Supplementary Materials for:

Using ocean surface paleo-density to evaluate PMIP3 and PMIP4 Last Glacial Maximum climate simulations.

Barathieu Héloïse[1], Caley Thibaut[1], Kageyama Masa[2], Swingedouw Didier[1], Braconnot Pascale[2]

[1]Univ. Bordeaux, CNRS, Bordeaux INP, EPOC, UMR 5805, 33600 Pessac, France

[2]Laboratoire des Sciences du Climat et de l'Environnement – Institut Pierre-Simon Laplace (LSCE-IPSL), UMR CEA-CNRS-UVSQ, Université Paris-Saclay, Gif-sur-Yvette, France.

Correspondence to: Héloïse Barathieu (*heloise.barathieu@u-bordeaux.fr*)

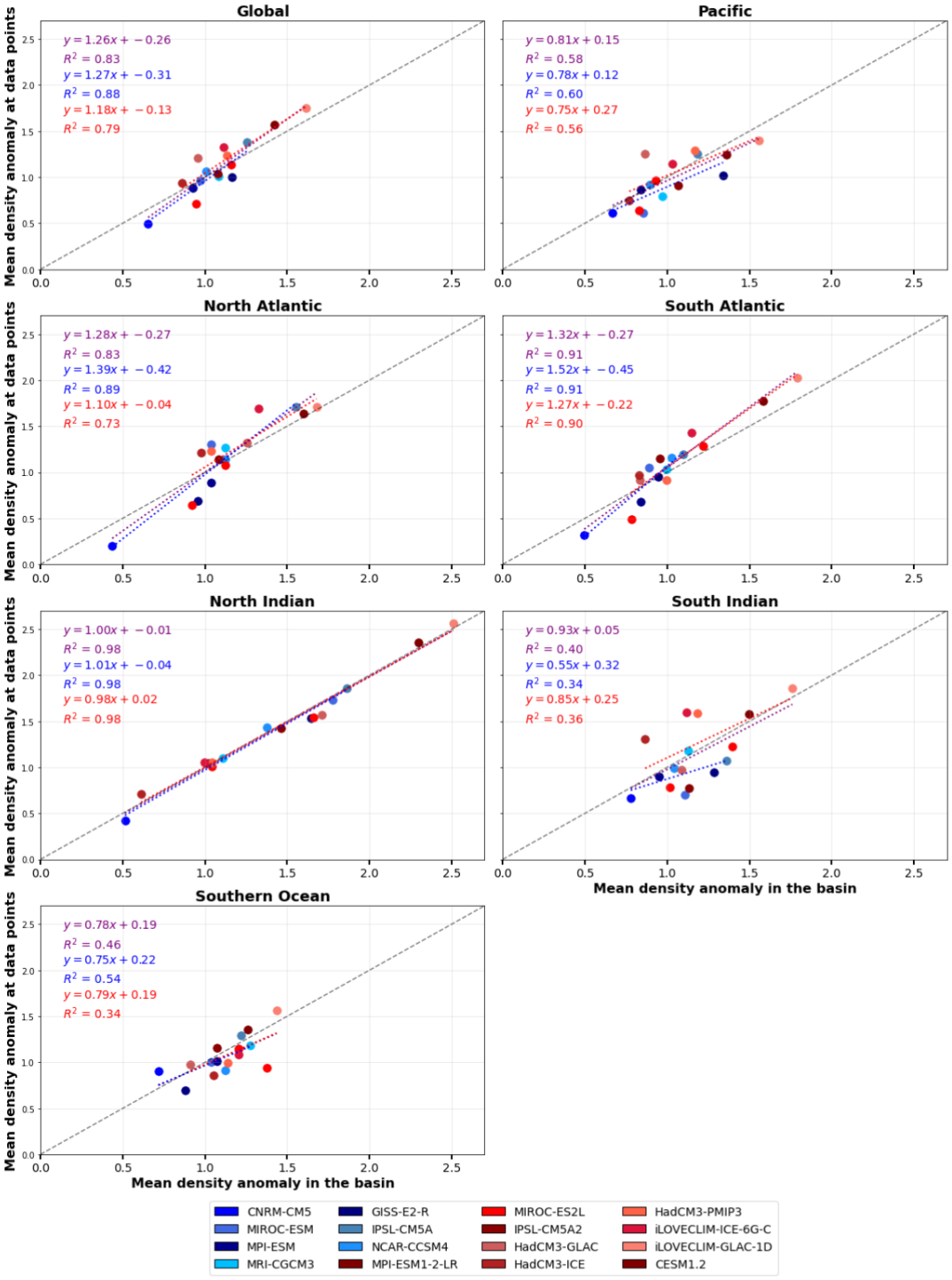


Figure S1: Pseudo-proxy test for ocean surface density. Comparison between the mean density anomalies (kg/m^3) averaged across all model grid points (x-axis) and the mean density anomalies (kg/m^3) averaged over proxy reconstruction sites (y-axis). PMIP3 simulations are shown in blue, and PMIP4 simulations in red. The purple regression line and R^2 represent the fit across all model simulations combined (PMIP3 + PMIP4). The "Global" panel includes all proxy locations from the individual basins shown.

