

~~The direct hand~~ Statistical mechanics of the ~~magnetosphere radar~~ aurora: emergent macroscopic transport in ~~controlling small-scale~~ auroral plasma Farley-Buneman turbulence: ~~Introducing the~~ renormalization group

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Abstract. ~~Studies of high-latitude plasma turbulence in~~ In Earth's upper atmosphere fundamentally focus on the differential response ~~E-region ionosphere, the differential magnetization~~ of electrons and ions ~~to strong external electric fields generated during geomagnetic storms. Because ions in the E-region are heavy and highly collisional, they remain largely tied to the neutral gas, whereas magnetized electrons undergo rapid $\mathbf{E} \times \mathbf{B}$ drift. Microscopic polarization electric fields are generated when the~~

5 ~~relative drift velocity between these streaming electrons and the background ions exceeds the local~~ allows strong auroral electric fields to drive Hall currents that, above the ion-acoustic speed, ~~triggering two-stream plasma instabilities, producing Farley-Buneman waves. We propose a new theory that explicitly considers thousands, or millions, of such waves being excited inside a limited volume of space around aurorae, subject to the renormalization group. The resulting theory constitutes an effective field theory for~~ break the plasma up into Farley-Buneman turbulence in the Martin-Siggia-Rose formalism.

10 ~~At the core of this theory is a statistical description of Farley-Buneman waves, where we allow each individual wave to produce a polarization electric field. We treat the sum total of these “~~ turbulence. We construct an effective field theory for the macroscopic transport arising from this turbulence, treating the aggregate polarization micro-fields ” ~~that occur inside a turbulent volume of the saturated wave structures~~ as a stochastic variable, ~~or simply noise. That noise, now a thermodynamic property, becomes the basis for anomalous diffusion, and an effective diffusion tensor, and we recover the expression for~~

15 ~~Bohm diffusion. In support of this theory in the Martin-Siggia-Rose formalism. A renormalization group analysis classifies the macroscopic dynamics (dynamical exponent $z=2$, infrared-stable Gaussian fixed point), while a stochastic closure fixes the transport coefficients at the Bohm order of magnitude. The theory is confronted with three-dimensional kinetic simulations from the literature, tested internally by a reduced Monte Carlo simulation, and exposed to a direct empirical test. Here, we~~ aggregate 272 million ICEBEAR radar echo detections concurrent with 530, ~~we present a large statistical analysis of how~~

20 ~~auroral electrojet turbulence responds to magnetospheric driving, revealing a clear tendency for the observed number density of turbulent waves to scale~~ 000 electric-field wave spectra from the Arase spacecraft: above a critical magnetospheric wave

power, the radar echo detection rate rises linearly with driving power, matching the predictions made by our field theory's overdamped equations of motion. Crucially, the (exponent 0.98 ± 0.09), while echo Doppler speeds remain saturated near the ion-acoustic speed, which selects the amplitude-saturated, echo count-carried response predicted by the effective field theory offers. The resulting closed-form calculations of macroscopic transport relations that are uniquely suitable for sub-grid parameterization in transport law brings to space weather modeling, mimicking the success of stochastic parameterization in hydrodynamic the sub-grid stochastic parameterization of unresolved turbulence that Earth-system models. The derivation of these parameterized equations demonstrates how Bohm diffusion arises from a statistical-mechanical treatment of turbulent volumes rely on for analogous scale-separation problems, at the expense cost of an explicit treatment of the turbulent cascade. The equations should be investigated further, and in future, they may model the evolution of momentum and energy in numerical treatments of global magnetohydrodynamic circulation, below the scale sizes normally considered accessible to fast, predictive models.

1 introductionIntroduction

In Earth's lower ionosphere, collisions between charged particles and neutral molecules become so frequent that the ratio of collision to cyclotron frequency is large enough to demagnetize ions below 120 km and electrons below 80 km. The resulting difference in charge carrier mobility between 80 and 120 km allows strong electric fields in the aurora to drive Hall currents—a reservoir of free energy for electrostatic instabilities (Huba et al., 1985)(Fejer and Kelley, 1980; Huba et al., 1985). For plasma drifts below around 400 m/s, the Hall currents feature little internal structure, owing to ordinary plasma diffusion.

However, given sufficiently strong electric fields, or, equivalently, sufficiently fast electron drifts, the laminar currents rapidly break up into eddies with sizes that vary across a spectrum. This occurs for imposed drift speeds exceeding the ion acoustic speed, C_s , usually around 400 m/s. This triggers the Farley-Buneman (FB) instability via the ion inertia that is excited by the motion of the irregular structures, driven by the electron plasma ($E \times B$) drift (Farley, 1963; Buneman, 1963). This generates an increase in the overall Joule heating rate through the nonlinear dissipation of electrostatic structures (St-Maurice and Goodwin, 2021), and the dissipation occurs soon after the unstable structures have reached maximum amplitude, having taken as much free energy as possible from the system, thereby bringing the electric field to a point of marginal stability (St.-Maurice and Hamza, 2001). The dissipation comes from the inevitable creation of increasingly important wave electric field components along the magnetic field, which erode the structures (Drexler et al., 2002; Oppenheim and Dimant, 2013) and thereby returns the energy to the particles via acceleration. For very strong ambient Hall currents, the additional heating rate from the energy going through FB waves becomes highly relevant through large increases in the ambient electron temperature above 105 km (Schlegel and St.-Maurice, 1981; St.-Maurice, 1985; St-Maurice et al., 1990; Hamza, 1998; St.-Maurice and Hamza, 2001; St-Maurice and Goodwin, 2021).

The foregoing describes a dynamic and complex conversion of ambient Hall currents into electrostatic energy, as demonstrated through in-situ measurements (Pfaff et al., 1987; Kelley et al., 1986b, a), ground-based radar studies (St.-Maurice et al., 1989; Foster et al., 1992; Foster and Tetenbaum, 1992) and hybrid fluid- and particle-simulations (Oppenheim et al.,

1995, 1996; Oppenheim and Dimant, 2013). Recently, radar studies have also shown that the turbulence is triggered preferentially on the edges of the precipitation regions, (Bahcivan et al., 2006; Huyghebaert et al., 2021) where the electric field intensifies (Fujii et al., 2011; Ivarsen et al., 2024b).

Building on this description of externally driven FB turbulence, we surmise a state of criticality, where the response of the non-laminar electrojets around diffuse aurorae is *saturated*, and the system is forced into an overdamped state. To capture the saturation dynamics, we adopt a formalism consistent with Hamza and St-Maurice (1993) and St-Maurice and Chau (2016): (1) the relevant driving term for FB turbulence is controlled by a Doppler-shifted eigenfrequency $\omega' = \omega - \mathbf{k} \cdot \mathbf{V}_d$ that plays an important role after we consider that (2) the FB instability itself is regulated by an effective damping term, γ_{eff} , which forces the growth rate to go from large at early stages where the structures are unstable, to zero when the wave is damped to saturation. After that, the growth rate of single structures goes to zero owing to the nonlinear association of the wave field with the perturbed density, in reaction at first (St.-Maurice and Hamza, 2001). In their final stage of evolution, the growth of the wave-field, along the magnetic field, damps the unstable structures and dissipates them.

In specific terms, the dynamics of the density fluctuations in the frame of the drifting plasma are governed by a dispersion relation, that favors growth at first and then becomes highly dissipative, all through the changes in the nonlinear evolution of the eigenfrequency, as given by, e.g., Hamza and St-Maurice (1993),

$$\omega^2 - k_{\perp}^2 C_s^2 + i\omega' \gamma_{\text{eff}} = 0, \quad (1)$$

where $k_{\perp}$ is the perpendicular wavenumber and γ_{eff} is large, owing to the small magnitude of Pedersen currents relative to electron Hall currents. Once we recognize the impacts of the non-linear effects exerted on the eigenfrequencies, the dynamics becomes quite complex. The large value of γ_{eff} forces ω' to remain very close to zero, so that the frequency remains locked at $\mathbf{k} \cdot \mathbf{V}_d$. The complexity emerges from the fact that $\mathbf{V}_d$ responds nonlinearly to wave amplitudes, and ends up being a function of time. The term $\omega^2 - k_{\perp}^2 C_s^2$ in individual structures therefore fluctuates between positive and negative. The system then sees repeated oscillating cycles occurring in intensely excited, limited volumes of space. From this complexity we observe the emergence of novel qualities.

1.1 The role of emergence

However, the exact meaning of this fact is difficult to define, as it is unclear whether or not statistical qualities that emerge in complex datasets exist in a directly observable sense. We are faced with two interpretations. (1) Under the assumption of *strong* emergence, statistically derived qualities are rigorously replicated in nature, arising uniquely through collective interaction, and this interpretation is usually confined to condensed matter physics (Drossel, 2021) and chemistry. (2) On the other hand, in the case of *weak* emergence, the statistically derived qualities are observable only in aggregate, and represent a descriptive, working mode of explaining complex phenomena.

In the present paper, we present a clear case of emergence from the complex interactions of FB turbulence, though we are unable to determine whether this result should be interpreted through the lens of strong emergence, and so we favour a weak in-

interpretation. The result is arrived at through [a stochastic closure combined with a renormalization group \(RG\) theory analysis](#), and we demonstrate that this theoretical application is **entirely**-consistent with observations in statistical aggregates. Under a hypothetical strong interpretation, this would imply that the turbulent ionosphere features noise-enabled transport of momentum and energy, a transport that is observable in statistical aggregates. The question becomes whether this transport of momentum and energy adheres to a set of defined macroscopic laws. We define such a macroscopic effective field theory, wherein a deterministic advection-diffusion equation allows for parametrized transport equations (Eq. 2 below), for dynamics smaller than around 100 km in cross-field scale-size.

We favour a weak interpretation, for merits that we shall soon espouse, and urge further research into the thermosphere-ionosphere-magnetosphere interaction through the lens of complexity.

Numerical magnetohydrodynamic solutions of the ~~magnetosphere-ionosphere~~ [magnetosphere-ionosphere](#) system are unable to resolve scales smaller than approximately 50–100 km, set by the computational cost of global fluid solvers (Wiltberger et al., 2017a, b). Below this “grid-cell”-scale, electrojet turbulence must be evaluated using expensive hybrid fluid-kinetic simulations (Oppenheim et al., 1995; Dimant and Sudan, 1995; Dimant et al., 2021), and the nature and magnitude of the turbulent cascade in this regime remain unknown in operational space weather forecasting, and replacements are found in terms of eddy diffusion (Wu et al., 2017, e.g.). This problem is echoed in recent history, in terrestrial weather and climate modeling, where sub-grid convective processes are likewise inaccessible to direct simulation. The atmospheric community has addressed this gap through two generations of parameterization: deterministic effective diffusion (Smagorinsky, 1963) and, more recently, stochastic parameterization that treats unresolved convective scales as noise, yielding demonstrable improvements in ensemble forecast skill (Palmer, 2012; Berner et al., 2017). No comparable framework currently exists for sub-grid electrojet turbulence in space weather models. In what follows, we construct one. We treat the sum total of polarization electric fields produced by individual Farley–Buneman waves as a stochastic variable, and we apply renormalization group theory, yielding closed-form transport coefficients—~~an~~ an effective diffusion tensor (Eq. 2) and an anomalous resistivity (Eq. 46)—~~that~~ that parameterize the sub-grid turbulent cascade in three dimensions. These relations are suitable for direct incorporation into global magnetohydrodynamic models as constitutive transport laws, and provide strong motivation for our investment in the *weak*, i.e., descriptive, interpretation of our field-theory.

