

## S1 Benchmarking

This section presents the complete benchmarking results for all principal coarse-grained quantities discussed in the main text. The benchmark case consists of a discrete element method (DEM) simulation of a static monodisperse cubic lattice subjected to vertical plate displacement with fixed side walls. Particles have diameter  $d = 0.02$  m and density  $\rho = 2500$  kg m<sup>-3</sup>.

The system is analysed at the sample centre,  $P = 0, 0, 0.42725$  m. Results are reported for a range of smoothing half-widths, expressed as multiples of the particle diameter ( $w/d$ ), and for different smoothing functions (Lucy, Gaussian, and Heaviside).

Coarse-graining is performed using Pysammos and compared against other established software packages: EDEM, Granulysed, Iota, and MercuryCG.

The definitions of the following quantities are given in Appendix C.

### S1.1 Mixture density

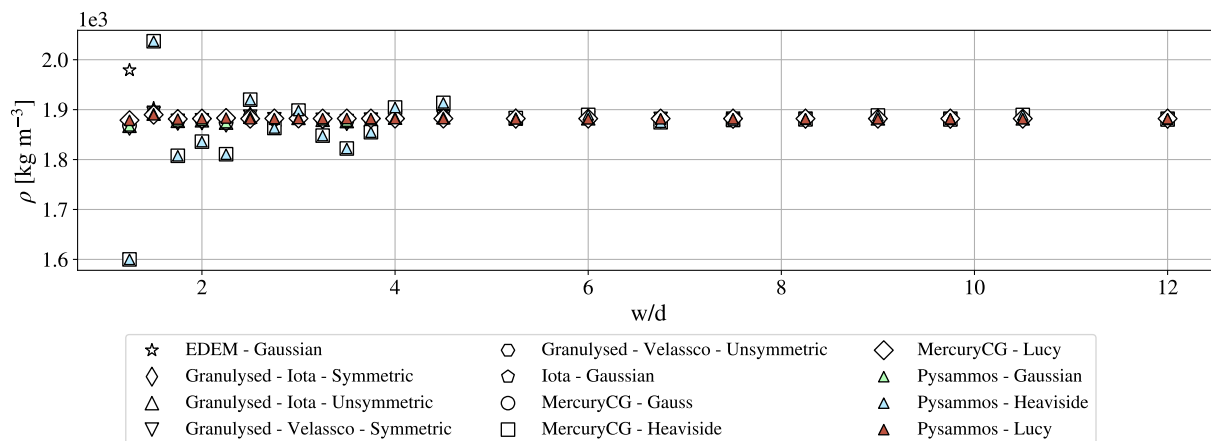


Figure S1: Benchmark of the mixture density of a monodisperse cubic lattice, of grains with a diameter of 0.02 m and density 2500 kg m<sup>-3</sup> (g), at a point  $P = 0, 0, 0.42725$  m. It was Coarse-grained with Pysammos (coloured triangles) and other Coarse-Graining software (void shapes): EDEM, Granulysed, Iota, and MercuryCG. Different smoothing functions (Gaussian, Heaviside and Lucy), are compared when available, for a range of smoothing half-width lengths multiples of the characteristic particle size (i.e.,  $w/d$ ).