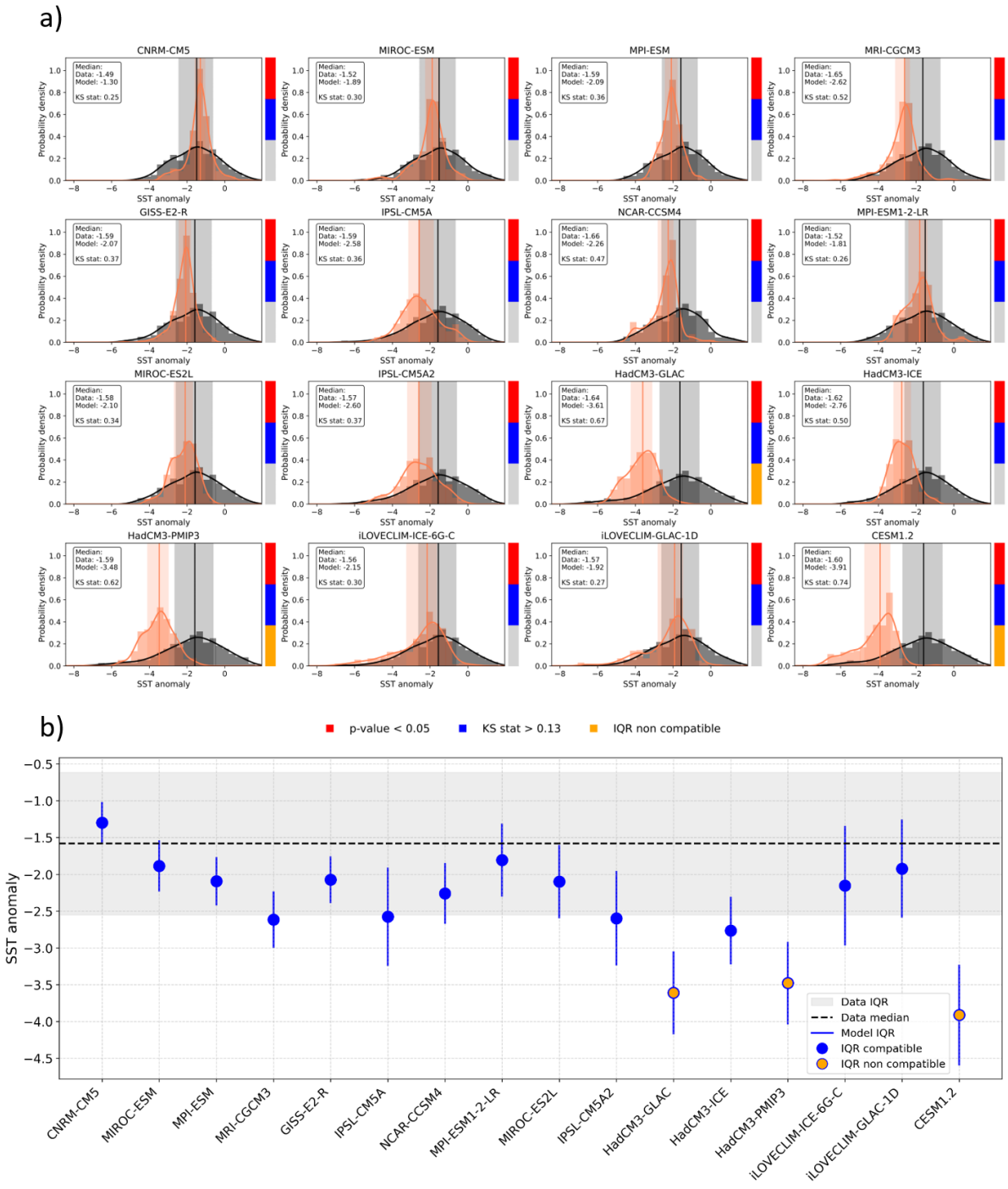


Figure S2: **(a)** Distribution histograms of SST anomalies (LGM–PI, °C) for each ocean basin. SST reconstructions from the MARGO database (MARGO project, 2009) are shown in black, and model simulations in orange. Kernel Density Estimates (KDEs) illustrate the central tendency and shape of the distributions. Vertical lines indicate medians, and shaded envelopes represent interquartile ranges (IQRs), capturing spread independent of outliers. Histograms show frequency and complement the KDEs. Extreme values were excluded via IQR filtering. Uncertainties from reconstructions and model spread are not shown, as the focus is on comparing distributional shapes. Colored indicators at the right of each panel show whether any of the following criteria are not met: p-value < 0.05, KS statistic ≥ 0.13, and absolute median difference < 0.2. A colored flag indicates a failed criterion. **(b)** Comparison of observed and simulated SST anomaly statistics at the global scale. The shaded grey band indicates the interquartile range (IQR) of the reconstructions data (from MARGO project, 2009), and the dashed black line marks the median of the reconstructions. For each model simulation, the blue vertical bars represent the modelled IQR, while the circular markers denote the model median. Blue markers indicate model simulations with IQR values consistent with reconstructions (IQR compatible), whereas orange markers highlight models with larger deviations (IQR non compatible).

