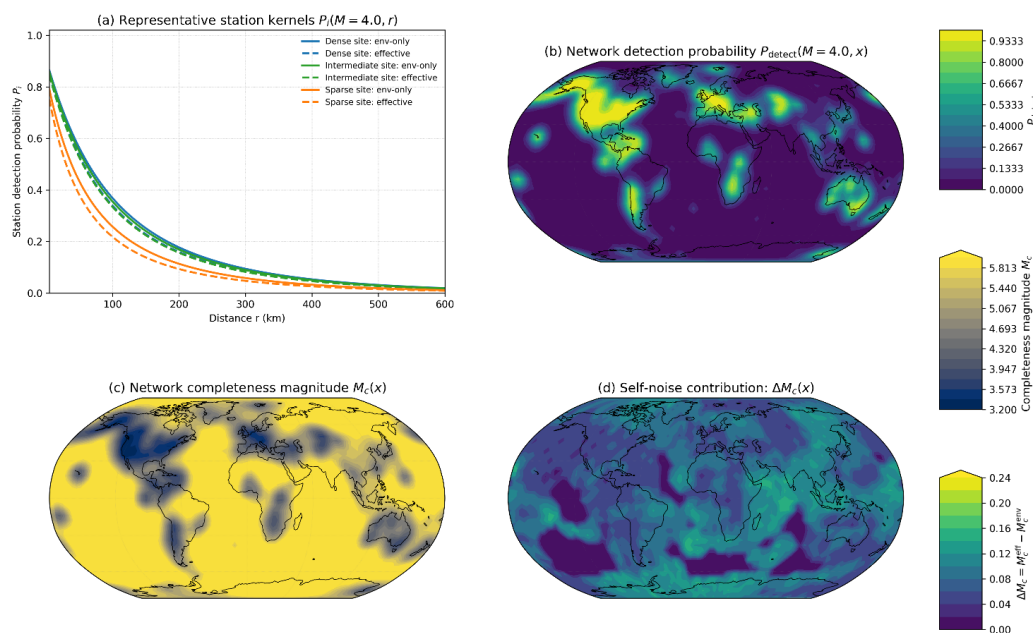
859 geometry and elevated effective station noise. Spatial fields are mildly smoothed for visualization purposes

860 only and do not affect the underlying detectability calculations.



861

862 **Figure 4.** Frequency-dependent propagation of effective station noise into signal-to-noise ratio and station-
 863 level detectability. Panel (a) shows representative environmental-noise spectra, propagated instrumental self-
 864 noise, effective station noise floor, and theoretical earthquake signal spectra for different magnitude–distance
 865 combinations. The effective station noise floor results from the combination of environmental and ground-
 866 referred instrumental self-noise after causal response correction. Panel (b) illustrates the frequency-dependent
 867 signal-to-noise ratio obtained using environmental noise only (blue) and the full effective noise model (red)
 868 for a representative event scenario. The shaded region indicates the integration band adopted for detectability
 869 calculations. Panel (c) shows the dependence of integrated signal-to-noise ratio on earthquake magnitude
 870 (solid curves) and epicentral distance (dashed curves), comparing environmental-noise-only and effective-
 871 noise conditions. Panel (d) presents the corresponding probabilistic station detection curves. The results
 872 demonstrate that propagated instrumental self-noise systematically reduces detectability, particularly near the
 873 lower signal-to-noise threshold and at larger source–receiver distances.



874

875 **Figure 5.** From station-level detectability kernels to network-scale completeness structure.

876 Panel (a) shows representative station-detection kernels for dense, intermediate, and sparse monitoring

877 configurations, comparing environmental-noise-only and effective-noise conditions. Detection probability

878 decreases with increasing source–receiver distance and is systematically reduced when propagated

879 instrumental self-noise is included. Panel (b) presents the resulting global network detection probability field

880 for a representative earthquake magnitude. High detectability is concentrated in densely instrumented

881 continental regions, whereas oceanic and poorly monitored areas exhibit substantially lower detection