1.2 Summary of the effective Farley-Buneman field theory

This paper is oriented around an effective field theory for electrojet turbulence. In this formulation, the spatial dynamics are governed by an effective diffusion tensor,

$$\mathbf{D}_{\text{eff}} = \gamma_{\text{eff}}^{-1} (C_s^2 \mathbb{I} - \mathbf{V}_d \otimes \mathbf{V}_d), \quad (2)$$

where C_s is the acoustic speed and $\mathbf{V}_d$ is the saturating drift velocity. Here, the instability manifests as negative diffusion (Ashinsky, 1988) instead of wave-growth, triggered when drifts surpass the ion acoustic speed ($D_{\parallel} \sim C_s^2 - V_d^2 < 0$). This unstable growth occurs only along the mean drift, while transverse modes, the overdamped secondary waves, (Hamza and

St-Maurice, 1993) remain subject to positive effective diffusion $D_{\perp} \sim C_s^2$. The instability grows until it reduces the driving
120 field V_d , flipping the diffusion from negative to positive and ensuring marginal stability.

Solving the resulting ~~advective-diffusive~~ advection-diffusion equation of motion ~~for the renormalized FB waves yields an Adler-Ohmic bifurcation in the system, where magnetospheric forcing elicits a~~ under periodic forcing yields the central macroscopic prediction of this paper: a thresholded, linear (Ohmic) response~~in the degree of-, in which magnetospheric forcing drives the~~ proliferation of FB turbulence. ~~We demonstrate the validity of the theory by showing that, during~~ We
125 test this prediction statistically. During intense diffuse aurorae, the ~~ionosphere-magnetosphere coupling exhibits the theorized Adler-Ohmic bifurcation, (Adler, 1946) with an Ohmic response where~~ rate of VHF radar echo detections, n , our measure of E-region turbulence intensity ~~n , scales linearly with~~ the electromagnetic power input ~~measured by the Arase spacecraft (Miyoshi et al., 2018b, 2022), $n \propto \Delta E$, but will only do so when and does so only once~~ ΔE exceeds a characteristic thresh-
old wave power ΔE_c , ~~measured by the Arase spacecraft (Miyoshi et al., 2018b, 2022),~~ We predict that the linear relationship
130 follows from the saturation of wave amplitude. The saturation of amplitudes forces the- The linearity follows from amplitude saturation: with amplitudes clamped near the acoustic speed, the number of turbulent parcels ~~(proliferation) to must~~ track the driver. ~~The~~ In this description, the ionosphere around diffuse aurorae ~~thus self-organizes into a driven-dissipative boundary, preserving a scaled-down structural imprint of the magnetospheric boundary turbulence, while optimizing energy dissipation~~ behaves as a driven, dissipative boundary that follows its magnetospheric driver without hysteresis; the synchronization (Adler) interpretation
135 of the threshold is developed in Section 2.1.3 and assessed, with its limits, in the Discussion.

2 Methodology

ICEBEAR is an auroral coherent radar that observed three-meter Farley-Buneman waves near the radar's horizon (Huyghebaert et al., 2019; Lozinsky et al., 2022), where those waves are perpendicular to the beamed radar signal. Echoes produced when the radar signal is Bragg-like-scattered by the Farley-Buneman waves, and a local analysis of the echo characteristics, correspond
140 directly to a quantitative description of Farley-Buneman turbulence. The radar can record thousands of echo locations per second, yielding exceedingly large point-cloud datasets (Ivarsen et al., 2023, 2024c). ICEBEAR combines software-defined radio with multiple frequency interferometry, which allows the radar to observe the bulk 3D shapes of the turbulent ionosphere near aurorae.

The radar operates every night near Saskatoon in Saskatchewan, Canada, regardless of the ongoing geomagnetic activity, and
145 it only observes auroral plasma turbulence during approximately 4% of the time spent operational. However, it observes the turbulent trails of *meteors* that burn up in Earth's atmosphere at all times, meaning that an ICEBEAR dataset in aggregate yields a statistical treatment of an *entire space weather system*. The dataset thus spans the transition from a static state, where the only observable turbulence stems from the perpetual rain of meteors into Earth's atmosphere, to a fully turbulent state wherein the aurora is *intermittently injecting* free energy that drives intense small-scale plasma turbulence (Ivarsen et al., 2024b).

150 In this paper, we shall analyze such a space weather system through a large database of observations by the ICEBEAR radar and the Japanese inner-magnetosphere spacecraft Arase (Miyoshi et al., 2018b, 2022). Arase's orbit features an apogee of

32,000 km, a perigee of 400 km, an inclination angle of 31° , and an orbital period of 570 minutes. We collected data from instances where both instruments were operational, and both were inside the nightside auroral region. This exercise yields 270 million radar echo detections and some 500,000 electric field wave spectra observed by Arase, covering such frequencies that
155 are known to interact with electrons causing particle precipitation.

A theoretical (renormalization group flow) and data-driven analysis converge on the existence of an emergent regime in auroral plasma turbulence, wherein FB waves in ensemble produce a *noise variable*, one that translates ~~directly to Bohm diffusion~~
into a finite, Bohm-order renormalization of the macroscopic diffusion tensor. This important result highlights the utility of complexity science to characterize space weather on scales below the typical “grid cell” of a global magnetohydrodynamic
160 model.

2.1 Renormalization group theory

In Section 1.1, we described emergence in complex systems, namely that order can emerge from otherwise efficiently dissipative processes, planting modes of fluctuation into a space weather system that are not predictable from deterministic theory alone, but rather completely dependent on a probabilistic treatment of an *ensemble*, whose sum total is unpredictable from an
165 isolated analysis of a single constituent process.

The foregoing represents a feedback mechanism that is completely missed by any theoretical analysis of the individual processes in isolation, and therefore we note the distinct merit to a confirmed observation of large-scale emergent behaviour in the statistical treatment of space weather systems around diffuse aurorae, the topic of the present paper. This observation justifies the construction of an effective field theory for auroral plasma turbulence, a step that decidedly acknowledges the vast
170 complexity that is being renormalized in such a construct. The potential feedback asserted by this renormalized complexity is the precise justification for the drastic step.

Having assiduously justified the step, we perform a Martin-Siggia-Rose (MSR) renormalization group (RG) theoretical deconstruction (Wilson, 1971; Hohenberg and Halperin, 1977) of Farley-Buneman turbulence, treating polarization electric fields produced by FB wave saturation (i.e., waves very briefly coasting at the ion sound speed), as a stochastic ensemble. As
175 we shall demonstrate, the ~~stochastic variable is directly renormalized into an anomalous transport coefficient, where we recover Bohm diffusion. This entails treating FB turbulence as noise, and that the effects of this noise necessitate a non-linear dynamic synchronization treatment of auroral plasma turbulence. We deduce from this observation that the noise is organized in a particular synchronization pattern, one that is irreducible from a fluid or kinetic, i.e., theoretical, treatment of FB turbulence. By this, we mean that the overall pattern~~
RG classifies this stochastic variable as an irrelevant coupling: it leaves the macroscopic advection-diffusion form of the theory intact, while contributing a finite, anisotropic renormalization to the effective diffusion tensor. The magnitude of that renormalization, which is of Bohm order, is fixed by a stochastic closure of the micro-field statistics, not by the RG itself. This entails treating FB turbulence as noise. The macroscopic pattern that results is created
180 ~~by numerous on-off trigger behaviours, cleanly describable by Kuramoto synchronization in terms of Josephson junctions (microelectrical superconductor chips) and in the Discussion we entertain, as a candidate interpretation rather than a derived~~

185 [result, a description of this pattern in the language of phase synchronization and Josephson junctions, with the Doppler velocity of the saturated waves supplying the measurable phase.](#)

In what follows, we construct an effective action in the path integral formalism, using the Martin-Siggia-Rose framework, and with this, we perform a dynamic scaling analysis with spatial rescaling $x' = b^{-1}x$ and temporal rescaling $t' = b^{-z}t$. We then derive the macroscopic “Adler-Ohmic” linear power law observed in the auroral E-region ionosphere, from the micro-
 190 scopic physics of FB turbulence. The derivation is performed in the neutral atmosphere’s frame of reference, but an equivalent derivation in the ions’ frame of reference yields identical results.

2.1.1 The dispersion relation

We write out the dispersion relation in the atmosphere frame, defining the wave frequency ω relative to the neutral gas,

$$\omega^2 - k_{\perp}^2 C_s^2 + i(\omega - \mathbf{k} \cdot \mathbf{V}_{d0})\gamma_{\text{eff}} = 0, \quad (3)$$

195 where $\mathbf{V}_{d0}$ is the mean $\mathbf{E} \times \mathbf{B}$ drift velocity. We note that the real part of the frequency corresponds to the convective derivative in the time domain, while the imaginary damping term acts on the Doppler-shifted frequency.

Next, we shall apply the Martin-Siggia-Rose formalism of describing statistical dynamics for classical systems (Phythian, 1976; Andersen, 2000), turning the stochastic differential equations of motion for our system into an effective, macroscopic field theory (Bonicelli et al., 2025). We start by constructing the unperturbed action in the path integral formalism, based on
 200 Eq. (3). In the neutral gas frame, this explicitly retains the mean advection of the plasma structures:

$$S_0 = \int dt d^d x \tilde{\psi} \left[\partial_t^2 + 2(\mathbf{V}_{d0} \cdot \nabla) \partial_t + (\mathbf{V}_{d0} \cdot \nabla)^2 - C_s^2 \nabla^2 + \gamma_{\text{eff}}(\partial_t + \mathbf{V}_{d0} \cdot \nabla) \right] \psi, \quad (4)$$

where $\psi = \delta N_e / N_{e,0}$ is the electron density variation divided by mean density, and $\tilde{\psi}$ is the auxiliary response field acting as a Lagrange multiplier to enforce the deterministic trajectory (Bonicelli et al., 2025). A two-step argument treats the non-linearities in the turbulent plasma:

205 **(1)** The linear propagator S_0 accounts for the mean flow but neglects the effects of turbulent mode-coupling. However, the physical mechanism at the level of individual structures involves the development of secondary polarization fields. A justification for treating these non-linearities stochastically can be derived from the work of St.-Maurice and Hamza (2001); Drexler et al. (2002). Based on Fourier analysis, St.-Maurice and Hamza (2001) showed that for an initial finite-size structure, polarization electric fields are created that oppose the driving field. These internal fields saturate the instability locally, explaining
 210 why VHF Doppler speeds cluster around C_s (Figure 3**bd**).