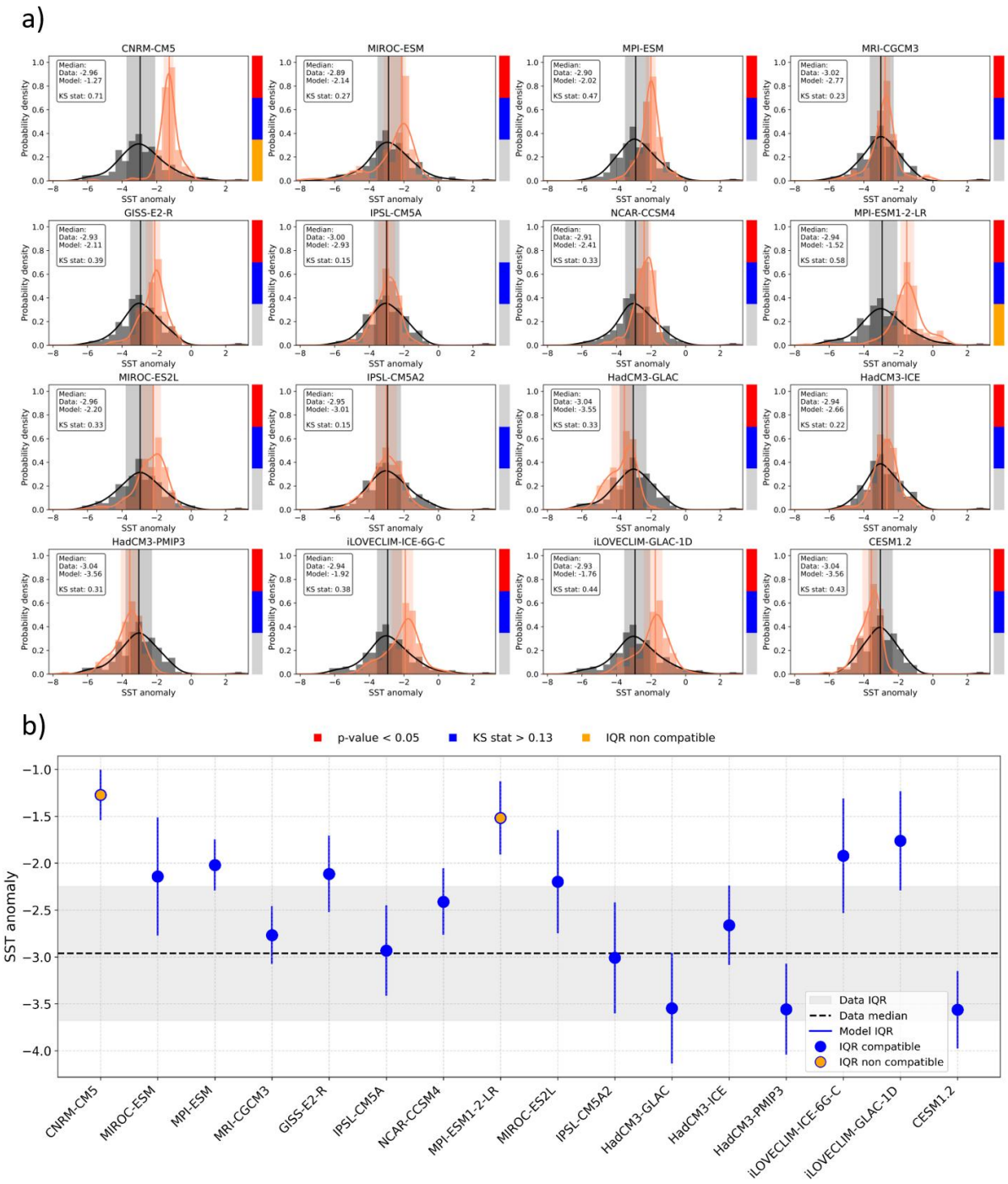
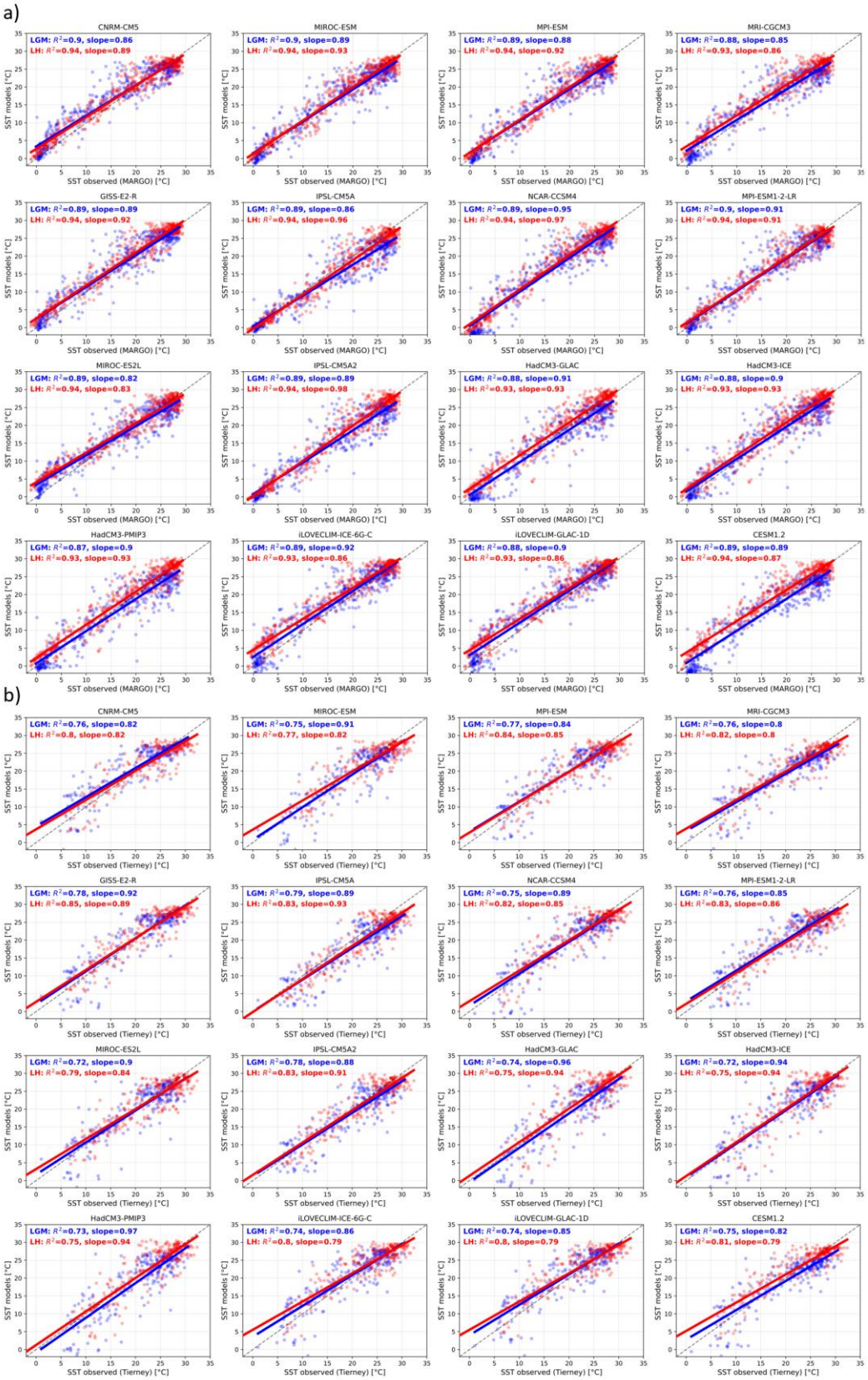
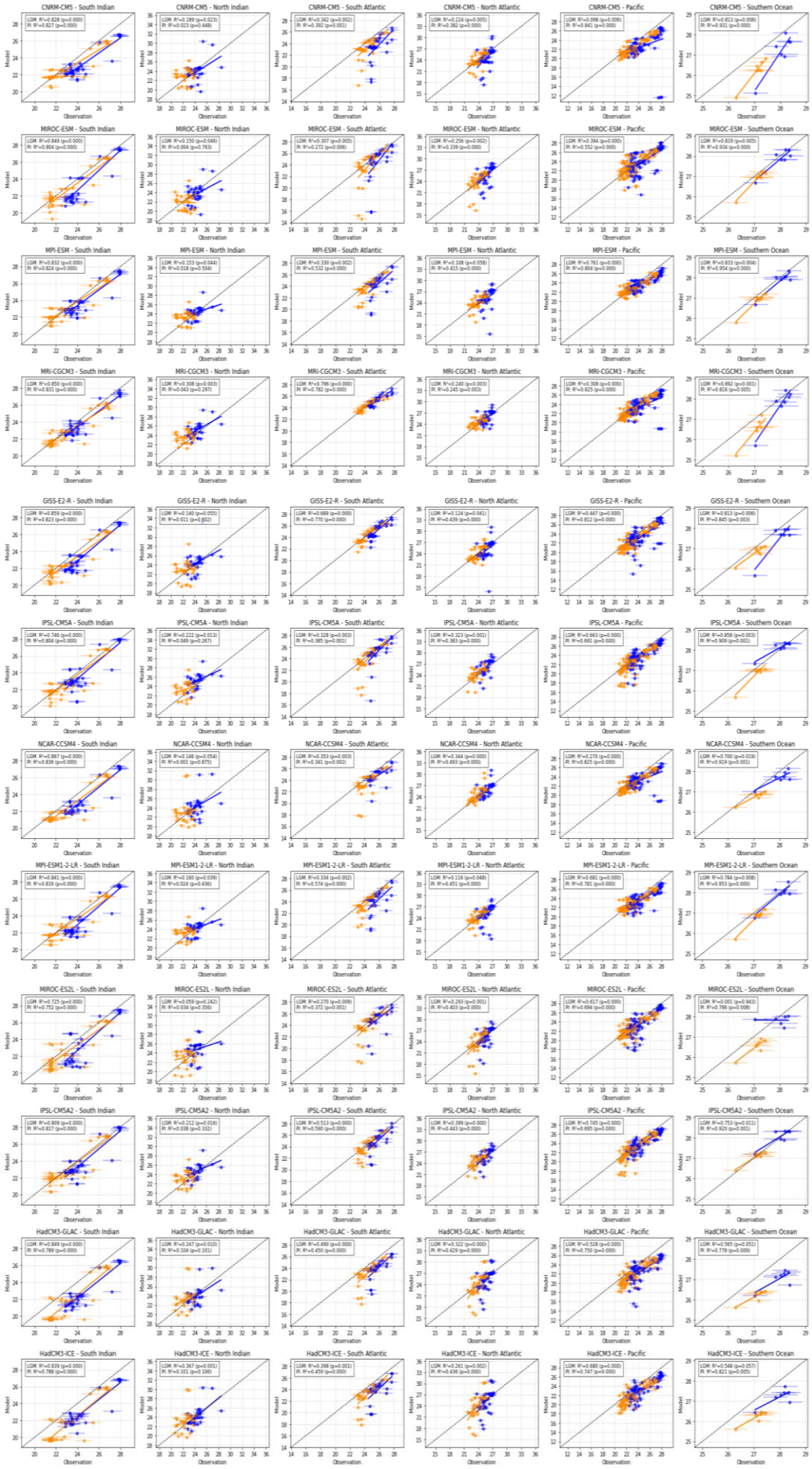