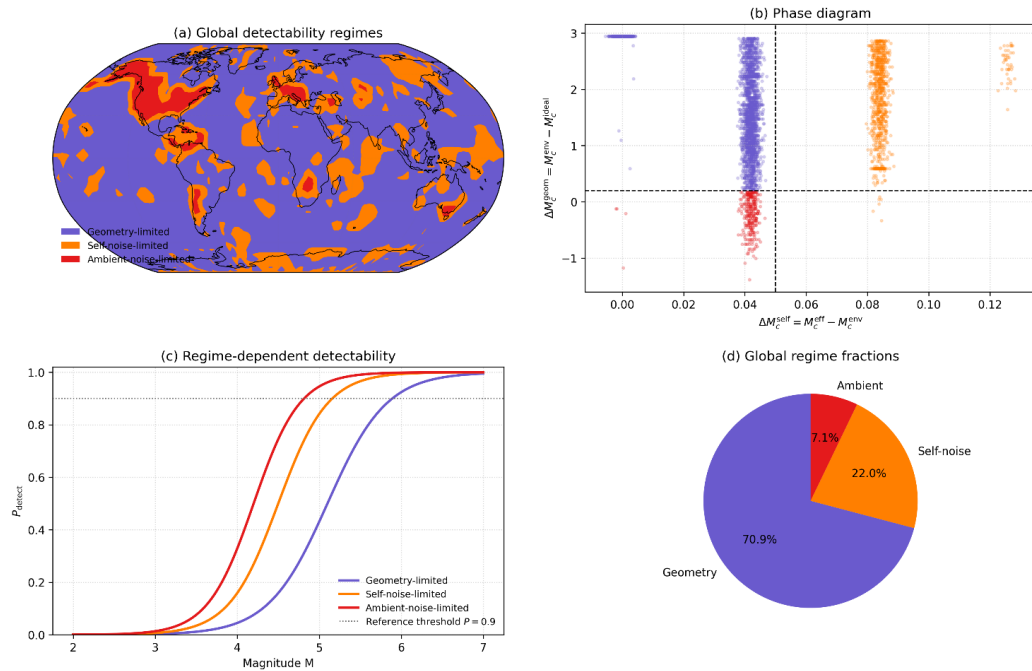
882 capability. Panel (c) shows the corresponding spatial completeness-magnitude field $M_c(x)$, highlighting the

883 strong geographical heterogeneity of global seismic-network performance. Panel (d) illustrates the increase

884 in completeness magnitude associated with propagated instrumental self-noise. Although generally moderate,

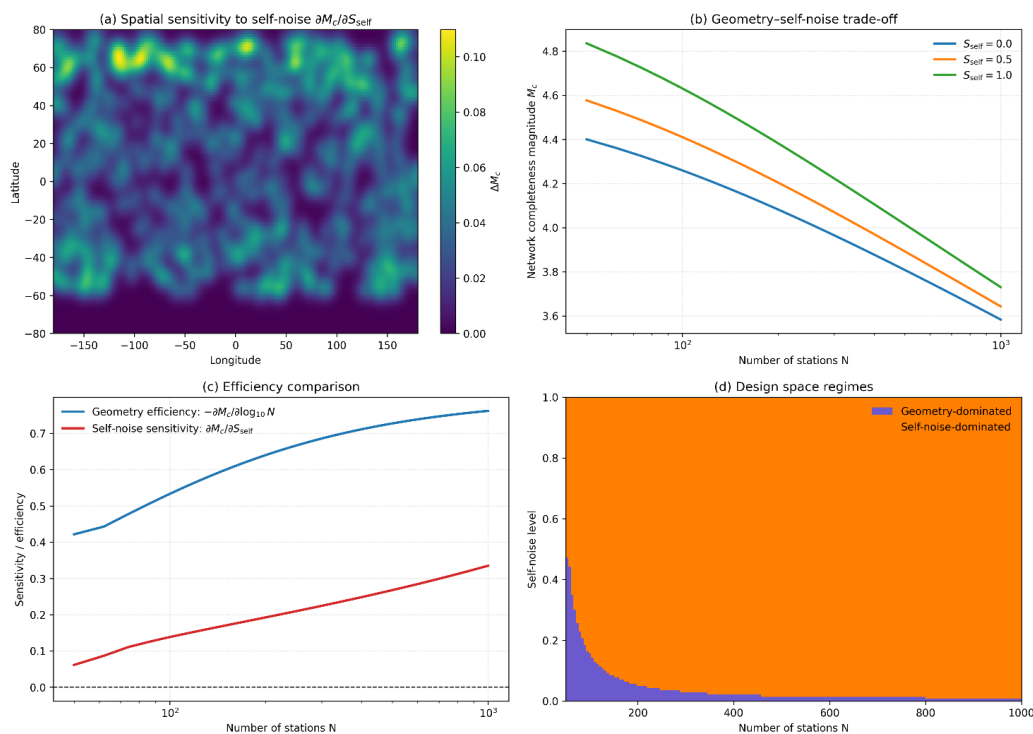
885 this degradation becomes spatially coherent and regionally significant when integrated over large-scale

886 network configurations.