(2) As a result of point (1), we construct a stochastic source term driven by these self-generated polarization fields. Rather than a slow modulation, we recognize that the total drift velocity $\mathbf{V}(x, t)$ at any point is the sum of the coherent mean drift $\mathbf{V}_{d0}$

and a fluctuating component $\delta\mathbf{v}(t)$ arising from the turbulent ensemble:

$$\mathbf{V}(x, t) = \mathbf{V}_{d0} + \delta\mathbf{v}(x, t). \quad (5)$$

215 Because these fluctuations are driven by the instability itself, they occur on fast timescales, and we write the time-dependent drift for an individual structure i as

$$\mathbf{V}_i(t) = \mathbf{V}_{d0} \left[1 - \varepsilon_v \sin(\Omega t + \alpha_i) \right], \quad (6)$$

where Ω represents the waves' life-cycle, α_i represents the random phase of the i^{th} structure, and ε_v is the amplitude of the non-linear [velocity](#) feedback. Furthermore, since the macroscopic turbulent electrojet is an ensemble of such structures spread
220 over 10s of km, the phase information of the fluctuations is effectively random across the volume. We therefore define the stochastic velocity field $\delta\mathbf{v}(x, t)$ via the random phase approximation (RPA) (Krommes, 2002),

$$\delta\mathbf{v}(x, t) = -\varepsilon_v \sum_i \mathbf{v}_i \sin(\Omega t + \alpha_i(x)). \quad (7)$$

By virtue of the central limit theorem, the sum of these uncorrelated, fast-oscillating terms converges to a Gaussian random variable. Thus, we formally replace the deterministic non-linear trajectory with a stochastic field $\delta\mathbf{v}$ governed by a Gaussian
225 probability distribution $P[\delta\mathbf{v}]$,

$$P[\delta\mathbf{v}] \propto \exp \left(- \int dt d^d x \frac{|\delta\mathbf{v}|^2}{2\sigma_v} \right). \quad (8)$$

Here, the variance σ_v is constrained by the power of the turbulent electric fields. Following the MSR formalism, this stochastic noise enters the equation of motion as a multiplicative source term acting on the gradient of the density field,

$$\hat{\mathcal{L}}_0 \psi = -\gamma_{\text{eff}} (\delta\mathbf{v} \cdot \nabla) \psi, \quad (9)$$

230 which introduces a linear coupling between the response field $\tilde{\psi}$ and the noise ensemble,

$$S_{\text{coupling}} = \int dt d^d x \tilde{\psi} [\gamma_{\text{eff}} (\delta\mathbf{v} \cdot \nabla) \psi]. \quad (10)$$

To derive the effective macroscopic action, we integrate over the ensemble of phase histories. We compute the expectation value of the coupling term over the Gaussian distribution,

$$I = \left\langle \exp \left(i \int dt d^d x (\gamma_{\text{eff}} \tilde{\psi} \nabla \psi) \cdot \delta\mathbf{v} \right) \right\rangle_{\delta\mathbf{v}}. \quad (11)$$

235 Using the Gaussian integral identities, we obtain,

$$I = \exp\left(-\frac{\sigma_v}{2} \int dt d^d x \left(\gamma_{\text{eff}} \tilde{\psi} \nabla \psi\right)^2\right). \quad (12)$$

The MSR path integral weight is defined as $\mathcal{Z} = \int \mathcal{D}\psi \mathcal{D}\tilde{\psi} e^{iS}$. To translate the real-valued Gaussian probability weight of the noise fluctuations into the complex-weighted path integral formalism, we identify the integration result with iS_{int} , the imaginary component of the action,

$$240 \quad iS_{\text{int}} = -\frac{\sigma_v}{2} \gamma_{\text{eff}}^2 \int dt d^d x (\tilde{\psi} \nabla \psi)^2. \quad (13)$$

~~It is crucial to note-~~

~~It is important to note that the Gaussian phase average performed in the foregoing is a stochastic closure of the specific mechanics of scale separation at play here. Unlike standard Wilsonian renormalization, which relies on spatial momentum-shell integration to explicitly eliminate fast, high-wavenumber fluctuation modes, our MSR approach achieves scale separation primarily in the time domain. This Gaussian path integral over the random temporal phase histories mathematically transforms the microscopic polarization feedback directly into a macroscopic anomalous diffusion tensor, bypassing the need for explicit k -space integration.~~ micro-field statistics under the random phase approximation, and not a renormalization step. The closure is controlled by the ordering $\epsilon \equiv \tau_c / T_{\text{driver}} \ll 1$, where $\tau_c \sim 1$ s is the observed lifetime of a saturated FB structure (Prikryl et al., 1988, 1990) and T_{driver} is the envelope timescale of the magnetospheric driver, of order minutes (e.g., Alfvénic timescales); on scales longer than τ_c the micro-field ensemble acts as a *stationary noise bath*, and the response subsequently carries no hysteresis. The construction is carried out in full in Appendix C. The closure keeps the mean advection, the damping, the acoustic stiffness, and the Gaussian statistics of $\delta \mathbf{v}$; it drops the $\psi \nabla \psi$ three-wave phase correlations, and with them the explicit spectral energy flux of the turbulent cascade. The division of labor with the renormalization group is then clean: the closure supplies the magnitude of the transport coefficient, a non-universal quantity, while the rescaling analysis of the next subsection establishes the universality class of the macroscopic theory and the status of the noise coupling under coarse-graining.

We can now combine the deterministic propagator (derived from the dispersion relation, Eq. 3), and the stochastic noise term, (Eq. 13), obtaining the total action,

$$S = \int dt d^d x \left\{ \tilde{\psi} \left[\partial_t^2 + 2(\mathbf{V}_{d0} \cdot \nabla) \partial_t + (\mathbf{V}_{d0} \cdot \nabla)^2 - C_s^2 \nabla^2 + \gamma_{\text{eff}} (\partial_t + \mathbf{V}_{d0} \cdot \nabla) \right] \psi + i \frac{\sigma_v}{2} \gamma_{\text{eff}}^2 (\tilde{\psi} \nabla \psi)^2 \right\} \quad (14)$$

2.1.2 Renormalization group flow

260 We determine the macroscopic phase of the system by analyzing the flow of the coupling constants under a rescaling of spacetime,

$$x \rightarrow bx,$$

$$t \rightarrow b^z t,$$

$$\psi \rightarrow b^\chi \psi,$$

265 $\tilde{\psi} \rightarrow b^{\tilde{\chi}} \tilde{\psi}.$

The integration measure therefore transforms as,

$$\int dt d^d x \rightarrow b^{z+d} \int dt d^d x. \quad (15)$$

We next choose the stiffness operator (diffusion) to be scale-invariant. In other words, with the scaling relations applied to $C_s^2 \tilde{\psi} \nabla^2 \psi$ (Eq. 14), we obtain the constraint,

270 $\chi + \tilde{\chi} = 2 - z - d. \quad (16)$

Applying ~~this constraint to the other operators~~ Eq. (16) to each operator in the action ~~to find their~~ classifies its scaling behavior relative to the stiffness, ~~we find that,~~

$$\int dt d^d x \tilde{\psi} \partial_t^2 \psi \sim b^{(2-z-d)+z+d-2z} = b^{2-2z},$$

$$\int dt d^d x \gamma_{\text{eff}} \tilde{\psi} \partial_t \psi \sim b^{(2-z-d)+z+d-z} = b^{2-z},$$

275 $\int dt d^d x \gamma_{\text{eff}} \tilde{\psi} (\mathbf{V}_{d0} \cdot \nabla) \psi \sim b^{(2-z-d)+z+d-1} = b^1,$

$$\int dt d^d x \tilde{\psi} (\mathbf{V}_{d0} \cdot \nabla)^2 \psi \sim b^{z+d+\chi+\tilde{\chi}-2} = b^0,$$

$$\int dt d^d x \frac{\sigma_v}{2} \gamma_{\text{eff}}^2 (\tilde{\psi} \nabla \psi)^2 \sim b^{z+d-2+2(\chi+\tilde{\chi})} = b^{2-z-d},$$

~~where we used Eq. (16). We observe that at the inertial fixed point ($z=1$); the full tabulation is given in Appendix D. Requiring the damping to be marginal selects $z=2$. At $z=2$, the inertial term (Eq. ??) is relevant. However, with increasing z the system flows to the dissipative fixed point at $z=2$, where the inertial term becomes irrelevant (scales as b^{-2}) and the damping term becomes marginal (b^0). Crucially, the linear advection term (Eq. ??) scales as b^1 , identifying it as a relevant operator:~~

280

macroscopic transport is dominated by the mean drift, and the cross term as b^{-1} ; both irrelevant. The second-order dynamics flows to an overdamped, first-order theory; the overdamped reduction is derived, not assumed. The noise coupling scales as $b^{2-z-d} = b^{-d}$, irrelevant in any dimension. The one relevant operator is the linear advection (b^{+1}), but it does not destabilize the fixed point: the full action is invariant under the Galilean boost $\psi(\mathbf{x}, t) \rightarrow \psi(\mathbf{x} - \mathbf{u}t, t)$, $\mathbf{V}_{d0} \rightarrow \mathbf{V}_{d0} + \mathbf{u}$, so the advection is a choice of reference frame, and the associated Ward identity forbids any renormalization of $\mathbf{V}_{d0}$ (see, e.g., Forster et al., 1977)

In the macroscopic limit, The fate of the noise coupling decides what the RG can and cannot achieve in our application of it. Forming the dimensionless ratio g of noise strength to diffusion at the running scale, the flow is dominated by the tree-level scaling dimension, $dg/d\ell = -dg + \mathcal{O}(g^2)$ with $b = e^\ell$, so that g decays as b^{-d} toward the infrared-stable Gaussian fixed point $g^* = 0$. An irrelevant coupling does not alter the fixed-point action, but it shifts the marginal couplings by a finite amount before it decays: integrating the flow yields a finite, anisotropic, drift-aligned addition to the diffusion tensor, $\delta\mathbf{D} \propto \hat{\mathbf{V}}_d \otimes \hat{\mathbf{V}}_d$, active below the crossover scale L_c derived in Section 2.1.4 and saturated above it. The magnitude of $\delta\mathbf{D}$ is the inertial terms (∂_t^2) and the stochastic driver fluctuations (σ_v) are renormalized. The stochastic variance σ_v generates the anomalous diffusion tensor, while the mean drift remains as a relevant transport term. This leaves a deterministic effective theory describing an mixing-length coefficient supplied by the stochastic closure; the RG adds that no further corrections survive coarse-graining, since they decay as b^{-d} . Because the fixed point is Gaussian, $z = 2$ holds exactly, with no anomalous exponents: the advection is linear in ψ and the advecting velocity field is short-range correlated (finite structure scale), the two properties that forbid anomalous scaling here. In the macroscopic limit the theory is therefore a deterministic, advecting, diffusing fluid,

$$S = \int dt d^d x \tilde{\psi} [\gamma_{\text{eff}}(\partial_t + \mathbf{V}_{d0} \cdot \nabla) + (\mathbf{V}_{d0} \cdot \nabla)^2 - C_s^2 \nabla^2] \psi. \quad (17)$$

The RG analysis thus establishes the $z = 2$ dissipative universality class and the irrelevance of the microscopic Farley-Buneman polarization electric field noise; the macroscopic transport is advection-diffusion with a diffusion tensor receiving a finite, anisotropic renormalization, and the Bohm-order value of that renormalization is fixed by the stochastic closure, not by the RG (Appendix D).