## S1.2 Volume fraction

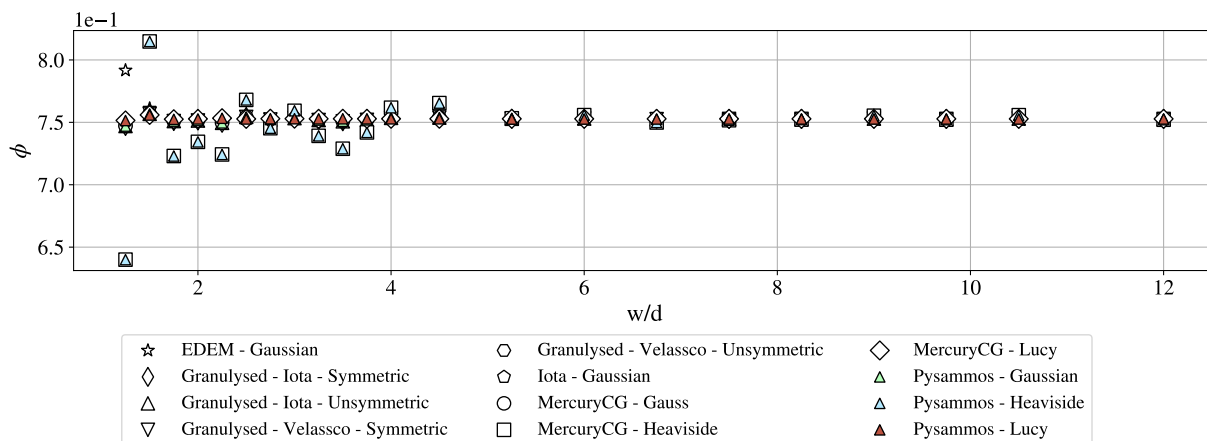


Figure S2: Benchmark of the volume fraction of a monodisperse cubic lattice, of grains with a diameter of 0.02 m and density  $2500 \text{ kg m}^{-3}$  (g), at a point  $P = 0, 0, 0.42725 \text{ m}$ . It was Coarse-grained with Pysammos (coloured triangles) and other Coarse-Graining software (void shapes): EDEM, Granulysed, Iota, and MercuryCG. Different smoothing functions (Gaussian, Heaviside and Lucy), are compared when available, for a range of smoothing half-width lengths multiples of the characteristic particle size (i.e.,  $w/d$ ).

## S1.3 Velocity

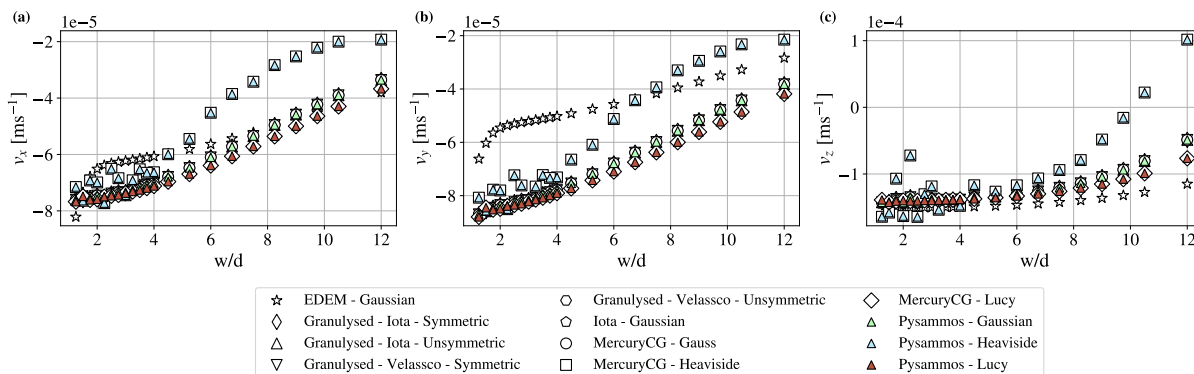


Figure S3: Benchmark of the velocity of a monodisperse cubic lattice, of grains with a diameter of 0.02 m and density  $2500 \text{ kg m}^{-3}$  (g), at a point  $P = 0, 0, 0.42725 \text{ m}$ . It was Coarse-grained with Pysammos (coloured triangles) and other Coarse-Graining software (void shapes): EDEM, Granulysed, Iota, and MercuryCG. Different smoothing functions (Gaussian, Heaviside and Lucy), are compared when available, for a range of smoothing half-width lengths multiples of the characteristic particle size (i.e.,  $w/d$ ). Panels (a)–(c) show the velocity components in the  $x$ ,  $y$ , and  $z$  directions.

