Response to Referee #2
Manuscript: *From Fragmentation to Spanning: Connectivity Regime Transitions in Equal-Probability Geohazard Fields*
Manuscript ID: egusphere-2026-2387

We thank the referee for the constructive assessment and helpful comments. The manuscript has been revised accordingly, including additional methodological clarification, a grid-refinement analysis, an expanded physical interpretation of the coherence scale, a clearer discussion of template dependence and interpretive limitations, and improved figure readability. The referee's comments are reproduced below, followed by our point-by-point responses. Corresponding changes in the revised manuscript are highlighted for clarity.

RC2: 'Comment on egusphere-2026-2387', Anonymous Referee #2, 04 Oct 2026 reply
Dear Editor,

This paper develops a computational framework for investigating connectivity transitions in spatially correlated binary fields at fixed occupation probability, using observed liquefaction manifestations as an empirical spatial template. The study shows that equal-probability fields are organizationally non-unique and that spatial coherence controls the transition from fragmented to dominant-connected and spanning states. After reviewing and comparing the paper, and despite the paper's strengths-such as the clear explanation of the proposed method-it requires some revisions. The necessary corrections and questions to be addressed by the authors are outlined below.

1- The description of the observational liquefaction dataset used in this paper can be extended. In particular, the authors should provide basic information on the data source, spatial extend, number of observations and the criteria used for identifying liquefaction manifestations. The choice of the 250 m grid should also be briefly justified in terms of spatial density and accuracy of the observations.

Author's Response: Thank you for the comment. Section 3.1 has been expanded to provide additional information on the observational dataset. The empirical template is based on the Canterbury CPT liquefaction case-history dataset described by Geyin et al. (2021), comprising 5,668 CPT records at 5,654 distinct locations and manifestation information from the 2010, 2011, and 2016 Canterbury earthquakes. The dataset covers an area of approximately 19.1 km east–west and 43.6 km north–south. Liquefaction manifestation was identified from the documented field classifications available in the original dataset, and a location was treated as positive when documented manifestation was reported in at least one of the three considered earthquakes. Indeterminate observations were not treated as positive manifestations.

During revision, the preprocessing of these classifications was rechecked and an implementation issue was identified in which indeterminate observations had inadvertently been included as positive manifestations in the initial analysis. This was corrected, and the empirical template and all affected downstream analyses were regenerated accordingly.

The choice of a 250 m grid has also been clarified. The grid provides a common spatial support for aggregating the irregularly distributed point observations and for subsequent connectivity analysis. The median nearest-neighbour distance between observation locations is approximately 47 m,

indicating that the 250 m cell size provides a suitably coarse regional representation while avoiding an unrealistically fine discretization relative to the spatial density of the observations. The 250 m spacing is therefore interpreted as the analytical support scale of the connectivity analysis rather than as an estimate of the positional accuracy of the original observations. These clarifications have been added to Section 3.1 of the revised manuscript.

2- Can the authors provide a physical interpretation of the transition band in terms of spatial scales?

Author's Response: Thank you for the comment. The physical interpretation of the coherence parameter and the transition band has been expanded in Sections 3.2 and 5.2 by relating the model coherence scale to correlation length and scale of fluctuation concepts commonly used in geotechnical random-field analysis (Phoon and Kulhawy, 1999; Cami et al., 2020).

In the present framework, the dimensional Gaussian smoothing scale is:

$\ell s = \sigma c \times \Delta$

where σc is the coherence parameter and $\Delta = 250$ m is the reference grid spacing. Accordingly,

$\sigma c = 1$ corresponds to $\ell s = 250$ m,

$\sigma c = 2$ corresponds to $\ell s = 500$ m,

and $\sigma c = 3$ corresponds to $\ell s = 750$ m.