At this point, we reinstate the internal feedback reaction that the unstable structures excite (St.-Maurice and Hamza, 2001), namely that the turbulent electric field is periodically brought down to the threshold,

$$\mathbf{V}_{d0} \rightarrow \mathbf{V}_d = \mathbf{V}_{d0} (1 - \tilde{\alpha} \langle |\psi| \rangle^2), \quad (18)$$

where we recognize that the variance of the microscopic velocity fluctuations (σ_v) manifests macroscopically as an anomalous drag force. The importance of this step is considerable, and it is equally important to point out that this step, that bringing namely that the electric field being brought back to the instability threshold is governed by amplitude saturation ($\langle |\psi| \rangle^2$), is firmly supported in the literature (St.-Maurice and Hamza, 2001).

A self-regulating feedback mechanism follows: as the turbulent amplitudes increase, the driving electric field is brought down towards the threshold velocity, the ion acoustic speed, C_s , which in turn kills the instability and lowers the turbulent amplitude ($\langle |\psi|^2 \rangle \rightarrow 0$). As we shall demonstrate, this description ensures that the system finds the fixed point where the effective drift **is** **sits** just above the ion acoustic speed, with the backscatter Doppler speeds correspondingly clustering near C_s (Figure 3**bd**), and this is likewise strongly supported in the literature (~~Foster and Erickson, 2000; Oppenheim and Dimant, 2013; Koustov et al., 2005; Chau and St-Maurice, 2016; St-Maurice et al., 2023~~) (Foster and Erickson, 2000; Oppenheim and Dimant, 2013; Koustov et al., 2005; Chau and St-Maurice, 2016; St-Maurice et al., 2023). We obtain the deterministic action,

$$S = \int dt d^d x \tilde{\psi} \left[\gamma_{\text{eff}} (\partial_t + \mathbf{V}_d \cdot \nabla) \text{--} \text{+} (\mathbf{V}_d \cdot \nabla)^2 - C_s^2 \nabla^2 \right] \psi. \quad (19)$$

Here, we can combine the two stiffness terms by way of tensor notation,

$$[(\mathbf{V}_d \cdot \nabla)^2 - C_s^2 \nabla^2] \psi = -\nabla \cdot (C_s^2 \mathbb{I} - \mathbf{V}_d \otimes \mathbf{V}_d) \cdot \nabla \psi, \quad (20)$$

which allows us to write the first-order, overdamped advection-diffusion equation,

$$S_{\text{eff}} = \int dt d^d x \tilde{\psi} \gamma_{\text{eff}} [(\partial_t + \mathbf{V}_d \cdot \nabla) - \gamma_{\text{eff}}^{-1} \nabla \cdot (C_s^2 \mathbb{I} - \mathbf{V}_d \otimes \mathbf{V}_d) \cdot \nabla] \psi. \quad (21)$$

We note that the spatial dynamics are governed by the effective diffusion tensor $\mathbf{D}_{\text{eff}} = \gamma_{\text{eff}}^{-1} (C_s^2 \mathbb{I} - \mathbf{V}_d \otimes \mathbf{V}_d)$, with $\mathbf{V}_d$ representing the saturated drift,

$$\mathbf{D}_{\text{eff}} = \frac{1}{\gamma_{\text{eff}}} \begin{bmatrix} C_s^2 - V_d^2 & 0 & 0 \\ 0 & C_s^2 & 0 \\ 0 & 0 & C_s^2 \end{bmatrix}. \quad (22)$$

(1) In directions parallel to the mean advection $\mathbf{V}_{d0}$, if the driving drift $V_{d0} > C_s$, the effective diffusion coefficient becomes *negative*, which we identify as the macroscopic signature of the instability. This is followed by saturation,

$$D_{\parallel}(\psi) \approx (C_s^2 - V_{d0}^2) + 2\tilde{\alpha} V_{d0}^2 |\psi|^2, \quad (23)$$

where we have linearized the effective drift squared ($V_d^2 \approx V_{d0}^2 - 2\tilde{\alpha} V_{d0}^2 |\psi|^2$) and neglected higher-order $\mathcal{O}(|\psi|^4)$ terms. The negative diffusion that commences when $V_{d0} > C_s$ leads to perturbation growth, which in turn activates the anomalous drag (saturation) through $|\psi|^2$, stabilizing the transport.

(2) In directions perpendicular to the drift, the diffusion simplifies to $\mathbf{D}_{\text{eff}} \approx C_s^2$, the transverse terms representing the ordinary diffusion of the transverse modes. These act as an energy sink, aligning with Hamza and St-Maurice (1993), who explicitly identified the mode-coupled secondary waves as acting akin to “driven, overdamped oscillators.”

2.1.3 Advective-diffusive dynamics

The foregoing subsection established that at macroscopic scales ($z = 2$), the inertial terms (∂_t^2) are irrelevant. The system flows universally to an advective-diffusive state, and so we write the effective equation of motion for the turbulent density fluctuation field $\psi(\mathbf{x}, t)$ as,

$$340 \quad \gamma_{\text{eff}} \left(\frac{\partial}{\partial t} + \mathbf{V}_{d0} \cdot \nabla \right) \psi - \nabla \cdot [(C_s^2 - V_{d0}^2) + 2\tilde{\alpha}V_{d0}^2|\psi|^2] \cdot \nabla \psi = S(\mathbf{x}, t), \quad (24)$$

where γ_{eff} is the renormalized effective damping. $S(\mathbf{x}, t)$ is a source term, which we decompose into an amplitude $\mathcal{E}$ and a phase $\theta(t)$,

$$S(\mathbf{x}, t) = \mathcal{E} e^{i(\mathbf{k} \cdot \mathbf{x} - \theta(t))}, \quad (25)$$

where $\mathcal{E}$ is the strength of the driver and θ is phase. Here, we consider the VHF radar's (and the instability's) natural preference
 345 for the driving modes that resonate with the ion acoustic speed. To account for this observational effect, we approximate the driver phase speed with the acoustic speed characteristic of the saturated state,

$$\dot{\theta} \approx kC_s. \quad (26)$$

We next express the turbulent plasma density field ψ in terms of a slowly varying amplitude $A(t)$ and phase $\phi(t)$:

$$\psi(\mathbf{x}, t) = A(t) e^{i(\mathbf{k} \cdot \mathbf{x} - \phi(t))}. \quad (27)$$

350 We substitute Eqs. (25) and (27) into the effective equation of motion. The convective derivative on the left-hand-side expands as,

$$\left(\frac{\partial}{\partial t} + \mathbf{V}_d \cdot \nabla \right) \psi = \left[(\dot{A} - iA\dot{\phi}) + iA(\mathbf{k} \cdot \mathbf{V}_d) \right] e^{i(\mathbf{k} \cdot \mathbf{x} - \phi)}. \quad (28)$$

Combining this with the diffusion term,

$$-\nabla \cdot \mathbf{D}_{\text{eff}} \cdot \nabla \psi = (\lambda_0 + \tilde{\alpha}V_{d0}^2|\psi|^2) \psi, \quad (29)$$

355 where λ_0 is the positive, renormalized, anomalous diffusion coefficient, representing the effective macroscopic damping at the stable, dissipative fixed point (post-saturation), and where we neglect higher-order non-linear interaction terms. With Eq. (25), the full macroscopic equation of motion reads,

$$\left[\dot{A} - iA(\dot{\phi} - \mathbf{k} \cdot \mathbf{V}_d) \right] + \lambda_0 A = \mathcal{E}(\cos \Theta + i \sin \Theta). \quad (30)$$

where we defined the phase mismatch $\Theta(t) \equiv \phi(t) - \theta(t)$. Finally, Eq. (30) can be expressed more sensibly by recognizing
 360 that,

$$\dot{\phi} - \mathbf{k} \cdot \mathbf{V}_d = \dot{\Theta} + \dot{\theta} - \mathbf{k} \cdot \mathbf{V}_d \approx \dot{\Theta} - k(V_d - C_s), \quad (31)$$

which, when substituted this into Eq. (30), yields the final, macroscopic equation of motion,

$$\left[\dot{A} - iA \left(\dot{\Theta} - k(V_d - C_s) \right) \right] + \lambda_0 A = \mathcal{E}(\cos \Theta + i \sin \Theta). \quad (32)$$

(1) The imaginary part of Eq. (32) reveals the phase dynamics relative to the mean flow.

$$365 \quad -A \left[\dot{\Theta} - k(V_d - C_s) \right] = \mathcal{E} \sin \Theta, \quad (33)$$

which we can rearrange to isolate $\dot{\Theta}$, yielding the Adler equation for phase-locking phenomena, (Adler, 1946; Ambegaokar and Halperin, 1969; Wiesenfeld, 1996)

$$\dot{\Theta} = k(V_d - C_s) - \frac{\mathcal{E}}{A} \sin \Theta. \quad (34)$$

(2) Taking the real part of Eq. (32) yields,

$$370 \quad \dot{A} + \lambda_0 A = \mathcal{E} \cos \Theta, \quad (35)$$

which we can solve for the steady state by applying singular perturbation theory (Jones, 1995). ~~The characteristic lifetime of FB turbulence is~~ Three timescales must be kept apart here. The phase coherence of an individual 3-m FB wave is lost within milliseconds; the saturated ~~short~~ structures, on millisecond timescales (< 1 s), while the driver that carry the backscatter persist for $\tau_c \sim 1$ s, the empirical correlation time of the turbulence (Prikryl et al., 1988, 1990); and the magnetospheric driver
 375 envelope $\mathcal{E}$ varies over seconds, enabling us to write $\epsilon = \tau/T_{\text{driver}} \ll 1$ evolves over T_{driver} of order minutes. The separation relevant to the amplitude equation is $\epsilon = \tau_c/T_{\text{driver}} \ll 1$. In the limit $\epsilon \rightarrow 0$, the time derivative $\epsilon \dot{A}$ vanishes. The system collapses onto the slow manifold,

$$A(t) \approx \frac{\mathcal{E}}{\lambda_0} \cos \Theta(t). \quad (36)$$

To relate this to the radar observations, we ~~approximate the echo intensity~~ recall that n is the radar echo detection rate.
 380 Backscattered power and scatterer count coincide as measures of turbulence intensity because the per-structure amplitude is clamped at saturation: each saturated structure presents a fixed scattering cross-section, so the total backscattered power in the radar volume is proportional to the number of saturated structures it contains. The echo detection rate thus measures a filling factor, equivalently a number density, of saturated structures, and we may approximate it with the turbulence power,

or the time-averaged square of the amplitude, $n = \langle A^2 \rangle_t$. Squaring Eq. (36) and time-averaging over the rapid phase slippage (where $\langle \cos^2 \Theta \rangle = 1/2$), we find that the turbulence intensity becomes proportional to the square of the driver strength,

$$n \propto \frac{1}{2\lambda_0^2} \mathcal{E}^2. \quad (37)$$

Since the source term $\mathcal{E}^2$ is proportional to the input wave power ΔE , this ~~strictly~~ recovers the Adler-Ohmic regime observed in Figure 3c): the turbulence intensity ~~seals-is predicted to scale~~ linearly with the driving power, ~~confirming that~~ the complex microscopic physics ~~linearize-linearizing~~ in the macroscopic view of the system.