## S1.4 Momentum density

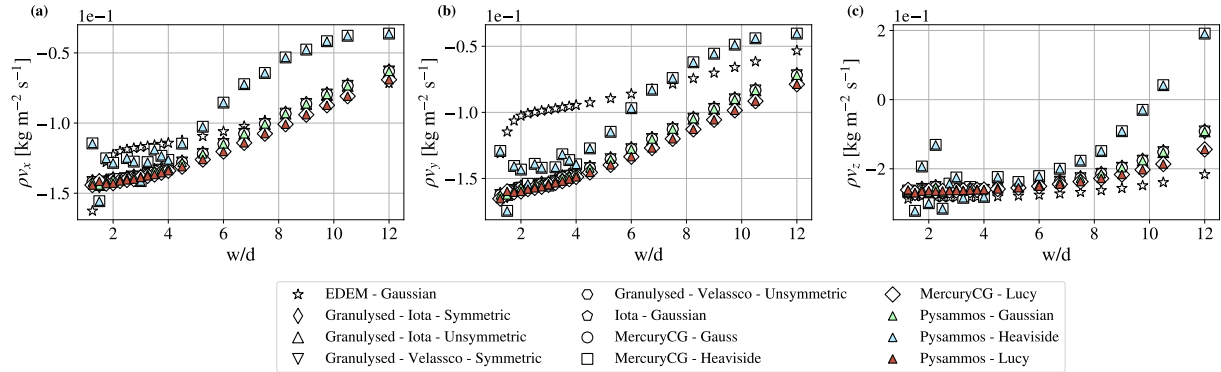


Figure S4: Benchmark of the momentum density of a monodisperse cubic lattice, of grains with a diameter of  $0.02\text{ m}$  and density  $2500\text{ kg m}^{-3}$  (g), at a point  $P = 0, 0, 0.42725\text{ m}$ . It was Coarse-grained with Pysammos (coloured triangles) and other Coarse-Graining software (void shapes): EDEM, Granulysed, Iota, and MercuryCG. Different smoothing functions (Gaussian, Heaviside and Lucy), are compared when available, for a range of smoothing half-width lengths multiples of the characteristic particle size (i.e.,  $w/d$ ). Panels (a)–(c) show the momentum density components in the  $x$ ,  $y$ , and  $z$  directions.