Figure S3: **(a)** Distribution histograms of SST anomalies (LGM–PI, °C) for each ocean basin. SST reconstructions from the Tierney database (Tierney et al. 2020) are shown in black, and model simulations in orange. Kernel Density Estimates (KDEs) illustrate the central tendency and shape of the distributions. Vertical lines indicate medians, and shaded envelopes represent interquartile ranges (IQRs), capturing spread independent of outliers. Histograms show frequency and complement the KDEs. Extreme values were excluded via IQR filtering. Uncertainties from reconstructions and model spread are not shown, as the focus is on comparing distributional shapes. Colored indicators at the right of each panel show whether any of the following criteria are not met: p-value < 0.05, KS statistic ≥ 0.13, and absolute median difference < 0.2. A colored flag indicates a failed criterion. **(b)** Comparison of observed and simulated SST anomaly statistics at the global scale. The shaded grey band indicates the interquartile range (IQR) of the reconstructions data (from Tierney et al. 2020), and the dashed black line marks the median of the reconstructions. For each model simulation, the blue vertical bars represent the modelled IQR, while the circular markers denote the model median. Blue markers indicate model simulations with IQR values consistent with reconstructions (IQR compatible), whereas orange markers highlight models with larger deviations (IQR non compatible).



<u>Figure S4:</u> Linear regressions between SST from proxy-based reconstructions (x-axis, °C) and model simulations (y-axis, °C), aggregated at the global scale (across all selected basins). Results are shown for the LGM period (blue) and the piControl period (red). The slope and R^2 values correspond to standard linear regressions, without accounting for uncertainties on the x-axis (the Monte Carlo method was not applied here). **(a)** Proxy-data reconstructions from MARGO project (2009). **(b)** Proxy-data reconstructions from Tierney et al. (2020).


CNRM-CM5 - South Indian
CNRM-CM5 - North Indian
CNRM-CM5 - South Atlantic
CNRM-CM5 - North Atlantic
CNRM-CM5 - Pacific
CNRM-CM5 - Southern Ocean
MIROC-ESM - South Indian
MIROC-ESM - North Indian
MIROC-ESM - South Atlantic
MIROC-ESM - North Atlantic
MIROC-ESM - Pacific
MIROC-ESM - Southern Ocean
MPI-ESM - South Indian
MPI-ESM - North Indian
MPI-ESM - South Atlantic
MPI-ESM - North Atlantic
MPI-ESM - Pacific
MPI-ESM - Southern Ocean
MRI-CGCM3 - South Indian
MRI-CGCM3 - North Indian
MRI-CGCM3 - South Atlantic
MRI-CGCM3 - North Atlantic
MRI-CGCM3 - Pacific
MRI-CGCM3 - Southern Ocean
GISS-E2-R - South Indian
GISS-E2-R - North Indian
GISS-E2-R - South Atlantic
GISS-E2-R - North Atlantic
GISS-E2-R - Pacific
GISS-E2-R - Southern Ocean
IPSL-CM5A - South Indian
IPSL-CM5A - North Indian
IPSL-CM5A - South Atlantic
IPSL-CM5A - North Atlantic
IPSL-CM5A - Pacific
IPSL-CM5A - Southern Ocean
NCAR-CCSM4 - South Indian
NCAR-CCSM4 - North Indian
NCAR-CCSM4 - South Atlantic
NCAR-CCSM4 - North Atlantic
NCAR-CCSM4 - Pacific
NCAR-CCSM4 - Southern Ocean
MPI-ESM1-2-LR - South Indian
MPI-ESM1-2-LR - North Indian
MPI-ESM1-2-LR - South Atlantic
MPI-ESM1-2-LR - North Atlantic
MPI-ESM1-2-LR - Pacific
MPI-ESM1-2-LR - Southern Ocean
MIROC-ES2L - South Indian
MIROC-ES2L - North Indian
MIROC-ES2L - South Atlantic
MIROC-ES2L - North Atlantic
MIROC-ES2L - Pacific
MIROC-ES2L - Southern Ocean
IPSL-CM5A2 - South Indian
IPSL-CM5A2 - North Indian
IPSL-CM5A2 - South Atlantic
IPSL-CM5A2 - North Atlantic
IPSL-CM5A2 - Pacific
IPSL-CM5A2 - Southern Ocean
HadCM3-GLAC - South Indian
HadCM3-GLAC - North Indian
HadCM3-GLAC - South Atlantic
HadCM3-GLAC - North Atlantic
HadCM3-GLAC - Pacific
HadCM3-GLAC - Southern Ocean
HadCM3-ICE - South Indian
HadCM3-ICE - North Indian
HadCM3-ICE - South Atlantic
HadCM3-ICE - North Atlantic
HadCM3-ICE - Pacific
HadCM3-ICE - Southern Ocean
Model
Observation