390 2.1.4 The cross-over scale

From Eq. (34), we define the phase-locking criterion,

$$|k(V_d - C_s)| \leq \frac{\mathcal{E}}{A}, \quad (38)$$

yielding the stability bandwidth. We derive the crossover-scale L_c separating the inertial drift regime from the macroscopic effective field theory. We utilize the slow-manifold solution to the amplitude equation, $A = \mathcal{E}/(\sqrt{2}\lambda_0)$. Substituting the renormalized diffusive linewidth $\lambda_0 = D_{\text{eff}}k^2$ into Eq. (38) yields the scale-dependent locking condition:

$$k(V_d - C_s) \leq \sqrt{2}D_{\text{eff}}k^2. \quad (39)$$

We solve for the critical wavenumber k_c , the minimum k required for the diffusive damping (right-hand-side) to overcome the linear drift drive (left-hand side):

$$k_c = \frac{V_d - C_s}{\sqrt{2}D_{\text{eff}}}, \quad (40)$$

400 corresponding to the cross-over scale for the RG flow,

$$L_c = \frac{2\sqrt{2}\pi D_{\text{eff}}}{V_d - C_s}. \quad (41)$$

This length scale demarcates the validity of the effective field theory. For small scales ($L < L_c$), the renormalized diffusion dominates, and the system is described by the macroscopic transport. In other words, the system's macroscopic correlation length is characteristic of a continuous phase transition, (Hohenberg and Halperin, 1977) diverging at $V_d = C_s$.

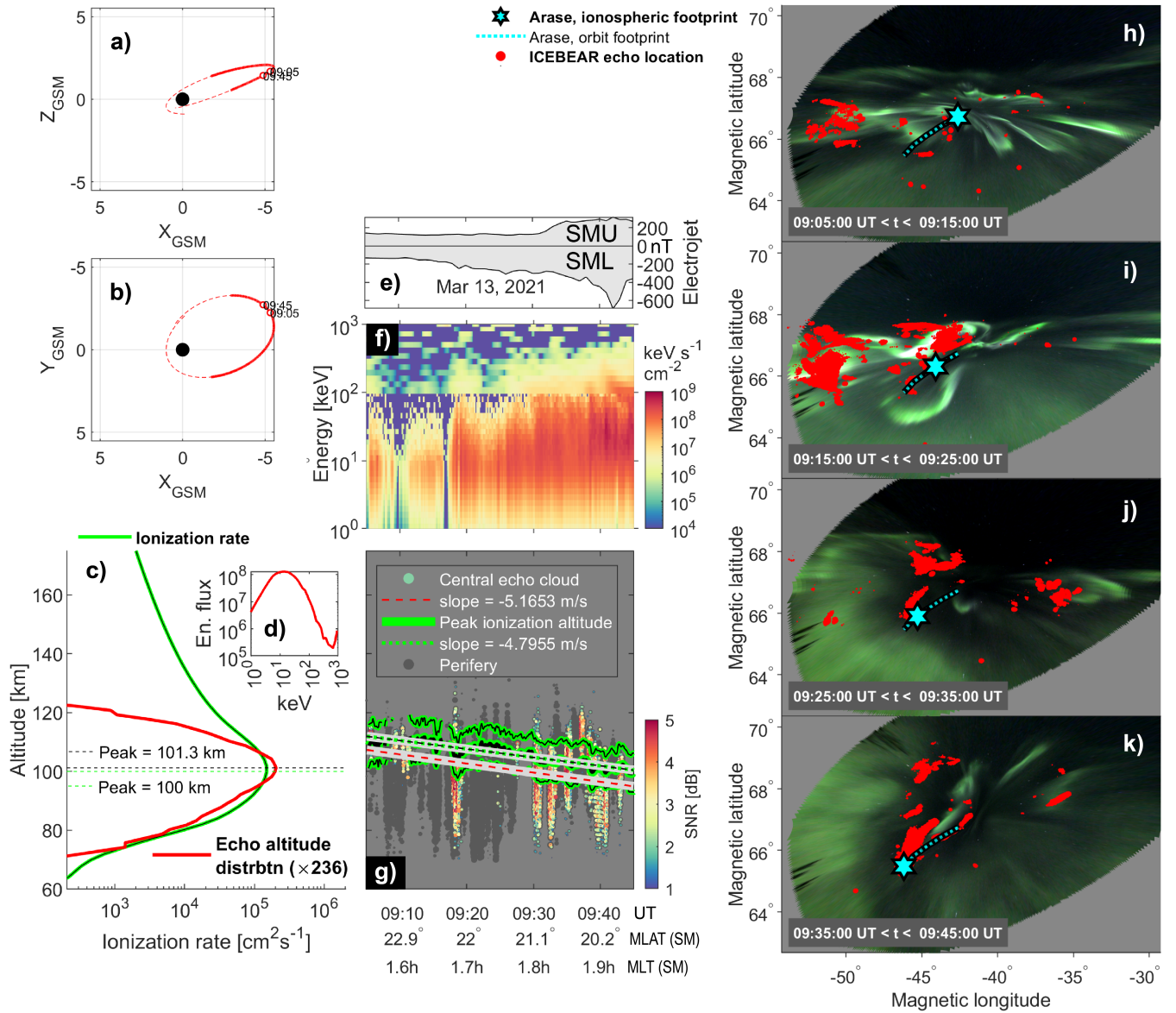


Figure 1. Panels a) and b) show Arase's position in the magnetosphere during the conjunction, in units of Earth radii. Panel c) compares the inferred ionization altitude profile from Arase (green), based on the accumulative precipitating energy flux through the interval, with the altitude distribution of radar echoes superposed (red line). Panel d) shows the median precipitating electron spectra, while Panel e) shows the upper and lower envelope of the auroral electrojet index. Panel f) shows the combined precipitating electron energy flux. Panel g) shows altitude-time-intensity point-clouds of echoes, color-coded by signal-to-noise ratio (SNR). Green and black lines indicate the peak ionization altitude (thick line) and a single standard deviation (thin lines). Echoes colored dark grey are observed outside of the radar's central field-of-view, whose altitudes are anomalous (Ivarsen et al., 2023). Panels g-j) show auroral images from the TREX RGB (Gillies et al., 2020) all-sky-imager at Rabbit Lake, with radar echo locations in red and Arase's ionospheric footprint as a cyan star.

In this section, we shall present results ~~form~~from case studies and from a large statistical aggregate, that directly support the foregoing description. First, we shall recount the nature of the particle precipitation found in the diffuse aurorae.

The kinetic energy of the particle precipitation in diffuse and pulsating auroral patches, created when wave-particle interactions leadsto pitch-angle scattering of hot electrons in Earth's radiation, or Van Allen, belts, (Thorne et al., 2010; Kasahara et al., 2018a) can be high. Significant precipitation is found even at energies larger than 30 keV (Nishimura et al., 2020), and Arase frequently observes significant particle fluxes approaching 1 MeV. This ionizes the ionosphere at low altitudes (Fang et al., 2010; Miyoshi et al., 2015, 2021), where the FB instability may become modulated by ohmic wave-heating (Dimant and Sudan, 1997; St.-Maurice and Chau, 2016) and the electron thermal instability (Dimant and Oppenheim, 2004; Dimant, 2024), as well as conventional FB waves, meaning electrojet turbulence will effectively accompany particle precipitation throughout the E-region; this is the radio, or radar, aurora.

Figure 1 shows measurements that confirm the foregoing, performed during the onset of a magnetospheric substorm, which was captured by a conjunction between the 3D VHF radar ICEBEAR and the inner-magnetosphere spacecraft Arase (Miyoshi et al., 2018a, b). Notably, during this ~~direct conjunction lasting 40-minute interval~~40-minute direct conjunction, we observe a clear and unequivocal relation between the estimated precipitating particle flux and the spatiotemporal distribution of auroral plasma turbulence.

We combine the electron particle detector data through the LEP-e, (Kazama et al., 2017) MEP-e, (Kasahara et al., 2018b) and HEP (Mitani et al., 2018) instruments onboard Arase, and we constrain the data to pitch-angles lower than 5° (LEP-e, MEP-e) and 10° (HEP), which we take as a proxy for the real precipitating energy flux. We convert this flux to an ionization rate altitude profile following Ivarsen (2026), and we compare this ionization altitude profile to the altitude-distribution of VHF backscatter turbulence echoes, ignoring echoes with azimuths $< 0^\circ$ and $> 20^\circ$, whose altitudes are anomalous (Ivarsen et al., 2023). Remarkably, throughout the 40-minute interval, the peak altitude from the two datasets match, and exhibit a very similar downward trend (Figure 1d, f), with the peak of the turbulent layer (as well as the peak impact altitude) decreasing at a rate of around 5 m/s, or 13 km through the 40-minute interval. This sinking of the ionosphere is caused by the onset of a moderate magnetospheric substorm (Newell and Gjerloev, 2011b), evidenced by the SuperMAG electrojet (SME) index, (Newell and Gjerloev, 2011a), shown in Figure 1e).

The event in Figure 1 featured a rare direct conjunction, of which there are few [and we direct the reader towards a notable such direct conjunction in Ivarsen et al. (2025a)]. Figure 2 shows an indirect conjunction, where Arase's ionospheric footprint was situated a few hundred kilometers west of ICEBEAR's field-of-view, akin to Ivarsen et al. (2025c), and following that study, we shift the Arase observations backward in time, finding that ICEBEAR observes the same dynamics 3 minutes and 20 seconds after Arase. However, whereas the previous conjunction event took place when Arase was situated in the generator region of the auroral magnetosphere (some 20° raised from magnetospheric equator), the conjunction event in Figure 2 took place when Arase was situated *inside the radiation belts*, $< 5^\circ$ from magnetospheric equator, some five Earth radii distant from

