Supplementary Material

Classification-Constrained Retrieval of $PM_{2.5}$ Vertical Profiles from Fluorescence-Raman-Mie Polarization Lidar

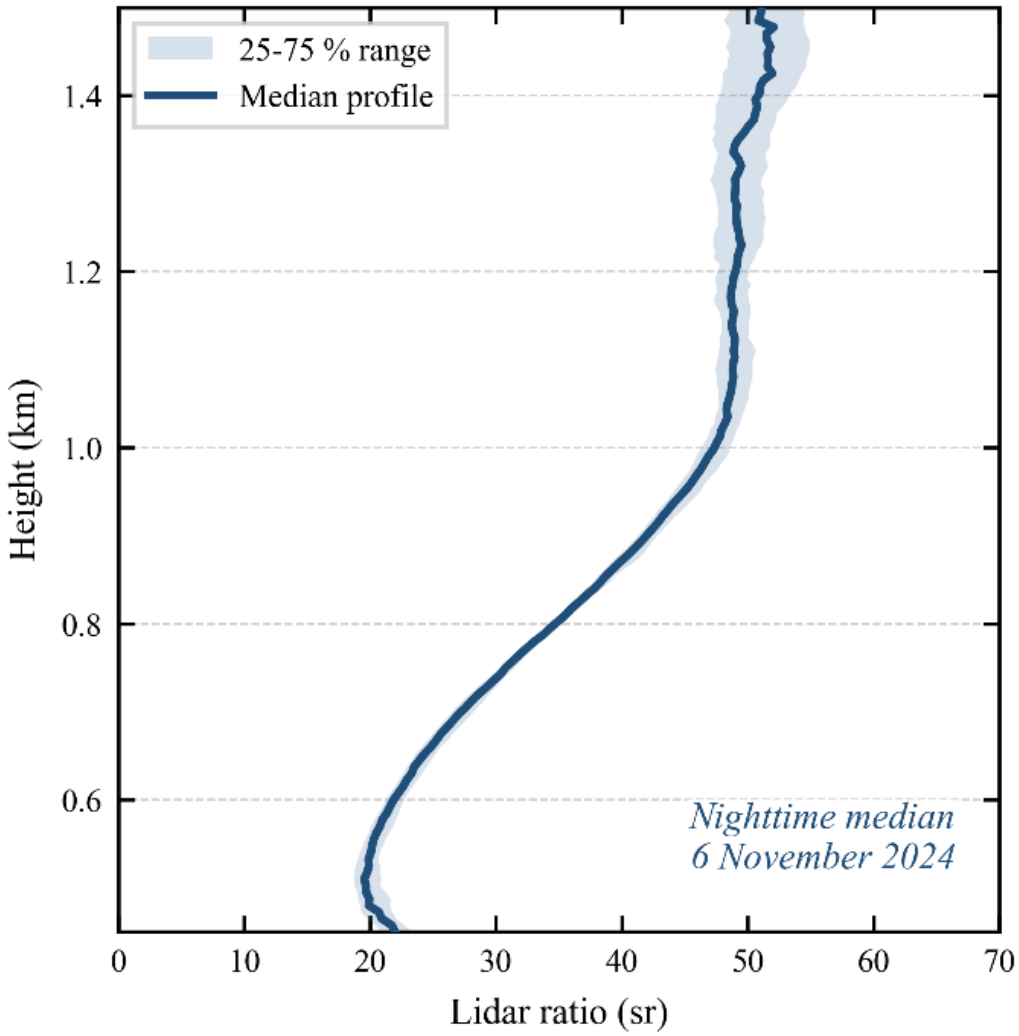

Figure S1. Nighttime median lidar-ratio (LR) profile on 6 November 2024. The shaded area represents the 25%-75% interquartile range. The continuous LR profile between 0.45 and 1.50 km provides supplementary information on vertical optical stratification during the observation period.

Table S1. Operational classification ranges, pure-component end-member parameters, and lidar-ratio (LR) values used in the uncertainty analysis.

Aerosol type	Operational PDR range	Operational G_f range ($\times10^{-4}$)	Pure-component depolarization potential, δ'	Pure-component G_f ($\times10^{-4}$)	Reference LR (sr)	LR sensitivity range (sr)
Anthropogenic pollution aerosol (APA)	0.11-0.16	0.60-1.10	0.1247	0.822	51.1	42.4-59.8
Desert dust (DD)	0.25-0.45	0.18-0.49	0.2238	0.403	43.0	31.0-60.7
Mineral dust (MD)	0.30-0.45	0.85-1.23	0.2616	0.969	48.5	39.3-57.7

Operational PDR and G_f ranges and the pure-component parameters were adopted from Jiang et al. (2024). LR reference values and sensitivity ranges were used only to assess the influence of aerosol-type-dependent LR on the component extinction allocation and retrieved $PM_{2.5}$.

Table S2. $PM_{2.5}$ uncertainty interval widths for the two mixed-aerosol case days.

Case date	Classification-related uncertainty interval width	LR-related uncertainty interval width	Model-parameter uncertainty interval width	Joint 95% interval width for the mean estimate	Prediction interval width including model residuals
2024-11-06	2.40	1.23	11.45	12.02	36.31
2024-11-07	2.76	1.13	10.54	12.79	36.85

All interval widths are in micrograms per cubic metre (μg m^{-3}). Classification-related uncertainty was obtained by Monte Carlo perturbation of the PDR and G_f end-member parameters; LR-related uncertainty reflects the sensitivity ranges in Table S1.

Table S3. Seasonal comparison of RMSE values for the global Model C and season-specific models using date-separated validation data.

Season	Global Model C RMSE	Seasonal model RMSE
Spring	18.19	18.96
Summer	21.49	16.38
Autumn	14.01	14.22
Winter	15.30	20.18

RMSE values are in μg m^{-3}. The global Model C is the single all-season model; each seasonal model was independently fitted using training dates separated from its validation dates.

Table S4. Independent validation performance of the linear relationship between near-surface extinction coefficient and ground-level PM_{10} for DD/MD-classified aerosol periods.

Training samples	Validation samples	Pearson r	RMSE (μg m^{-3})	MAE (μg m^{-3})	Bias (μg m^{-3})
48	309	0.310	104.61	88.98	69.11

The model was fitted using 48 hourly samples from selected DD/MD-dominated periods and evaluated using 309 date-separated hourly samples. The low correlation and large errors indicate that a stable quantitative PM_{10} retrieval relationship was not obtained under the available mineral-aerosol conditions.