Brief communication: Delayed Ice Avalanches Triggered by Earthquakes: A Strain-rate Dependent Strengthening Mechanism

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Contents of this file

Supplement S1 to S3

Introduction

This Supplement presents the derivation processes of Eqs. (2) to (4) in the main text. Specifically, Supplement S1 provides the derivation process of Eq. (2), Supplement S2 that of Eq. (3), and Supplement S3 that of Eq. (4). The physical meanings of the symbols used in the Supplement are consistent with those in the main text.

Supplement S1.

To describe the localized deformation at the ice–ice contact interface, we consider a simplified axisymmetric contact model in which a circular rigid body is vertically displaced onto an elastic–plastic half-space (Fig. S1). The ice particle has a radius R, and the downward displacement is denoted by d. The radial coordinate is represented by r. As the displacement progresses, the contact region sequentially undergoes purely elastic and then elastic–plastic deformation. The schematic in Fig. S1 illustrates the contact geometry and defines two key boundary parameters: the elastic contact radius a_e, and the plastic contact radius a_p. These define the extents of the stress distribution under elastic and plastic regimes, respectively. The analytical expressions for these boundaries are given by:

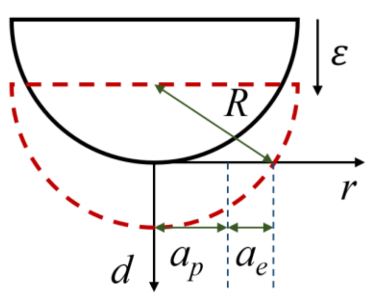

Figure S1. Geometry of the elastic–plastic contact model and definitions of the elastic and plastic contact radii.

$$\begin{cases} a_e = \sqrt{2Rd - d^2} \\ a_p = \sqrt{2R(d - \frac{\sigma_f}{k}R) - (d - \frac{\sigma_f}{k}R)^2} \end{cases} \tag{S1}$$

where a_e and a_p are the elastic and plastic contact radii, (mm), R is the radius of ice particle, (mm), d is the imposed vertical displacement, (mm); k is normal stiffness, (MPa); σ_f is yield strength, (MPa).

The surface displacement z(r) at any arbitrary radial distance r satisfies the governing equation:

$$z(r) = d - R + \sqrt{R^2 - r^2} \tag{S2}$$

The contact force response in the model evolves as a function of strain and strain rate, transitioning from a purely elastic regime to an elastic–plastic regime as displacement increases. The governing equations for each regime are provided below.

(1) Elastic regime

When the contact stress remains below the material's yield strength σ_f, the deformation is purely elastic. The contact force F can be expressed as:

$$F = \int_0^{a_e} 2\pi rk\left(\varepsilon - 1 + \sqrt{1 - (r/R)^2}\right)dr + \int_0^{a_e} 2\pi r\eta\varepsilon' dr \tag{S3}$$

where F is the contact force, (N); k is normal stiffness, (MPa); η is viscosity, (MPa·s); ε' is the strain rate, (s^{-1}). This expression assumes linear elasticity within the contact zone.

(2) Elastic-plastic regime

When the applied strain exceeds the elastic limit, plastic deformation initiates, and the force–displacement relationship becomes nonlinear. The contact force in this regime is given by:

$$F = \int_0^{a_p} 2\pi r\sigma_f dr + \int_{a_p}^{a_e} 2\pi rk\left(\varepsilon - 1 + \sqrt{1 - (r/R)^2}\right)dr + \int_0^{a_e} 2\pi r\eta\varepsilon' dr \tag{S4}$$

(3) General expression

Across both regimes, the strain-dependent contact force can be described in piecewise form as:

$$\sigma = \frac{F}{\pi R^2} = \begin{cases} k\varepsilon^2(1 - \frac{\varepsilon}{3}) + \eta\varepsilon'\varepsilon(2-\varepsilon), & \varepsilon \le \frac{\sigma_f}{k} \\ \sigma_f\left[\varepsilon(2-\varepsilon) - \frac{\sigma_f}{k}(1-\varepsilon) - \frac{\sigma_f^2}{3k^2}\right] + \eta\varepsilon'\varepsilon(2-\varepsilon), & \frac{\sigma_f}{k} \le \varepsilon \end{cases} \tag{S5}$$

Equation (S5) captures the transition from elastic to plastic behavior and incorporates both stiffness- and rate-dependent effects in the model.

Supplement S2.

We decompose the total energy during particle contact into three components: damping energy, plastic energy, and heat exchange energy. The corresponding energy densities are derived below.