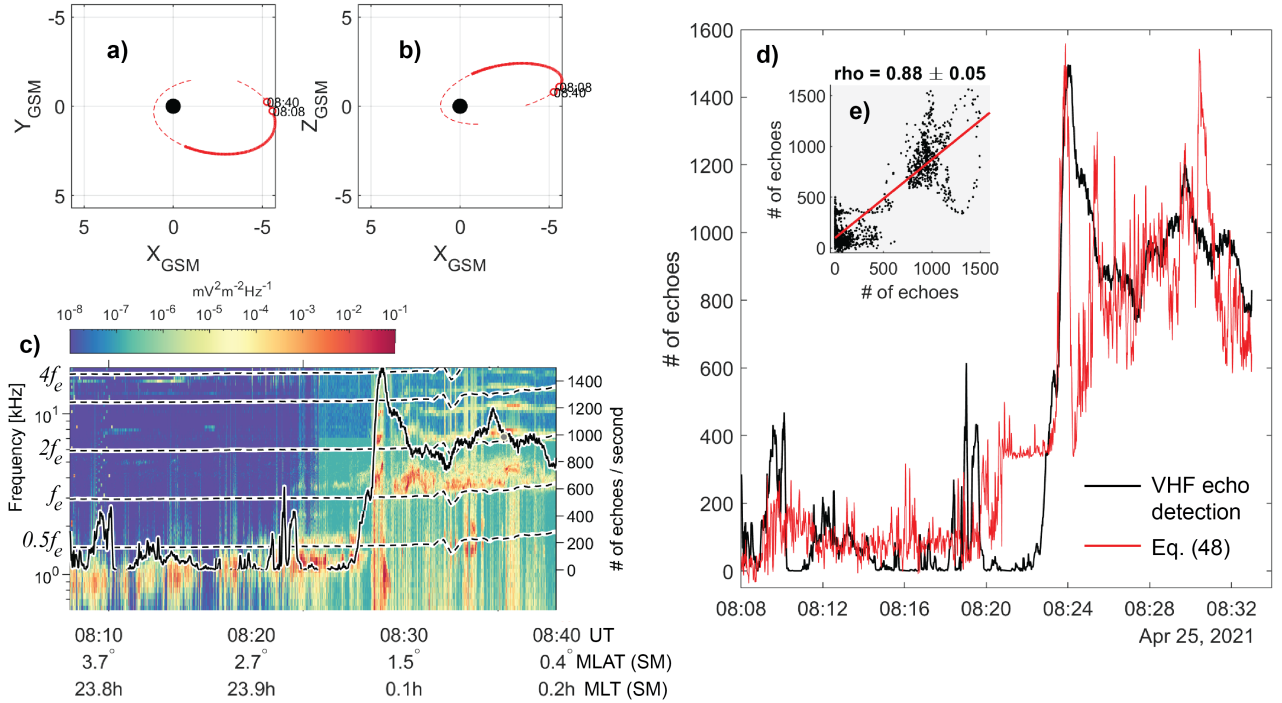


Figure 2. Panels a) and b) show Arase's position in the magnetosphere during the conjunction, in units of Earth radii. Panel c) shows the electric field wave power measured by the PWE instrument (Kasahara et al., 2018c), as a spectrogram, with frequency along the y -axis and time along the x -axis. Multiples of the local electron cyclotron frequency f_e [measured using the MGF instrument (Matsuoka et al., 2018)] is shown in dashed black lines, identifying four frequency bands. The rate of observed VHF echoes is superposed in black, and the Arase-data is shifted 3 m 20 s backwards in time. Panel d) shows the multivariable linear regression model (Eq. 42) in red, compared with the observed echo detection rate in black; the two timeseries have a resolution of 1 s and are highly correlated, with a linear regression analysis shown in the inset Panel e) (Pearson correlation coefficient ρ is indicated at 0.88 ± 0.05 , with error margin given by the 95% confidence interval of the linear regression, or 3σ). The model Eq. (42) defines the response variable as the radar echo rate and the predictor variables as the base-10 logarithms of the wave power integrated across the four distinct frequency bands on display in Panel c); see Figure A1 in the Appendix for details of the frequency-dependent power, as well as the geospatial locations of the instruments.

the E-region. Such conjugate data contains both the driver (plasma wave interactions with electrons, observed by Arase) and the resulting turbulence (FB waves observed by ICEBEAR), and the two can evolve in lockstep (Ivarsen et al., 2025a).

440 We observe such lockstep evolution in Figure 2, with a tight correlation, albeit with a 3 minute 20 second lag. To demonstrate this lockstep evolution, we have implemented a constrained multivariable linear regression, to model the occurrence rate of ionospheric VHF radar echoes, $\hat{n}(t)$, as a function of magnetospheric wave power inside certain frequency bands, building on Ivarsen et al. (2025a),

$$\hat{n}(t) = \sum_{i=1}^4 [\beta_{2i-1} + \beta_{2i} \log_{10}(x_i(t - \tau_i))], \quad (42)$$

where β_{2i-1} and β_{2i} are intercept and slope respectively, for the four frequency bands, with x_i being the magnetospheric wave power (root-mean-square) for the i^{th} frequency band on display in Figure 2c). As we detail in Appendix A, highest correlation is found in the $f_e < f < 2f_e$ band, characteristic of electrostatic cyclotron harmonic waves. τ_i is an additional lag (deviating from the stated 3 m 20 s); and we observe that the first frequency interval, which corresponds to the lower-band chorus waves with $0.1f_e < f < 0.5f_e$, is shifted an additional 20 s backwards, whereas the three other frequency bands are best correlated at the stated lag of 3 m 20 s. To estimate the eight β -parameters in Eq. (42), we apply a constrained optimization routine (MATLAB's `fmincon` algorithm), which minimizes the mean squared error between the observed n and predicted $\hat{n}$. The parameter space is strictly bounded to enforce non-negativity across all slopes and intercepts.

The analysis reveals a striking Pearson correlation coefficient between n and $\hat{n}$ of $\rho = 0.88 \pm 0.05$. This result echoes several recent investigations that all demonstrate that auroral turbulence in the E-region can be used as proxy observations for magnetospheric wave processes, and vice versa (Ivarsen et al., 2024a; Shen et al., 2024; Ivarsen et al., 2025a, b). This proxy status is explained by the theoretical foundation built in Sections 1 and 2: any excess driving electric field that surpasses ~ 20 mV/m is near-instantly counteracted by the saturation of the Farley-Buneman instability, forcing radar observations of the latter to evolve in close accordance with the magnetospheric driving signal, thereby leading to directly driven small-scale plasma turbulence without hysteresis or inertia. Here, it is appropriate to note what the correlation alone can and cannot establish: simple covariance with a common geomagnetic driver is not excluded by the correlation coefficient itself, and the directional reading rests on the lag structure, in which the ionospheric response trails the magnetospheric signal by 3 m 20 s with the lower-band chorus channel shifted a further 20 s; a common large-scale driver would impose no such frequency-ordered delay.

3.1 Statistical aggregate

Conjunctions like those shown in Figures 1 and 2 are rare. However, a considerable database of observations have been amassed for both ICEBEAR and Arase, and we can discern clear trends by comparing observations that were made within the same region of geospace; i.e., the nightside auroral oval, during a wide variety of geomagnetic conditions. For the four years between 2020 and 2023 inclusive, we have collected 1.2 million one-second intervals of VHF echo detection, corresponding to the identification of a total of 272 million echoes, and we compare this database with the concurrent observation by Arase of some 530,000 electric wave-spectra, observed when Arase was within 5° of magnetospheric equator, on nights during which ICEBEAR was operational.

The wide variety of conditions that the foregoing datasets cover allows us to statistically describe the energy transfer of ionosphere-magnetosphere coupling during such observations of intense auroral plasma turbulence. In Figure 3 we show median values of the VHF radar echo detection rate n and the total integrated wave power ΔE , for 15 geomagnetic activity bins [see the Appendix B in Ivarsen et al. (2025a) for details about this dataset]. In Figure 3a), we observe a distinct elbow in the curve at an auroral electrojet value of around 150 nT, below which the echo detection rate is flat (meteor trail echoes (Oppenheim et al., 2003; Hussey et al., 2000; Ivarsen et al., 2023)). For bins above this elbow, the bin-median echo detection rate is near-perfectly correlated with the auroral electrojets ($\rho = 0.99$), and at the same threshold, we observe that the wave power locks into a consistent scaling with the electrojet index ($\rho = 0.95$). We label this elbow with the critical wavepower ΔE_c .

When the magnetospheric wave power exceeds this threshold, the two quantities exhibit a similar response to enhancements in the electrojets, facilitating the linear ($\beta = 0.98 \pm 0.09$) relationship shown in Figure 3c). The raw fit slopes in Figures 3a) and b), 1.64 and 2.53 against the electrojet index, should not be combined into a naive prediction for β : the meteor echo floor suppresses the slope in panel a) at low activity, and the fit in panel c) applies only to the branch above ΔE_c , where both quantities respond to the same driving.

4 Discussion

~~Penetrating electric fields produced by Hall currents and corresponding energetic particle precipitation carry Alfvén waves to the upper atmosphere, the ionospheric E-region. Here, impedance matching and its effect on Joule heating (as well as turbulent heating), allow for a self-organized criticality to emerge in the interplay between the driver and the immediately responding FB turbulence. This immediacy, or ephemeral quality, is accompanied by a reduction in electric field (inside the turbulent regions) to the ion-acoustic speed, which limits the amplitude of FB waves—but their numbers. Any response beyond the saturation to the ion sound speed is immediately counter-acted by polarization electric fields, whose multitude we have analyzed in the present study. In a companion paper CITE COMPOSITE SPECTRA PAPER HERE, we provide evidence for a persistent kinetic Alfvén signature in composite radar-GNSS measurements of auroral plasma turbulence.~~

~~In the present paper, the~~ The statistical aggregates in Figure 3 clearly identifies a linear, statistical scaling between the packing of FB waves within the auroral electrojets inside ICEBEAR’s observational volume and the driving wave power observed near the radiation belts at conjugate locations. The specific physical mechanism enforcing this linear scaling between the driver and the proliferation of FB turbulence ($\beta \approx 1$) can be derived from the non-linear evolution of finite plasma structures. Following St.-Maurice and Hamza (2001), we consider that an initial density perturbation evolves as a finite structure that generates internal polarization electric fields. These fields oppose the driver, effectively reducing the internal electric field until the drifts barely exceed the threshold condition (C_s). ~~(Oppenheim and Dimant, 2013)~~ (Oppenheim and Dimant, 2013)

~~This feedback mechanism explains the pervasive observation of phase velocities saturated near the ion-acoustic speed (Foster and Erickson, 2000; Oppenheim and Dimant, 2013)~~ (Foster and Erickson, 2000; Koustov et al., 2005; Oppenheim and Dimant, 2013). Crucially, ~~the~~ because the per-structure amplitude is clamped at saturation, the aggregate intensity of the turbulence is determined by the density of these structures within the unstable volume: the echo detection rate n counts saturated structures rather than measuring their individual amplitudes. We posit a space-time equivalence where the number of active scatterers, n , is proportional to the linear growth rate γ_{lin} . ~~(Drexler et al., 2002)~~ (Drexler et al., 2002). Since the fluid growth rate scales as $\gamma_{\text{lin}} \propto (V_d^2 - C_s^2)$, and the drift velocity V_d is proportional to the external electric field E , the strongly driven regime ($V_d^2 \gg C_s^2$) yields a scatterer density $n \propto E^2$ (see Section 2.1.3). The linear response is therefore a direct consequence of the instability actively populating the volume in proportion to the available free energy.

A rigorous ~~The~~ measured exponent $\beta = 0.98 \pm 0.09$ (Figure 3c) therefore discriminates between two saturation scenarios. If amplitudes are clamped and the driver populates the volume with more saturated structures, the echo rate tracks the number density and the response in wave power is linear, $\beta = 1$. If, instead, amplitudes were free to grow with the driver,

Panel a) aggregates the ICEBEAR echo detection rate, for some 1.2 million one-second intervals (containing a total of 272 million echoes), while **Panel b)** aggregates Arase-observed wave power (integrated wave spectra for frequencies $f > 0.1f_e$, f_e being the electron cyclotron frequency) for some 530,000 one-second wave spectra, measured using the Plasma Wave Experiment on board Arase (Kasahara et al., 2018c), using measurements from when the satellites was within 5° off magnetospheric equator, and during magnetic local times (Baker and Wing, 1989) between 22h and 06h, with data collected on days when the radar was operational (between January 2020 and June 2023). In both panels c) and f), red triangles represent the median value inside 15 logarithmically spaced geomagnetic activity bins (using the SME-index, (Newell and Gjerloev, 2011a) or auroral electrojet index). Vertical errorbars denote upper/lower quartile distributions. Linear fits are indicated, by non-linear least squares minimization of the root-mean-square error. **Panel c)** shows the same 15 bins in a scatterplot, with a black dashed line denoting $n \propto \Delta E^\beta$ (see Eq. 47), with $\beta = 0$ when $\Delta E < \Delta E_c$ and $\beta = 0.98 \pm 0.09$ otherwise, determined through non-linear least squares minimization, where $\Delta E_c \sim 10^{-4} \text{ mV}^2\text{m}^{-2}$ represents the critical wave power. Statistical error margin is posted showing 95-percent confidence intervals for β (3-sigma), using a Monte Carlo-based bootstrapping routine with 500 iterations and a randomized data selection for each iteration.