## S1.5 Contact stress

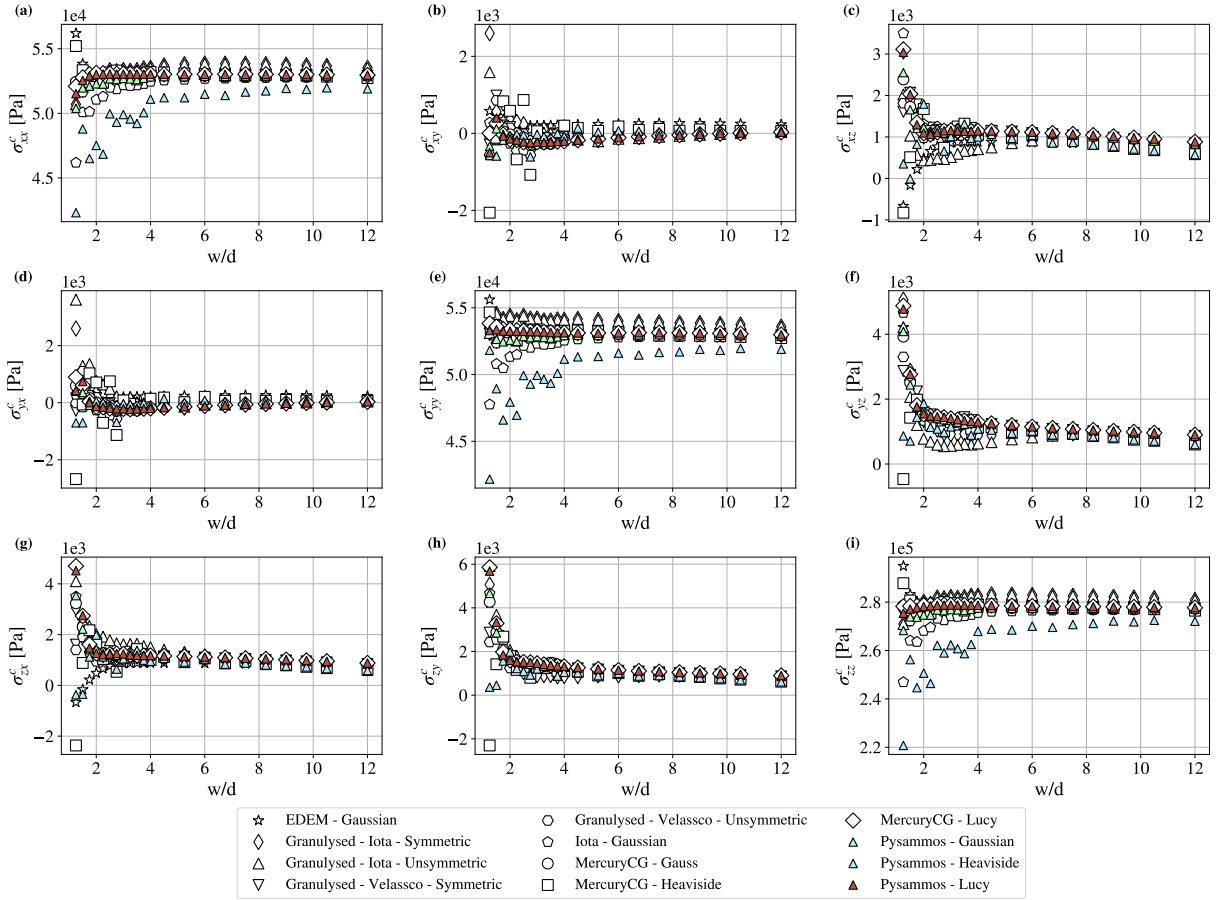


Figure S5: Benchmark of the contact stress of a monodisperse cubic lattice, of grains with a diameter of 0.02 m and density  $2500 \text{ kg m}^{-3}$  (g), at a point  $P = 0, 0, 0.42725 \text{ m}$ . It was Coarse-grained with Pysammos (coloured triangles) and other Coarse-Graining software (void shapes): EDEM, Granulysed, Iota, and MercuryCG. Different smoothing functions (Gaussian, Heaviside and Lucy), are compared when available, for a range of smoothing half-width lengths multiples of the characteristic particle size (i.e.,  $w/d$ ). Panels (a)–(i) show the components of  $T_{ij}$ , arranged in matrix form, with rows corresponding to  $i = x, y, z$  and columns to  $j = x, y, z$ .

