Review of *A computationally efficient TVD-FFSL hybrid tracer transport scheme on spherical centroidal voronoi tessellations in iAMAS (v2.6.3)*

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The authors describe a 3D transport scheme for the IAMAS model that employs a nominally 2nd-order TVD scheme for horizontal transport on the horizontal SCVT meshes and a PPM based FFSL scheme for vertical transport. The scheme is of potential interest to users of IAMAS and perhaps other models that employ icosahedral meshes or SCVT meshes. There are two main areas that need attention in the current manuscript to allow readers to evaluate the effectiveness of the authors' new scheme. The first area regards the description and properties of the scheme. The second concerns the performance of the horizontal TVD scheme and comparison of results from the new scheme with published results from other schemes.

Concerning the formulation and properties of the 3D scheme:

1. What is the integration procedure for the full 3D scheme that combines the horizontal TVD scheme and the vertical PPM-based FFSL scheme? For the horizontal TVD scheme, the authors present the integration in equation (11):

$$\begin{aligned} \rho\tilde{\psi}^{(1)} &= \rho\tilde{\psi}^{n} + \Delta t \mathrm{L}(\rho\tilde{\psi}^{n}) \\ \rho\tilde{\psi}^{n+1} &= \frac{1}{2}\rho\tilde{\psi}^{n} + \frac{1}{2}\rho\tilde{\psi}^{(1)} + \frac{1}{2}\Delta t \mathrm{L}(\rho\tilde{\psi}^{(1)}) \end{aligned} \tag{11}$$

If we denote the vertical scheme operator as P, the question is how is it combined with the scheme (11)? One possibility is

$$\begin{aligned} \rho\tilde{\psi}^{(1)} &= \rho\tilde{\psi}^{n} + \Delta t \mathrm{L}(\rho\tilde{\psi}^{n}) + \Delta t \mathrm{P}(\rho\tilde{\psi}^{n}), \\ \rho\tilde{\psi}^{n+1} &= \frac{1}{2}\rho\tilde{\psi}^{n} + \frac{1}{2}\rho\tilde{\psi}^{(1)} + \frac{1}{2}\Delta t \mathrm{L}(\rho\tilde{\psi}^{(1)}) + \frac{1}{2}\Delta t \mathrm{P}(\rho\tilde{\psi}^{n}). \end{aligned}$$

One could imagine other variants, for example where the FFSL update is left out of the first step and the full FFSL update is included in the final TVD step, i.e.

$$\begin{aligned} \rho\tilde{\psi}^{(1)} &= \rho\tilde{\psi}^{n} + \Delta t \mathrm{L}(\rho\tilde{\psi}^{n}), \\ \rho\tilde{\psi}^{n+1} &= \frac{1}{2}\rho\tilde{\psi}^{n} + \frac{1}{2}\rho\tilde{\psi}^{(1)} + \frac{1}{2}\Delta t \mathrm{L}(\rho\tilde{\psi}^{(1)}) + \Delta t \mathrm{P}(\rho\tilde{\psi}^{n}). \end{aligned}$$

The authors need to clearly state how the full 3D scheme is implemented.

2. Is the limiter applied on both steps in (11) for the TVD scheme?

3. The TVD schemes and the PPM-based FFSL scheme are derived and analyzed for one-dimensional transport, and usually they use some form of splitting to handle multi-dimensional transport. Furthermore, the TVD scheme is usually a single-step scheme -

the first step equation in (11). Presumably, the two-step time integration (11) renders the 2D scheme stable? Is the 2D scheme stable without the application of the limiter ($r \equiv 1$). Is the full 3D scheme stable without the limiters applied in the TVD scheme and the PPM scheme? Stability analyses are needed for (11) for the unlimited scheme (at least in 1D, ideally in 2D), and for the unlimited version of the full combined TVD-FFSL scheme, perhaps in 2D (x,z).

4. Is the horizontal TVD scheme guaranteed to be TVD using the time integration (11)? If the scheme is not spatially split then the 1D analysis isn't conclusive.

Concerning the results and scheme performance:

5. It is difficult to assess how well this TVD scheme performs in comparison to other published schemes. The authors use significantly smaller Courant numbers in their tests compared to results for other published scheme. For example, the existing IAMAS scheme comes from Skamarock and Gassman (2012). The phase errors for the authors' second-order scheme appear to be much less than those reported for the second order scheme considered in Skamarock and Gassman, however, the authors tests use much-lower Courant numbers and higher resolution than used to produce the results in Skamarock and Gassman. For example, compare the authors' figure 3 for the solid-body rotation test using the cosine bell with Skamarock and Gassman figure 3. Skamarock and Gassman use a nominally 240 km horizontal SCVT mesh for this test with a centerline Courant number of ~ 0.58 (the bell is located over the equator), whereas the authors place the bell at 30N and use a much smaller time step (assuming the 6Δ rule the authors note earlier in the manuscript?) on a 60 km mesh with the resulting Courant number being ~ 0.2. Are the lower Courant numbers and higher resolution used in the authors' tests the reason for the different error characteristics for their 2nd order scheme compared to Skamarock and Gassman, or does the different time integration used in the authors' new scheme result in significantly reduced phase errors (and now similar to the other higher-order published schemes). Test with higher Courant numbers and lower resolution are needed to answer this question and give readers the information needed to evaluate the scheme.

6. Why does the 2HIFCT scheme show overshoots (values greater than 1000) in Figure 5 for the slotted cylinder test problem? The results in Skamarock and Gassman (2012) do not show these overshoots (see Skamarock and Gassman figure 12) - the FCT limiter should eliminate all overshoots and undershoots. One possibility is that the discrete flow field is not divergence free (it is divergence free in the Skamarock and Gassman tests).

7. Why are the TVD results and the 2HIFCT results shown in figure 5 positive definite but not monotonic (they contain overshoots)? If a nonzero constant value were added to the state would undershoots appear?

8. The scheme convergence rates for the cosine bell test case are computed with the limiters on (figures 4 and 8). The less than 2nd-order convergence is caused by the limiters but may also be caused by the cosine bell being only only C1 continuous. The authors should report results with both limiters on and off for test cases using a continuous function such as the Gaussian hills test cases reported in Lauritzen et al (2014) - doi:10.5194/gmd-7-105-2014.