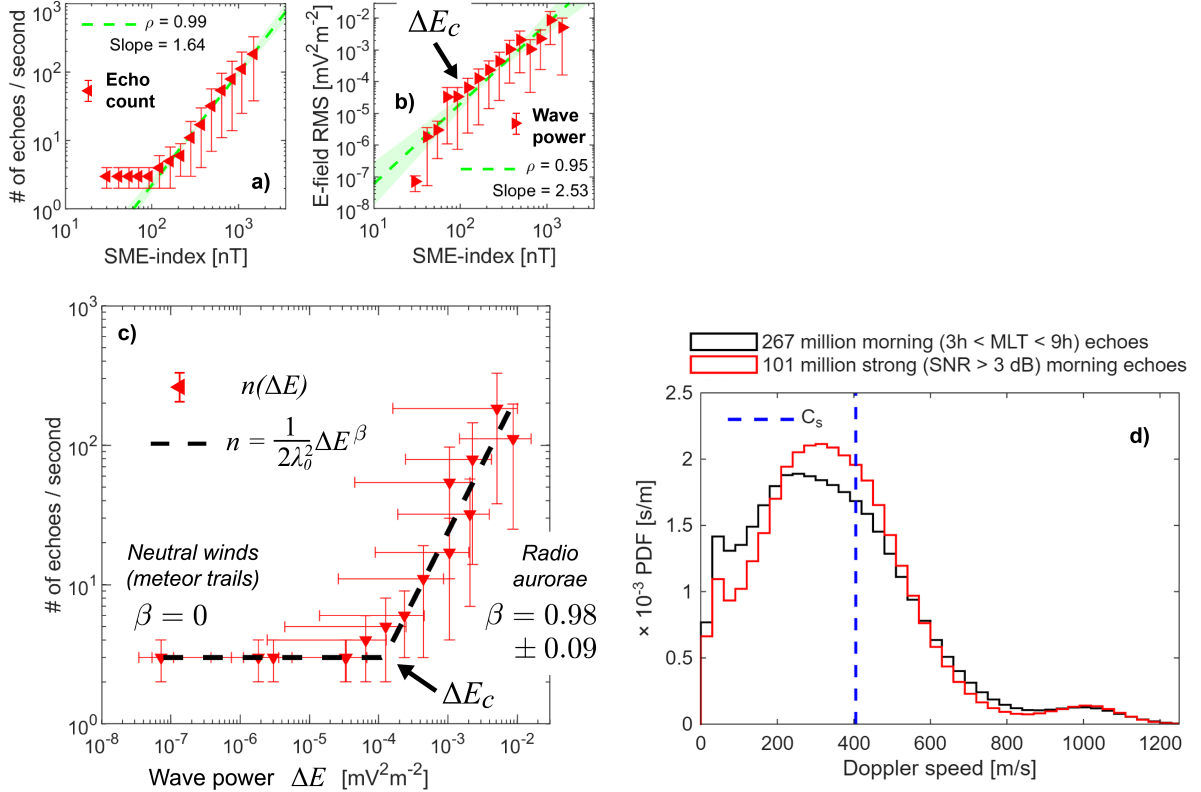


Figure 3. **Panel a)** aggregates the ICEBEAR echo detection rate, for some 1.2 million one-second intervals (containing a total of 272 million echoes), while **Panel b)** aggregates Arase-observed wave power (integrated wave spectra for frequencies $f > 0.1f_e$, f_e being the electron cyclotron frequency) for some 530,000 one-second wave spectra, measured using the Plasma Wave Experiment on board Arase (Kasahara et al., 2018c), using measurements from when the satellites was within 5° off magnetospheric equator, and during magnetic local times (Baker and Wing, 1989) between 22h and 06h, with data collected on days when the radar was operational (between January 2020 and June 2023). In both panels a) and b), red triangles represent the median value inside 15 logarithmically spaced geomagnetic activity bins (using the SME-index, (Newell and Gjerloev, 2011a) or auroral electrojet index). Vertical errorbars denote upper/lower quartile distributions. Linear fits are indicated, by non-linear least squares minimization of the root-mean-square error. **Panel c)** shows the same 15 bins in a scatterplot, with a black dashed line denoting $n \propto \Delta E^\beta$ (see Eq. 47), with $\beta = 0$ when $\Delta E < \Delta E_c$ and $\beta = 0.98 \pm 0.09$ otherwise, determined through non-linear least squares minimization, where $\Delta E_c \sim 10^{-4} \text{ mV}^2\text{m}^{-2}$ represents the critical wave power. **Panel d)** shows the probability density of Doppler speeds for morning-sector ($3 < \text{MLT} < 9$) ICEBEAR echoes observed between January 2020 and January 2025, an interval extending beyond the Arase-concurrent period of panels a)–c): all 267 million detections ($\text{SNR} > 1 \text{ dB}$) in black, and the 101 million strong detections ($\text{SNR} > 3 \text{ dB}$) in red. The blue dashed line marks the isothermal ion acoustic speed $C_s = \sqrt{k_B(T_e + T_i)/m_i} \approx 405 \text{ m/s}$ (for $T_e + T_i = 600 \text{ K}$ and $m_i = 30.5 \text{ u}$); line-of-sight projection places the distribution mode below this edge. The clustering of Doppler speeds near C_s , robust to the doubled detection threshold, is the saturation signature reported across the literature (Foster and Erickson, 2000; Koustov et al., 2005; Oppenheim and Dimant, 2013; Chau and St.-Maurice, 2016); the small population near 1000 m/s consists of weakly turbulent, narrow-spectrum type IV echoes (St-Maurice et al., 2023). The error margin

the backscattered power would compound the growth of both number and amplitude, and the response in wave power would be superlinear. The observed linearity is thus a necessary consequence of amplitude-clamped proliferation, satisfied here to within the fit uncertainty. The reduced Monte-Carlo of Appendix E quantifies this discrimination: the clamped model yields $\beta = 1.01$, the unclamped alternative $\beta = 2.08$ (Figure E1). We stress that the test is necessary rather than sufficient: linearity is consistent with the proposed causal chain from magnetospheric wave power to structure count, but does not by itself exclude other routes to a linear response.

An application of renormalization group theory (Wilson, 1971) completes the foregoing phenomenological description (see Section 2.1 and Appendix D). By reconstructing an action in the path-integral formulation, working backwards from the dispersion relation Eq. (1), we apply a spacetime rescaling ($x' \rightarrow b^{-1}x$), after which the inertial operator responsible for acceleration becomes irrelevant, decaying as b^{-2} at macroscopic scales. Consequently, the system flows inevitably toward a stable, *overdamped* fixed point (dynamic exponent $z = 2$).

The overdamped equations of motion define an effective field theory with an inferred cutoff (correlation scale) given by (see Section 2.1.4):

$$L_c = \frac{2\sqrt{2\pi}D_{\text{eff}}}{V_d - C_s}, \quad (43)$$

where V_d is the saturated drift. L_c features a divergence at the threshold (C_s), characteristic of a continuous phase transition (Hohenberg and Halperin, 1977), but the excess drift saturates at around $1.1C_s$ inside the turbulent regions (Hamza and St-Maurice, 1993; Oppenheim and Dimant, 2013), keeping Eq. (43) finite.

The turbulent amplitude field $\psi = \delta N_e / N_{e,0}$ is governed by an effective transport equation. Here, $V_d > C_s$ triggers turbulence growth via negative diffusion. (Ashinsky, 1988) The net stability condition decomposes into a driving term (source) and a saturation term (sink) (Eq. 21):

$$D_{\parallel} \approx \gamma_{\text{eff}}^{-1} [(C_s^2 - V_{d0}^2) + 2\tilde{\alpha}V_{d0}^2|\psi|^2]. \quad (44)$$

Marginal stability implies a balance where the net diffusion vanishes ($D_{\parallel} \rightarrow 0$, Eq. C17 of Appendix C), meaning the anomalous saturation term must exactly cancel the unstable driver. Physically, this corresponds to an anomalous electric field,

$$E_{\text{anom}} \approx (V_{d0} - C_s)B, \quad (45)$$

that acts as a macroscopic drag force. This drag underpins the resistive non-linear currents (Dimant et al., 2021) and wave-heating (St.-Maurice and Chau, 2016) observed *in-situ* and in kinetic simulations (Appendix E). Figure 3c) offers clues as to how this drag manifests in the dynamics. The observed linear relationship $n \propto \Delta E$ implies that the *number density of turbulent waves* obeys Ohm's law as a constant-resistance load. This validates the definition of an anomalous resistivity $\eta \equiv E_{\text{anom}}/J$, which scales as $\eta \approx (B/n_e e)(1 - C_s/V_{d0})$ given the free energy source $J = n_e e V_{d0}$. To close the macroscopic transport model, we consider the generalized Einstein relation for non-equilibrium steady states, (Harada and Sasa, 2005; Loi et al., 2011)

$D_{\text{eff}} \approx \mu k_B T_{\text{eff}}$, where k_B is the Boltzmann constant. ~~Since the turbulent fluctuations are spectrally limited by the thermal saturation scale (C_s), we~~ The choice of effective temperature is constrained by the saturation physics: the fluctuation velocities are clamped at the ion acoustic speed, and since $C_s^2 = k_B(T_e + T_i)/m_i$, the energy scale of the saturated fluctuations is pinned to the ambient temperatures, making T_{eff} of order T_e the natural closure. We adopt $T_{\text{eff}} \approx T_e$ and obtain $D_{\text{eff}} \approx T_e/n_e e^2 \eta$. For the density fluctuations ψ , this yields the effective diffusion coefficient,

$$D_{\parallel} = \frac{T_e}{eB} \left(1 - \frac{C_s}{V_{d0}}\right)^{-1} \xrightarrow{V_{d0} \gg C_s} \frac{T_e}{eB}. \quad (46)$$

Thus, the friction generated by the electron slip stream is identified as the anomalous transport coefficient ‘*Bohm*’, (Kaufman, 1990; Treumann and Baumjohann, 1997; Burch et al., 2016; Braginskii, 1965; Ott and Bonitz, 2011) a phenomenological staple in plasma studies.