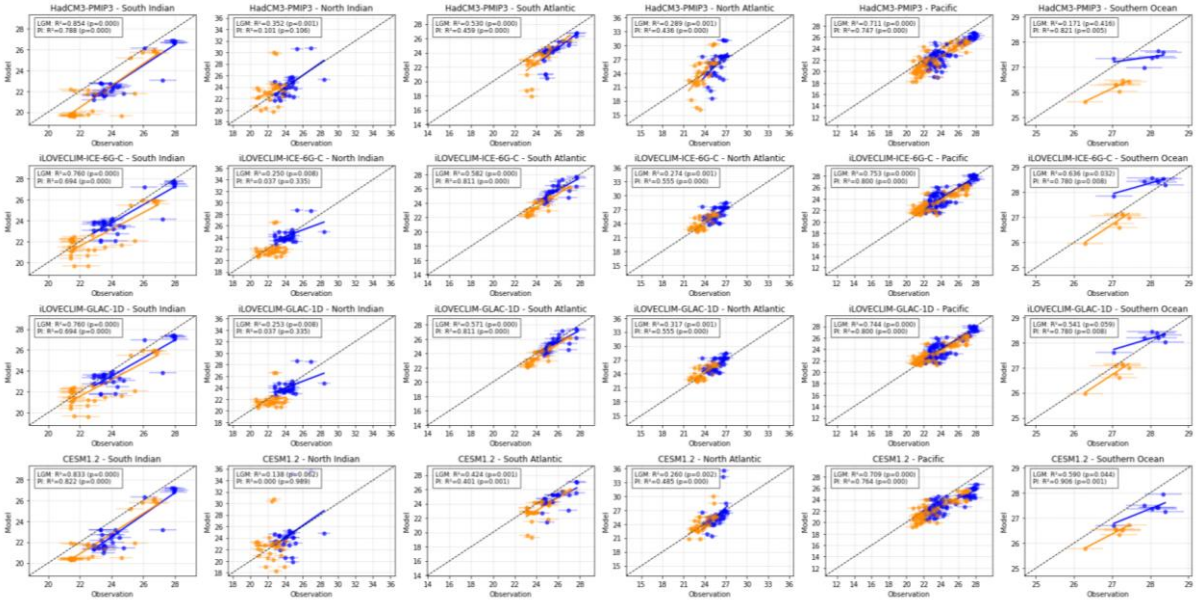


Figure S5: Linear regressions between surface absolute density (kg/m^3) from proxy-based reconstructions (x-axis) and model simulations (y-axis), for each basins and each model simulation. Results are shown for the LGM period (blue) and the piControl period (orange). Error bars on the x-axis represent the 95% confidence intervals of the reconstructed values. The R^2 values correspond to standard linear regressions, without accounting for uncertainties on the x-axis (the Monte Carlo method was not applied here).

Ocean: South Indian

Period	Model	N points	R^2 (Ref)	MC $R^2 \pm \sigma$	p-val R^2	Slope (Ref)	MC Slope $\pm \sigma$
LGM	CNRM-CM5	31	0.83	0.72 ± 0.06	< 0.05	0.89	0.78 ± 0.05
LGM	MIROC-ESM	28	0.85	0.74 ± 0.05	< 0.05	1.19	1.04 ± 0.07
LGM	MPI-ESM	31	0.83	0.71 ± 0.07	< 0.05	0.93	0.80 ± 0.07
LGM	MRI-CGCM3	31	0.85	0.73 ± 0.05	< 0.05	0.94	0.81 ± 0.05
LGM	GISS-E2-R	31	0.86	0.74 ± 0.06	< 0.05	1.08	0.93 ± 0.07
LGM	IPSL-CM5A	31	0.75	0.64 ± 0.06	< 0.05	1.07	0.93 ± 0.07
LGM	NCAR-CCSM4	29	0.87	0.76 ± 0.05	< 0.05	1.03	0.90 ± 0.06
LGM	MPI-ESM1-2-LR	31	0.84	0.73 ± 0.06	< 0.05	1.09	0.94 ± 0.07
LGM	MIROC-ES2L	29	0.72	0.62 ± 0.06	< 0.05	1.13	0.97 ± 0.08
LGM	IPSL-CM5A2	31	0.81	0.71 ± 0.05	< 0.05	1.07	0.93 ± 0.06
LGM	HadCM3-GLAC	30	0.85	0.74 ± 0.06	< 0.05	1.01	0.88 ± 0.05
LGM	HadCM3-ICE	30	0.84	0.73 ± 0.06	< 0.05	1.00	0.87 ± 0.07
LGM	HadCM3-PMIP3	30	0.85	0.74 ± 0.06	< 0.05	0.99	0.86 ± 0.06
LGM	iLOVECLIM-ICE-6G-C	30	0.76	0.66 ± 0.06	< 0.05	0.87	0.76 ± 0.06
LGM	iLOVECLIM-GLAC-1D	30	0.76	0.65 ± 0.06	< 0.05	0.86	0.75 ± 0.06
LGM	CESM1.2	30	0.83	0.71 ± 0.06	< 0.05	1.06	0.91 ± 0.07
PI	CNRM-CM5	31	0.83	0.74 ± 0.05	< 0.05	0.76	0.67 ± 0.05
PI	MIROC-ESM	28	0.80	0.72 ± 0.05	< 0.05	0.95	0.86 ± 0.06
PI	MPI-ESM	31	0.82	0.73 ± 0.05	< 0.05	0.81	0.72 ± 0.04
PI	MRI-CGCM3	31	0.83	0.74 ± 0.06	< 0.05	0.80	0.71 ± 0.05
PI	GISS-E2-R	31	0.82	0.73 ± 0.04	< 0.05	0.92	0.82 ± 0.06
PI	IPSL-CM5A	31	0.80	0.71 ± 0.05	< 0.05	0.93	0.83 ± 0.06
PI	NCAR-CCSM4	30	0.84	0.75 ± 0.05	< 0.05	0.92	0.83 ± 0.05
PI	MPI-ESM1-2-LR	31	0.82	0.72 ± 0.05	< 0.05	0.86	0.77 ± 0.05
PI	MIROC-ES2L	31	0.75	0.67 ± 0.05	< 0.05	0.90	0.79 ± 0.05
PI	IPSL-CM5A2	31	0.82	0.73 ± 0.06	< 0.05	0.95	0.84 ± 0.05
PI	HadCM3-GLAC	30	0.79	0.71 ± 0.05	< 0.05	1.06	0.95 ± 0.07
PI	HadCM3-ICE	30	0.79	0.70 ± 0.06	< 0.05	1.04	0.92 ± 0.07
PI	HadCM3-PMIP3	30	0.79	0.71 ± 0.05	< 0.05	1.04	0.94 ± 0.06
PI	iLOVECLIM-ICE-6G-C	31	0.69	0.61 ± 0.06	< 0.05	0.80	0.70 ± 0.05
PI	iLOVECLIM-GLAC-1D	31	0.69	0.61 ± 0.06	< 0.05	0.80	0.72 ± 0.06
PI	CESM1.2	30	0.82	0.73 ± 0.05	< 0.05	0.97	0.86 ± 0.06