887

888 **Figure 6.** Global classification of detectability regimes. Panel (a) shows the spatial distribution of the
 889 dominant detectability regimes derived from the relative contributions of network geometry, environmental
 890 noise, and propagated instrumental self-noise. Three principal regimes are identified: geometry-limited, self-
 891 noise-limited, and ambient-noise-limited. Panel (b) presents the corresponding phase diagram in the space
 892 defined by effective self-noise contribution and geometry-related detectability degradation. The dashed lines
 893 denote reference transition thresholds separating the dominant-control domains. Panel (c) illustrates
 894 representative detectability curves for the three regimes, demonstrating the systematic shift in completeness
 895 magnitude associated with changing dominant controls. Panel (d) summarizes the global fractional occupancy
 896 of each regime. The results show that sparse monitoring geometries dominate most large-scale detectability
 897 limitations, whereas instrumental self-noise becomes increasingly important in dense and low-noise
 898 monitoring environments.



899

900 **Figure 7.** Sensitivity analysis of geometry-driven and instrumentation-driven detectability controls.

901 Panel (a) shows the spatial sensitivity of completeness magnitude to propagated instrumental self-noise,

902 illustrating the heterogeneous geographical response of detectability to sensor-quality degradation. Panel (b)

903 presents the trade-off between network densification and instrumental self-noise reduction for different

904 effective self-noise levels. Increasing station density systematically improves detectability, although the relative

905 gain decreases as networks become progressively denser. Panel (c) compares the relative efficiency of

906 geometry improvement and self-noise reduction as functions of network size. Geometry-driven gains dominate

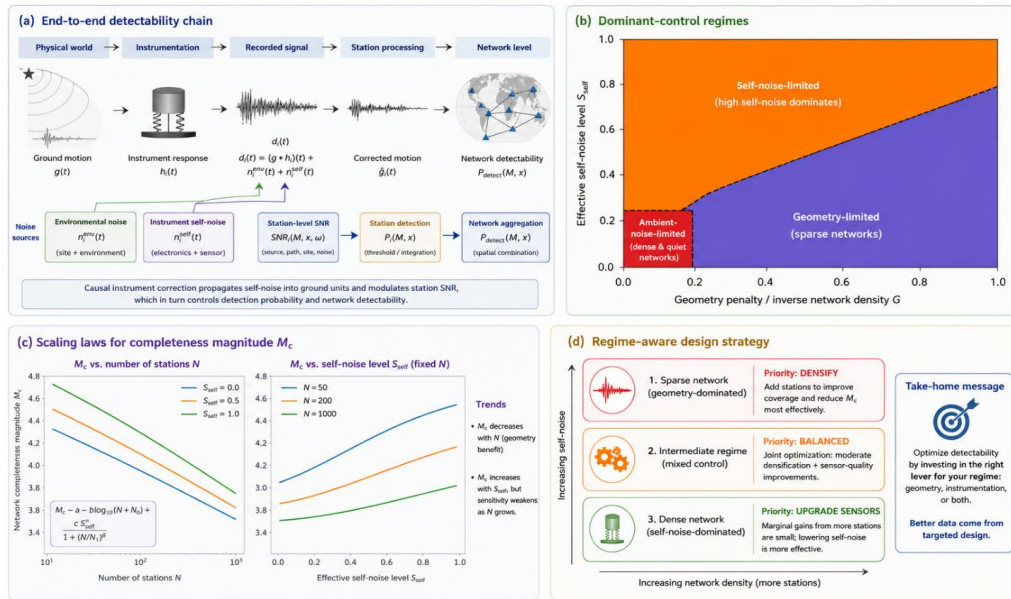
907 sparse configurations, whereas instrumentation-driven improvements become progressively more relevant in

908 dense networks. Panel (d) summarizes the resulting regime-aware design space, separating geometry-

909 dominated and self-noise-dominated optimization domains. The figure illustrates the transition from sparse

910 geometry-controlled systems toward dense monitoring configurations in which instrumental self-noise

911 becomes an increasingly important detectability constraint.



912

913 **Figure 8.** Unified conceptual synthesis of the proposed detectability framework. Panel (a) summarizes the end-
 914 to-end physical and computational chain linking ground motion, instrumental response, environmental noise,
 915 propagated instrumental self-noise, station-level signal-to-noise ratio, probabilistic station detection, and
 916 network-scale detectability. Panel (b) illustrates the dominant detectability-control regimes in the space
 917 defined by geometry-related degradation and effective self-noise level, highlighting the transition from
 918 ambient-noise-limited to geometry-limited and self-noise-limited monitoring conditions. Panel (c) shows
 919 representative scaling laws for completeness magnitude as functions of network density and propagated self-
 920 noise. Panel (d) summarizes the corresponding regime-aware optimization strategy for seismic-network
 921 design. Sparse networks primarily benefit from densification, whereas dense and low-noise monitoring systems
 922 increasingly benefit from reductions in instrumental self-noise. Together, the panels synthesize the central
 923 conceptual result of the study: seismic-network detectability emerges from the coupled multiscale interaction
 924 between instrument physics, environmental noise, and network geometry.