For a Gaussian spatial correlation representation, the autocorrelation function associated with this smoothing scale can be written as:

$\rho(r) = \exp[-r^2 / (4\ell s^2)]$

so that the distance at which the correlation decreases to 1/e is:

$r1/e = 2\ell s.$

The corresponding one-dimensional scale of fluctuation, defined from the integral of the autocorrelation function, is:

$\theta h = 2\sqrt{\pi}\ \ell s.$

Therefore, the principal coherence range $\sigma c = 1–3$ corresponds approximately to 1/e correlation distances of 0.5–1.5 km and horizontal scales of fluctuation of approximately 0.9–2.7 km.

These values can be placed within the established hierarchy of geotechnical spatial variability. Phoon and Kulhawy (1999) showed that inherent soil variability can be characterized through spatial correlation and scale of fluctuation. For CPT-derived local soil properties, reported

horizontal scales of fluctuation span approximately 0.14–80 m (Ching et al., 2018). Lenz and Baise (2007) similarly noted horizontal correlation distances of the order of tens of metres for local CPT properties, while showing that the liquefaction potential index, which integrates the soil profile and is evaluated at a regional scale, can exhibit horizontal correlation distances of several kilometres.

The transition band identified in the present study therefore occupies the mesoscale-to-regional part of this spatial hierarchy. In particular, at p = 0.35, the strongest second-largest-cluster and finite-cluster responses occur around σc = 3, corresponding to a Gaussian smoothing scale of approximately 750 m, a 1/e correlation distance of approximately 1.5 km, and an equivalent horizontal scale of fluctuation of approximately 2.7 km. This provides a physical interpretation of the transition as the spatial scale at which separated manifestation patches increasingly reorganize into larger connected structures and begin to develop regional-scale continuity.

3- Is the non-monotonic spanning probability in Figure 6 a finite-size effect, or does it persist as the grid resolution is refined?

Author's Response: Thank you for the comment. An additional grid-refinement analysis was performed to examine whether the non-monotonic spanning response in Figure 6 persists as the grid spacing is reduced. The procedure and results have been added to Sections 3.3 and 4.1 of the revised manuscript, and the corresponding resolution comparison is provided below in Figure R1.