Ocean: South Atlantic

Period	Model	N points	R^2 (Ref)	MC $R^2 \pm \sigma$	p-val R^2	Slope (Ref)	MC Slope ± σ
LGM	CNRM-CM5	26	0.34	0.22 ± 0.09	< 0.05	1.32	0.84 ± 0.21
LGM	MIROC-ESM	24	0.31	0.20 ± 0.09	< 0.05	1.61	1.03 ± 0.25
LGM	MPI-ESM	26	0.33	0.21 ± 0.09	< 0.05	1.13	0.70 ± 0.18
LGM	MRI-CGCM3	26	0.80	0.52 ± 0.12	< 0.05	0.83	0.53 ± 0.08
LGM	GISS-E2-R	26	0.69	0.43 ± 0.11	< 0.05	1.10	0.69 ± 0.10
LGM	IPSL-CM5A	25	0.33	0.22 ± 0.10	< 0.05	1.29	0.84 ± 0.19
LGM	NCAR-CCSM4	23	0.35	0.24 ± 0.09	< 0.05	0.82	0.55 ± 0.13
LGM	MPI-ESM1-2-LR	26	0.33	0.21 ± 0.09	< 0.05	1.20	0.76 ± 0.20
LGM	MIROC-ES2L	24	0.27	0.17 ± 0.09	< 0.05	1.00	0.61 ± 0.18
LGM	IPSL-CM5A2	25	0.51	0.36 ± 0.11	< 0.05	1.21	0.82 ± 0.15
LGM	HadCM3-GLAC	25	0.49	0.32 ± 0.12	< 0.05	1.21	0.77 ± 0.15
LGM	HadCM3-ICE	25	0.40	0.27 ± 0.09	< 0.05	1.18	0.78 ± 0.16
LGM	HadCM3-PMIP3	25	0.53	0.36 ± 0.10	< 0.05	1.15	0.76 ± 0.14
LGM	iLOVECLIM-ICE-6G-C	26	0.58	0.38 ± 0.12	< 0.05	0.84	0.54 ± 0.09
LGM	iLOVECLIM-GLAC-1D	26	0.57	0.38 ± 0.12	< 0.05	0.82	0.54 ± 0.10
LGM	CESM1.2	24	0.42	0.28 ± 0.11	< 0.05	0.77	0.50 ± 0.12
PI	CNRM-CM5	26	0.39	0.27 ± 0.10	< 0.05	1.13	0.76 ± 0.17
PI	MIROC-ESM	26	0.27	0.20 ± 0.09	< 0.05	1.43	0.99 ± 0.26
PI	MPI-ESM	26	0.53	0.36 ± 0.11	< 0.05	1.09	0.73 ± 0.13
PI	MRI-CGCM3	26	0.78	0.52 ± 0.10	< 0.05	0.71	0.49 ± 0.07
PI	GISS-E2-R	26	0.77	0.50 ± 0.11	< 0.05	1.04	0.69 ± 0.10
PI	IPSL-CM5A	26	0.38	0.28 ± 0.11	< 0.05	1.27	0.87 ± 0.21
PI	NCAR-CCSM4	25	0.34	0.24 ± 0.10	< 0.05	1.07	0.72 ± 0.15
PI	MPI-ESM1-2-LR	26	0.57	0.40 ± 0.11	< 0.05	1.17	0.78 ± 0.13
PI	MIROC-ES2L	26	0.37	0.25 ± 0.10	< 0.05	1.29	0.87 ± 0.20
PI	IPSL-CM5A2	26	0.59	0.40 ± 0.11	< 0.05	1.14	0.77 ± 0.13
PI	HadCM3-GLAC	25	0.45	0.30 ± 0.09	< 0.05	1.23	0.83 ± 0.16
PI	HadCM3-ICE	25	0.46	0.33 ± 0.09	< 0.05	1.22	0.83 ± 0.14
PI	HadCM3-PMIP3	25	0.46	0.31 ± 0.09	< 0.05	1.22	0.81 ± 0.16
PI	iLOVECLIM-ICE-6G-C	26	0.81	0.55 ± 0.11	< 0.05	0.96	0.66 ± 0.09
PI	iLOVECLIM-GLAC-1D	26	0.81	0.53 ± 0.11	< 0.05	0.96	0.65 ± 0.08
PI	CESM1.2	25	0.40	0.29 ± 0.10	< 0.05	0.90	0.63 ± 0.13

Ocean: North Atlantic

Period	Model	N points	R^2 (Ref)	MC $R^2 \pm \sigma$	p-val R^2	Slope (Ref)	MC Slope ± σ
LGM	CNRM-CM5	34	0.22	0.14 ± 0.07	< 0.05	1.31	0.80 ± 0.23
LGM	MIROC-ESM	34	0.26	0.16 ± 0.08	< 0.05	1.24	0.77 ± 0.20
LGM	MPI-ESM	34	0.11	0.08 ± 0.06	0.06	0.88	0.56 ± 0.26
LGM	MRI-CGCM3	34	0.24	0.15 ± 0.07	< 0.05	0.81	0.50 ± 0.13
LGM	GISS-E2-R	34	0.12	0.09 ± 0.06	< 0.05	1.00	0.64 ± 0.26
LGM	IPSL-CM5A	33	0.32	0.22 ± 0.08	< 0.05	1.18	0.76 ± 0.16
LGM	NCAR-CCSM4	34	0.34	0.22 ± 0.08	< 0.05	1.05	0.65 ± 0.13
LGM	MPI-ESM1-2-LR	34	0.12	0.07 ± 0.05	< 0.05	0.86	0.51 ± 0.22
LGM	MIROC-ES2L	34	0.29	0.19 ± 0.08	< 0.05	1.39	0.88 ± 0.21
LGM	IPSL-CM5A2	33	0.40	0.25 ± 0.09	< 0.05	1.28	0.76 ± 0.16
LGM	HadCM3-GLAC	34	0.32	0.20 ± 0.08	< 0.05	1.66	1.01 ± 0.23
LGM	HadCM3-ICE	34	0.26	0.15 ± 0.06	< 0.05	1.54	0.91 ± 0.23
LGM	HadCM3-PMIP3	34	0.29	0.19 ± 0.07	< 0.05	1.58	1.01 ± 0.21
LGM	iLOVECLIM-ICE-6G-C	34	0.27	0.19 ± 0.08	< 0.05	0.78	0.50 ± 0.13
LGM	iLOVECLIM-GLAC-1D	34	0.32	0.20 ± 0.09	< 0.05	0.82	0.50 ± 0.13
LGM	CESM1.2	34	0.26	0.16 ± 0.08	< 0.05	1.41	0.86 ± 0.23
PI	CNRM-CM5	34	0.38	0.28 ± 0.09	< 0.05	1.01	0.72 ± 0.13
PI	MIROC-ESM	34	0.34	0.25 ± 0.08	< 0.05	1.27	0.90 ± 0.16
PI	MPI-ESM	34	0.41	0.31 ± 0.09	< 0.05	0.98	0.71 ± 0.12
PI	MRI-CGCM3	34	0.24	0.17 ± 0.07	< 0.05	0.46	0.32 ± 0.07
PI	GISS-E2-R	34	0.44	0.31 ± 0.08	< 0.05	1.06	0.74 ± 0.12
PI	IPSL-CM5A	34	0.38	0.27 ± 0.07	< 0.05	0.93	0.65 ± 0.09
PI	NCAR-CCSM4	34	0.49	0.36 ± 0.09	< 0.05	1.13	0.82 ± 0.13
PI	MPI-ESM1-2-LR	34	0.45	0.32 ± 0.08	< 0.05	0.96	0.68 ± 0.11
PI	MIROC-ES2L	34	0.40	0.28 ± 0.08	< 0.05	1.53	1.07 ± 0.18
PI	IPSL-CM5A2	34	0.44	0.30 ± 0.08	< 0.05	0.97	0.66 ± 0.10
PI	HadCM3-GLAC	34	0.43	0.30 ± 0.09	< 0.05	1.84	1.28 ± 0.20
PI	HadCM3-ICE	34	0.44	0.31 ± 0.09	< 0.05	1.85	1.30 ± 0.22
PI	HadCM3-PMIP3	34	0.44	0.32 ± 0.09	< 0.05	1.85	1.32 ± 0.19
PI	iLOVECLIM-ICE-6G-C	34	0.56	0.41 ± 0.09	< 0.05	0.90	0.65 ± 0.08
PI	iLOVECLIM-GLAC-1D	34	0.56	0.40 ± 0.09	< 0.05	0.90	0.63 ± 0.09
PI	CESM1.2	34	0.48	0.35 ± 0.09	< 0.05	1.10	0.79 ± 0.13