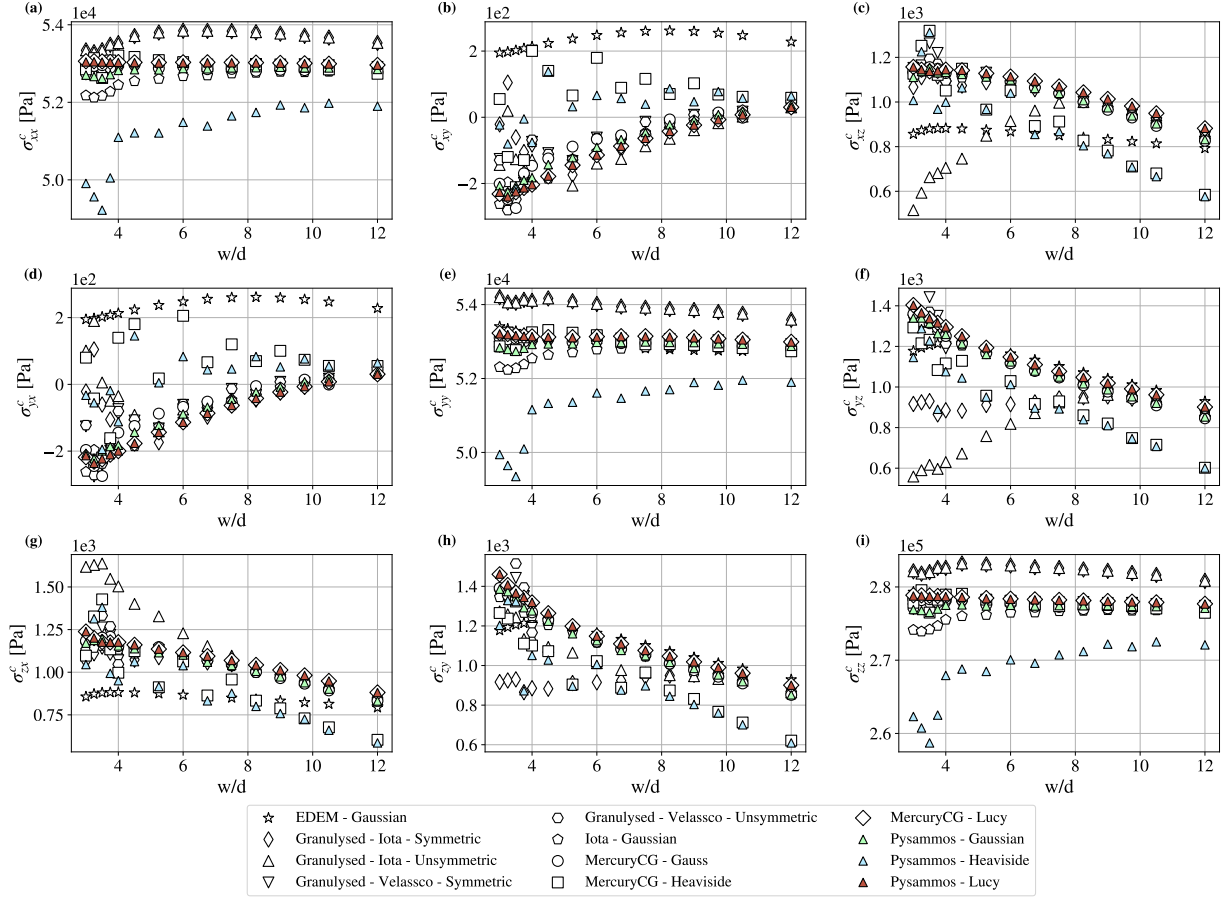


Figure S6: Benchmark of the contact stress of a monodisperse cubic lattice, of grains with a diameter of  $0.02 \text{ m}$  and density  $2500 \text{ kg m}^{-3}$  (g), at a point  $P = 0, 0, 0.42725 \text{ m}$ . It was Coarse-grained with Pysammos (coloured triangles) and other Coarse-Graining software (void shapes): EDEM, Granulysed, Iota, and MercuryCG. Different smoothing functions (Gaussian, Heaviside and Lucy), are compared when available, for a range of smoothing half-width lengths multiples of the characteristic particle size (i.e.,  $w/d$ ) above 2.5. Panels (a)–(i) show the components of  $T_{ij}$ , arranged in matrix form, with rows corresponding to  $i = x, y, z$  and columns to  $j = x, y, z$ .