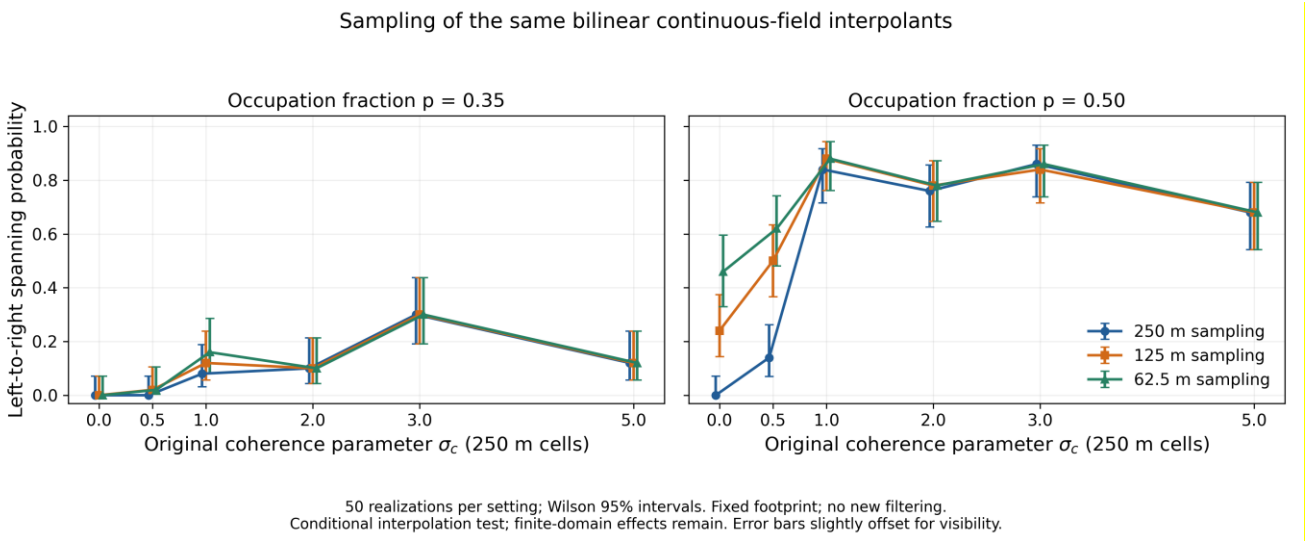


Figure R1. Grid-refinement sensitivity of left-to-right spanning probability for p = 0.35 and p = 0.50. The same continuous-field realizations were sampled at grid spacings of 250, 125, and 62.5 m while retaining the same physical domain. Points show spanning probabilities from 50 realizations and error bars indicate 95% Wilson confidence intervals.

For each coherence level, the same 50 continuous-field realizations were sampled at grid spacings of 250, 125, and 62.5 m while keeping the physical domain and the underlying continuous fields unchanged. Each field was subsequently thresholded at the prescribed occupation fraction, and left-to-right spanning was evaluated using four-neighbour connectivity.

The non-monotonic response is consistent with the fixed-occupation design. Increasing coherence does not increase the number of occupied cells; instead, it redistributes the same occupied area. This reorganization can produce a larger and more dominant component while simultaneously removing narrow connections or shifting the connected structure away from one of the opposite domain boundaries. Consequently, increasing coherence does not necessarily produce a monotonic increase in spanning probability.

The decrease in spanning probability at higher coherence persists at all three sampling resolutions. At $p = 0.35$, spanning probability decreases from 0.30 at $\sigma c = 3$ to 0.12 at $\sigma c = 5$ for all three grid spacings. At $p = 0.50$, the corresponding changes are 0.86 to 0.68, 0.84 to 0.68, and 0.86 to 0.68 for grid spacings of 250, 125, and 62.5 m, respectively. Thus, the non-monotonic decrease is retained as the grid is refined and is not simply a consequence of the original 250 m discretization.

The analysis also shows greater resolution sensitivity at low coherence. For example, at $p = 0.50$ and $\sigma c = 0$, spanning probability changes from 0.00 to 0.24 and 0.46 as the grid spacing decreases from 250 to 125 and 62.5 m. This indicates that fragmented configurations are more sensitive to spatial discretization than the high-coherence response. Because the physical domain dimensions were held fixed in this comparison, the analysis specifically addresses grid-resolution sensitivity rather than changes in physical domain size.

4- Since the simulated fields are obtained from an observational template and then thresholded to keep the same occupation probability, the authors should mention briefly if some connectivity features could depend on the spatial morphology of the template selected. This is an important limitation to keep in mind when considering the generality of the framework. A further comparison with another template, if available, may be useful, but is not required.

Author's Response: Thank you for the comment. We agree that some connectivity characteristics may depend on the spatial morphology of the observational template. Although the occupation fraction is fixed, the spatial arrangement of the active cells remains influenced by the inherited template geometry, including patch compactness, separation, orientation, and proximity to domain boundaries. These features may affect spanning probability, fluctuation peaks, and the location of regime boundaries.

Section 5.2 has therefore been revised to state explicitly that the quantitative transition characteristics reported here are conditional on the selected empirical template. The framework itself can be applied to other spatial templates, while comparison across independent templates would be a useful direction for assessing the transferability of the quantitative results.

5- The labels and legends of some of the figures are small, and the authors need to enlarge them to make them readable.

Author's Response: Thank you for the comment. All figures have been revised to improve readability. Axis labels, tick labels, legends, panel titles, and color-bar annotations have been enlarged and standardized throughout the manuscript. Multi-panel layouts and spacing were also adjusted where necessary to reduce crowding, and overlapping curves were distinguished using

different line and marker styles. The revised figures have replaced the corresponding original versions in the manuscript.