Ocean: Pacific

Period	Model	N points	R^2 (Ref)	MC $R^2 \pm \sigma$	p-val R^2	Slope (Ref)	MC Slope $\pm \sigma$
LGM	CNRM-CM5	75	0.10	0.09 ± 0.02	< 0.05	0.46	0.40 ± 0.05
LGM	MIROC-ESM	71	0.39	0.35 ± 0.04	< 0.05	0.81	0.71 ± 0.05
LGM	MPI-ESM	75	0.76	0.68 ± 0.04	< 0.05	0.67	0.60 ± 0.03
LGM	MRI-CGCM3	75	0.31	0.28 ± 0.03	< 0.05	0.56	0.50 ± 0.03
LGM	GISS-E2-R	75	0.45	0.39 ± 0.04	< 0.05	0.84	0.74 ± 0.05
LGM	IPSL-CM5A	75	0.66	0.59 ± 0.04	< 0.05	0.91	0.81 ± 0.04
LGM	NCAR-CCSM4	75	0.27	0.24 ± 0.03	< 0.05	0.49	0.43 ± 0.04
LGM	MPI-ESM1-2-LR	75	0.68	0.60 ± 0.04	< 0.05	0.77	0.69 ± 0.04
LGM	MIROC-ES2L	75	0.62	0.54 ± 0.04	< 0.05	1.01	0.90 ± 0.04
LGM	IPSL-CM5A2	75	0.74	0.66 ± 0.04	< 0.05	0.80	0.71 ± 0.03
LGM	HadCM3-GLAC	74	0.53	0.47 ± 0.04	< 0.05	0.86	0.76 ± 0.04
LGM	HadCM3-ICE	74	0.68	0.60 ± 0.04	< 0.05	0.85	0.76 ± 0.04
LGM	HadCM3-PMIP3	71	0.71	0.63 ± 0.04	< 0.05	0.91	0.81 ± 0.04
LGM	iLOVECLIM-ICE-6G-C	73	0.75	0.67 ± 0.03	< 0.05	0.91	0.81 ± 0.04
LGM	iLOVECLIM-GLAC-1D	73	0.74	0.67 ± 0.04	< 0.05	0.88	0.79 ± 0.04
LGM	CESM1.2	75	0.71	0.63 ± 0.04	< 0.05	0.69	0.62 ± 0.03
PI	CNRM-CM5	75	0.84	0.75 ± 0.03	< 0.05	0.82	0.73 ± 0.03
PI	MIROC-ESM	73	0.55	0.49 ± 0.04	< 0.05	0.80	0.72 ± 0.04
PI	MPI-ESM	75	0.80	0.72 ± 0.03	< 0.05	0.68	0.62 ± 0.02
PI	MRI-CGCM3	75	0.82	0.74 ± 0.03	< 0.05	0.84	0.76 ± 0.03
PI	GISS-E2-R	75	0.81	0.73 ± 0.03	< 0.05	0.90	0.80 ± 0.03
PI	IPSL-CM5A	75	0.69	0.63 ± 0.03	< 0.05	0.94	0.85 ± 0.04
PI	NCAR-CCSM4	75	0.82	0.74 ± 0.03	< 0.05	0.78	0.70 ± 0.02
PI	MPI-ESM1-2-LR	75	0.78	0.70 ± 0.03	< 0.05	0.71	0.63 ± 0.02
PI	MIROC-ES2L	75	0.69	0.62 ± 0.03	< 0.05	0.78	0.71 ± 0.03
PI	IPSL-CM5A2	75	0.69	0.62 ± 0.04	< 0.05	0.98	0.88 ± 0.04
PI	HadCM3-GLAC	75	0.75	0.67 ± 0.03	< 0.05	0.86	0.77 ± 0.03
PI	HadCM3-ICE	75	0.75	0.67 ± 0.04	< 0.05	0.86	0.78 ± 0.03
PI	HadCM3-PMIP3	75	0.75	0.68 ± 0.03	< 0.05	0.86	0.78 ± 0.03
PI	iLOVECLIM-ICE-6G-C	75	0.80	0.71 ± 0.03	< 0.05	0.71	0.63 ± 0.03
PI	iLOVECLIM-GLAC-1D	75	0.80	0.72 ± 0.03	< 0.05	0.71	0.64 ± 0.03
PI	CESM1.2	75	0.76	0.69 ± 0.03	< 0.05	0.76	0.69 ± 0.03