## S1.6 Kinetic stress

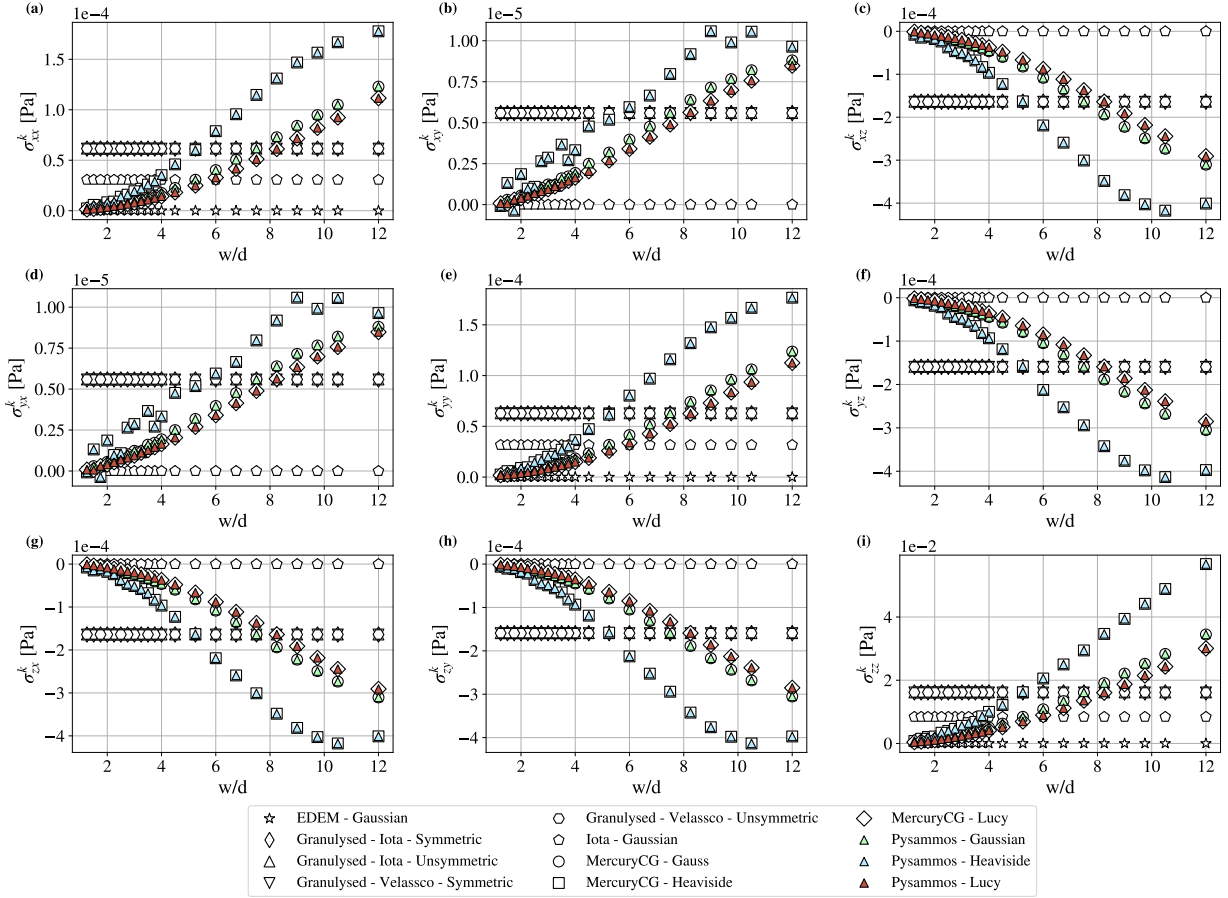


Figure S7: Benchmark of the kinetic stress of a monodisperse cubic lattice of grains with a diameter of  $0.02 \text{ m}$  and density  $2500 \text{ kg m}^{-3}$  (g), at a point  $P = 0, 0, 0.42725 \text{ m}$ . It was Coarse-grained with Pysammos (coloured triangles) and other Coarse-Graining software (void shapes): EDEM, Granulysed, Iota, and MercuryCG. Different smoothing functions (Gaussian, Heaviside and Lucy), are compared when available, for a range of smoothing half-width lengths multiples of the characteristic particle size (i.e.,  $w/d$ ). Panels (a)–(i) show the components of  $T_{ij}$ , arranged in matrix form, with rows corresponding to  $i = x, y, z$  and columns to  $j = x, y, z$ .