(1) The damping energy density

Assuming a constant strain rate, the damping force is constant throughout compression. The total damping energy is:

$$W_\eta = \int_0^{a_e} 2\pi r \eta \varepsilon' z(r) dr \tag{S6}$$

The corresponding energy density is:

$$U_\eta = \frac{3}{4}\eta \varepsilon' \varepsilon^2 (1 - \frac{\varepsilon}{3}) \tag{S7}$$

(2) The plastic energy density

The plastic energy is generated by frictional sliding in the contact zone. For a point at radial distance *r*, the local plastic displacement $Z_p(r)$ is:

$$Z_p(r) = Z(r) - \frac{\sigma_f}{k} R \tag{S8}$$

Integrating over the contact surface, the total plastic energy becomes:

$$W_p = \int_0^{a_p} 2\pi r \sigma_f Z_p(r) dr \tag{S9}$$

Neglecting higher-order terms, the plastic energy density can be approximated as:

$$U_p = \frac{3}{4}\sigma_f \left[\left(\varepsilon + \frac{\sigma_f}{k} \right)^2 - 4\frac{{\sigma_f}^2}{k^2} \right] \tag{S10}$$

(3) The heat exchange energy density

During plastic deformation, part of the internal energy is stored, while a portion is dissipated to the surrounding environment through heat conduction. The temperature evolution of the ice particle T_t relative to the ambient temperature T_s is governed by Newton's Law of Cooling:

$$\frac{dT_t}{dt} = -\psi (T_t - T_s) \tag{S11}$$

where ψ is the thermal exchange coefficient. Integrating Eq. (S11) yields the temperature evolution as:

$$\ln(T_t - T_s) = -\psi t + C \tag{S12}$$

where $C = \ln(T_0 - T_s)$, T_0 is the initial particle temperature at t = 0. The heat transferred to the environment is calculated as:

$$T_0 - T_s = \frac{U_p}{c\rho} \tag{S13}$$

where c is the specific heat capacity, (J/kg·°C); ρ is the density, (kg/m^3). Eq. (S13) defines the heat exchange energy density U_t at time t:

$$U_t = (T_0 - T_t)c\rho = (1 - e^{-\psi t})U_p \tag{S14}$$

Assuming uniaxial compression with constant compression rate, the heat exchange duration t is inversely proportional to the strain rate. Then the heat exchange energy density U_t as a function of the strain rate can be expressed as:

$$U_t = \left(1 - e^{-\frac{\psi}{\varepsilon'}}\right)U_p \tag{S15}$$

By combining Eqs. S10 and S15, the internal energy density of the model becomes:

$$U_T = U_p - U_t = e^{-\frac{\psi}{\varepsilon'}}U_p \tag{S16}$$

Supplement S3.

In this model, ice is treated as an isotropic material. Under uniaxial stress, the distortion energy density U is defined as:

$$U = \tfrac{(1+\upsilon)}{3E}\sigma^2 \tag{S17}$$

Where U is the distortion energy density, (J/m^3); υ is the Poisson's ratio; E is the elastic modulus, (Pa); σ is the axial stress, (Pa).

In classical strength theory (the von Mises or fourth strength criterion), material failure occurs when the distortion energy reaches a critical threshold, beyond which no further elastic deformation is sustainable. Traditionally, all strain energy is assumed to be recoverable and non-dissipative. However, in our formulation, we account for energy dissipation through both internal energy and damping energy , and define the critical condition as:

$$U - U_T - U_\eta = U_0 \tag{S18}$$

where U_0 is the critical distortion energy density, (J/m^3).

Substituting Eqs. (S7), (S16), and (S17) into Eq. (S18), we obtain a modified strength criterion that incorporates the effects of both strain rate and thermal dissipation:

$$\frac{(1+\upsilon)}{3E}\sigma^2 - \frac{3}{4}e^{-\frac{\psi}{\varepsilon'}}\sigma_f\left[\left(\varepsilon+\frac{\sigma_f}{k}\right)^2 - 4\frac{\sigma_f^{\ 2}}{k^2}\right] - \frac{3}{4}\eta\varepsilon'\varepsilon^2(1-\frac{\varepsilon}{3}) = \frac{(1+\upsilon)}{3E}\sigma_0^{\ 2} \quad \text{(S20)}$$

where σ_0 is the uniaxial strength of ice under low strain rates. This expression describes the dynamic uniaxial strength of ice as a coupled function of mechanical energy storage and dissipation, generalizing the classical distortion energy theory for transient loading conditions.