64

Ocean: Southern Ocean

Period	Model	N points	R^2 (Ref)	MC $R^2 \pm \sigma$	p-val R^2	Slope (Ref)	MC Slope ± σ
LGM	CNRM-CM5	7	0.81	0.42 ± 0.23	< 0.05	1.91	0.94 ± 0.40
LGM	MIROC-ESM	7	0.82	0.41 ± 0.25	< 0.05	1.09	0.51 ± 0.25
LGM	MPI-ESM	7	0.83	0.44 ± 0.25	< 0.05	1.07	0.51 ± 0.31
LGM	MRI-CGCM3	7	0.89	0.47 ± 0.25	< 0.05	1.88	0.93 ± 0.41
LGM	GISS-E2-R	7	0.81	0.40 ± 0.25	< 0.05	1.59	0.67 ± 0.39
LGM	IPSL-CM5A	7	0.86	0.46 ± 0.22	< 0.05	0.72	0.35 ± 0.15
LGM	NCAR-CCSM4	7	0.70	0.39 ± 0.24	< 0.05	0.64	0.31 ± 0.18
LGM	MPI-ESM1-2-LR	7	0.78	0.40 ± 0.22	< 0.05	1.06	0.52 ± 0.25
LGM	IPSL-CM5A2	7	0.75	0.42 ± 0.21	< 0.05	0.79	0.42 ± 0.18
LGM	iLOVECLIM-ICE-6G-C	7	0.64	0.34 ± 0.24	< 0.05	0.43	0.21 ± 0.11
LGM	CESM1.2	7	0.59	0.31 ± 0.22	< 0.05	0.61	0.29 ± 0.19
PI	CNRM-CM5	7	0.93	0.38 ± 0.25	< 0.05	1.63	0.60 ± 0.32
PI	MIROC-ESM	7	0.93	0.38 ± 0.23	< 0.05	1.33	0.52 ± 0.32
PI	MPI-ESM	7	0.95	0.42 ± 0.26	< 0.05	1.25	0.48 ± 0.24
PI	MRI-CGCM3	7	0.82	0.33 ± 0.24	< 0.05	1.52	0.54 ± 0.31
PI	GISS-E2-R	7	0.85	0.33 ± 0.24	< 0.05	0.91	0.31 ± 0.21
PI	IPSL-CM5A	7	0.91	0.41 ± 0.24	< 0.05	1.29	0.52 ± 0.24
PI	NCAR-CCSM4	7	0.92	0.39 ± 0.27	< 0.05	0.67	0.24 ± 0.15
PI	MPI-ESM1-2-LR	7	0.95	0.41 ± 0.24	< 0.05	1.28	0.51 ± 0.29
PI	MIROC-ES2L	7	0.79	0.32 ± 0.23	< 0.05	0.96	0.31 ± 0.23
PI	IPSL-CM5A2	7	0.92	0.38 ± 0.25	< 0.05	0.79	0.30 ± 0.20
PI	HadCM3-GLAC	7	0.78	0.36 ± 0.23	< 0.05	0.69	0.25 ± 0.17
PI	HadCM3-ICE	7	0.82	0.40 ± 0.24	< 0.05	0.72	0.29 ± 0.14
PI	HadCM3-PMIP3	7	0.82	0.34 ± 0.23	< 0.05	0.72	0.28 ± 0.14
PI	iLOVECLIM-ICE-6G-C	7	0.78	0.42 ± 0.26	< 0.05	1.00	0.41 ± 0.23
PI	iLOVECLIM-GLAC-1D	7	0.78	0.32 ± 0.24	< 0.05	1.00	0.32 ± 0.25
PI	CESM1.2	7	0.91	0.36 ± 0.27	< 0.05	0.79	0.30 ± 0.17

Ocean: North Indian

Period	Model	N points	R^2 (Ref)	MC $R^2 \pm \sigma$	p-val R^2	Slope (Ref)	MC Slope $\pm \sigma$
LGM	CNRM-CM5	27	0.19	0.13 ± 0.06	< 0.05	0.80	0.56 ± 0.16
LGM	MIROC-ESM	27	0.15	0.12 ± 0.06	< 0.05	0.67	0.49 ± 0.14
LGM	MPI-ESM	27	0.15	0.13 ± 0.07	< 0.05	0.42	0.33 ± 0.10
LGM	MRI-CGCM3	27	0.31	0.22 ± 0.08	< 0.05	0.72	0.52 ± 0.10
LGM	GISS-E2-R	27	0.14	0.12 ± 0.05	0.05	0.69	0.54 ± 0.13
LGM	IPSL-CM5A	27	0.22	0.17 ± 0.08	< 0.05	0.59	0.44 ± 0.12
LGM	NCAR-CCSM4	26	0.15	0.11 ± 0.06	0.05	0.74	0.53 ± 0.19
LGM	MPI-ESM1-2-LR	27	0.16	0.13 ± 0.07	< 0.05	0.46	0.35 ± 0.10
LGM	MIROC-ES2L	25	0.06	0.05 ± 0.04	0.24	0.43	0.31 ± 0.15
LGM	IPSL-CM5A2	27	0.21	0.18 ± 0.08	< 0.05	0.56	0.44 ± 0.10
LGM	HadCM3-GLAC	26	0.25	0.19 ± 0.08	< 0.05	0.82	0.61 ± 0.14
LGM	HadCM3-ICE	27	0.37	0.27 ± 0.08	< 0.05	1.05	0.76 ± 0.14
LGM	HadCM3-PMIP3	27	0.35	0.27 ± 0.08	< 0.05	1.07	0.81 ± 0.17
LGM	iLOVECLIM-ICE-6G-C	27	0.25	0.19 ± 0.08	< 0.05	0.56	0.42 ± 0.11
LGM	iLOVECLIM-GLAC-1D	27	0.25	0.20 ± 0.08	< 0.05	0.58	0.43 ± 0.10
LGM	CESM1.2	26	0.14	0.11 ± 0.06	0.06	1.06	0.77 ± 0.23
PI	HadCM3-GLAC	27	0.10	0.07 ± 0.06	0.10	0.76	0.46 ± 0.22
PI	HadCM3-ICE	27	0.10	0.08 ± 0.06	0.11	0.76	0.50 ± 0.23
PI	HadCM3-PMIP3	27	0.10	0.07 ± 0.05	0.11	0.76	0.50 ± 0.22

Table S1: Coefficients of determination (R^2) and regression slopes for each ocean basin and each model simulation, evaluated separately for the LGM and pre-industrial (PI) periods. These statistics correspond to the linear relationships between surface absolute density (kg/m³) from proxy-based reconstructions (x-axis) and model simulations (y-axis), as shown in the previous figure. Values are reported for both standard (ref) least-squares regression and uncertainty-aware estimates using Monte Carlo (MC) simulations (n = 10,000), which propagate uncertainties from the reconstructions. Standard errors for MC estimates reflect variability across iterations. Only results with significant relationships ($p \leq 0.05$) for slope (MC) and/or R^2 (MC) are shown. Shading indicates significance and performance: light green for p-values ≤ 0.05, for R^2 (Ref or MC) > 0.5, and for slope (Ref or MC) values between 0.8 and 1.2.