Here, we note the following. (1) we use the convention $D_B = T_e/eB$, which evaluates to approximately 2000 m²/s for $T_e \sim 0.1$ eV and $B \approx 5 \times 10^{-5}$ T; the classical Bohm convention carries an additional factor 1/16 and gives approximately 125 m²/s. The closure claims to recover the order of magnitude and not the phenomenological prefactor. (2) the correspondence with Bohm is structural: $T_e/eB = v_{\text{th},e} \rho_e$, a thermal speed multiplied by a gyroradius, is a mixing-length coefficient, and the eddy diffusivity of the micro-field ensemble, a fluctuation velocity multiplied by a structure scale, has the same form ($D_{\parallel}^v \sim \delta v_{\parallel} \ell_c$, Eq. C15). That two independent routes, the Einstein relation above and the mixing-length closure of Appendix C, converge on the same Bohm-order magnitude is a mutual consistency check. (3) the identification is testable against the Doppler distribution itself: since C_s scales with $\sqrt{T_e + T_i}$, the saturated Doppler speed acts akin to a thermometer, and the Doppler speeds of the fully turbulent population under strong driving reach approximately 700 m/s, matching the heated ion acoustic speed near 105 km altitude (St-Maurice et al., 2023): the clamp migrates with the electron temperature, as the $T_{\text{eff}} \approx T_e$ closure implies. The faster population near 1000 m/s in Figure 3d is of a different character: weakly turbulent, narrow-spectrum eigenmodes excited near threshold on the edge of the instability cone, whose Doppler shifts acquire an ion-drift contribution above 110 km and can approach the $\mathbf{E} \times \mathbf{B}$ drift (St-Maurice et al., 2023). These modes lie outside the saturated ensemble that the effective theory coarse-grains; their coexistence with the clamped bulk delimits the theory’s domain of validity.

We note that while equating the effective noise temperature to the ambient electron temperature provides a mathematically elegant closure to recover macroscopic Bohm diffusion, it should be treated as an idealized phenomenological parameterization rather than a rigorously derived kinetic certainty. A further robustness property is that the mixing-length value $D_{\parallel}^v \sim \delta v_{\parallel} \ell_c$ does not depend on the structure lifetime τ_c , the most uncertain parameter: the Kubo analysis of Appendix C places the physical regime ($Ku \gtrsim 1$) on the τ_c -independent plateau of the two-branch diffusivity, a result verified by direct tracer dispersion in the reduced Monte-Carlo (Appendix E).

We can use Eq. (46) to demarcate the field theory’s area of validity. We write $L_c \approx (2\sqrt{2}\pi/0.1C_s)(T_e/eB)$. With $T_e \sim 0.1$ eV and $B \sim 50,000$ nT, and, for somewhat active conditions, $C_s = 500$ m/s, we obtain $L_c \sim 350$ m; C_s varies with the ambient ion and electron temperatures, and L_c shifts with it. For scales $L \lesssim 350$ m, the effective field theory is valid, and the stochastic sum total of polarization electric fields renormalizes into Bohm resistivity in the electron slip stream, in the Hall

575 currents. Because the electron fluid is in a critical state of thermal saturation, the anomalous response field E_{anom} is near-instantly generated to maintain marginal stability, forcing the electron transport to strictly track the spatio-temporal structure of the magnetospheric driver (V_{d0}).

We next follow our effective field theory to its logical conclusion. In the macroscopic limit, the phase dynamics of the overdamped turbulent waves reduce to the Adler equation for synchronization phenomena (Adler, 1946). Consequently, we
580 recover the observed linear response (Eq. 37),

$$n \propto \frac{1}{2\lambda_0^2} \Delta E, \quad (47)$$

where λ_0 is the effective damping at the dissipative fixed point, which retains the Bohm scaling, $\lambda_0 = D_{\text{eff}} k^2 = (4\pi^2/9)(T_e/eB)$ at the radar wavelength. This response matches the empirical result (Figure 3c), ~~allowing us to identify~~ inviting an identification of the system’s behavior with the current-voltage (“I-V”) characteristics of an overdamped Josephson junction. (Stewart, 1968;
585 Ambegaokar and Halperin, 1969; Wiesenfeld et al., 1996) In this analogy, the transition from a static ‘locked’ state to a phase-slipping ‘running’ state corresponds to the onset of the electron slip stream ($V_{d0} > C_s$). Crucially, the response in Figure 3c) exhibits the signature of a *noise-broadened bifurcation* (Ambegaokar and Halperin, 1969; Danner et al., 2021), ~~confirming that~~ consistent with the onset of the ‘running’ state, $V_d > C_s$, ~~is acting as~~ a critical point ~~created by the renormalized~~ broadened by the *noise* of the ensemble’s saturation electric fields.

590 This analogy is testable, because the phase variable is measurable: the Doppler velocity of the backscattered waves, taken relative to the driver, plays the role of the junction phase. The synchronization interpretation requires the running state to carry Doppler velocities clamped near the ion acoustic speed, with phase slippage confined to driving above ΔE_c ; a Doppler distribution that failed to organize around C_s above the threshold, or that showed the same organization below it, would falsify the interpretation. Figure 3d presents the above-threshold half of this diagnostic: the aggregate Doppler distribution
595 organizes around C_s and holds its shape when the detection threshold is doubled, so the organization is a property of the saturated waves rather than of detectability. The below-threshold half, conditioning the same distribution on $\Delta E < \Delta E_c$ over the Arase-concurrent subset, remains an explicit test. Three further diagnostics, addressable by existing kinetic simulation groups, are stated in Appendix E: the Gaussianity of binned PIC output, a drive-level scan of the response exponent, and second-scale structure-lifetime statistics.

600 The Adler-Ohmic bifurcation at ΔE_c denotes a characteristic behaviour in physical systems where oscillating constituents create a voltage drop against an effective tilted washboard potential, one that facilitates a ‘running’, or phase slipping, state, following a threshold of disorder. (Ambegaokar and Halperin, 1969) That we are able to observe this macroscopic description in the particular magnetosphere-ionosphere coupling created by Farley-Buneman waves ~~implies that~~ is consistent with the constituents of *that* system likewise ~~lock or slip~~ locking or slipping against an effective tilted washboard potential. Subsequently,
605 the magnetosphere-ionosphere coupling near intense diffuse aurorae is kept in a critical state sustained by efficient, turbulent energy dissipation, and its macroscopic description is ultimately governed by ‘Model A’ universal dynamics, (Hohenberg and

Halperin, 1977) a class of purely dissipative systems with dynamical exponent $z = 2$ (Appendix D); since ψ is a conserved density away from criticality, the label denotes the nearest Hohenberg-Halperin analogue rather than a strict assignment.

However, we highlight that the measurements that enabled this insight span *multiple systems* in geospace; the locked state in Figure 3c) is represented by meteor trail echoes, (Ivarsen et al., 2023) created when meteors burn up in the atmosphere. Friction creates intense, short-lived plasma turbulence (Oppenheim et al., 2003), and the backscatter echoes from this turbulence act as tracers for the neutral winds (Hall et al., 1997; Hussey et al., 2000; Kumar, 2007). ICEBEAR is capable of detecting and distinguishing meteor trail echoes (Ivarsen et al., 2023), but ~~When~~when parameterized by observed magnetospheric wavepower, the meteor trail turbulence shows no response, and therefore functions as a latent tracer for the turbulence within the system.

The association of Adler-Ohmic bifurcation at ΔE_c is therefore a characterization of *the auroral ionosphere as a system*, and specifically its turbulent response to external driving.

While the foregoing interpretation must be substantiated and elucidated by observations in closer detail, the immediate implications of our results can aid ongoing efforts. When saturation feedback effectively governs the system under study, the turbulent regions evolve without hysteresis, providing an explanation for observations of directly driven auroral plasma turbulence with zero or near-zero lag, (Shen et al., 2024; Ivarsen et al., 2025a; Brindley et al., 2025) as well as justification for the omission of hysteresis in global turbulence models (Wiltberger et al., 2017a).

Our field theory, however, represented by the derivation of complete, closed-form macroscopic transport relations (culminating in Eq. 46), offers a useful tool. It implies that, in two-stream plasmas, whenever the driving field attempts to push the drift velocity beyond the sound barrier, the system automatically generates Bohm-order resistivity to clamp the flow to the ion acoustic speed, providing strong justification for the application of Bohm diffusion in numerical models of the auroral ionosphere. The expenditure of Bohm diffusion is accomplished through saturation electric fields, in a stochastic ensemble. *The FB turbulence generates exactly enough noise, or effective temperature ($T_{\text{eff}} \approx T_e$) to assemble a thermal diffusion process at the acoustic scale.* This provides a useful constitutive relation for modeling anomalous currents and heating in global ionospheric simulations.

Lastly, we stress that, while interpreting the observed linear proliferation of turbulence in the auroral ionosphere system (Figure 3c) as an Adler-Ohmic bifurcation provides an exceptionally well-fitting statistical envelope for large-scale modeling, it remains highly uncertain whether the plasma structures genuinely achieve the concerted macroscopic phase-locking characteristic of microelectrical devices.

5 Conclusion

In this investigation, we ~~have pinpointed a clear emergent regime in auroral plasma turbulence, which we posit is produced by Alfvénic impedance matching establishing a dynamic equilibrium, or criticality. We explore the core mechanism in this emergent effect, namely the~~ explore the statistical effect of exciting thousands or millions of ~~FB-turbulent Farley-Buneman~~ (FB) waves inside a limited volume ~~inside of~~ the auroral electrojets.

~~We explore this mechanism~~ The exploration is performed in terms of a renormalization group (RG) theoretical treatment of
640 ~~Farley-Buneman~~ FB turbulence, as well as a massive dataset of observations by radar (final energy expenditure) and spacecraft
(driver). The dataset shows ~~clearly~~ that the driver and ~~expenditure produces~~ the expenditure produce a statistical “I-V” curve
~~that is the exact expectation of the form expected~~ from an interconnected array of Josephson junctions. This central result is
interpreted in one of two ways:

(1, *strong*) First, we treat the theoretical possibility that the striking emergent behaviour that we have described in Figure 3
645 leads directly to space weather changes through concerted fluctuations around locked frequencies akin to Josephson synchro-
nization in superconductivity theory. In other words, this is the interpretation of the curve in Figure 3c) that it *is* an I-V curve;
and this interpretation ~~follows directly from the renormalization performed~~ is motivated by the analysis in Section 2.1: switch-
ing between the quiescent and turbulent states ~~neecessitates that the~~ would then correspond to a space weather system ~~is~~ subject
to a tilted washboard potential (Ambegaokar and Halperin, 1969), opening up completely novel strands of theoretical investi-
650 gation of auroral plasma turbulence. Although justified by theory, and supported by statistical evidence, there is not sufficient
reason to claim that the parcels of turbulent plasma around aurorae function as interconnected Josephson junctions.

(2, *weak*) The second interpretation is practical, and concerns space weather modeling. Numerical, fluid solutions of the
magnetohydrodynamics around Earth are unable to resolve scales smaller than a certain boundary, dictated largely by reasons
of computational cost, referred to as a grid-cell. Below this scale, simulations must take both kinetic and fluid eventualities into
655 account, and are accordingly *very* computationally costly, the so-called hybrid fluid-kinetic solutions to the Farley-Buneman
instability (Oppenheim et al., 1995; Wiltberger et al., 2017b; Dimant and Sudan, 1995; Dimant et al., 2021). This is an instance
where space weather shares a known vulnerability with conventional hydrodynamic weather modeling of Earth’s atmosphere:
the nature and magnitude of the turbulent cascade remains unknown in space weather modeling for scales smaller than around
50-100 km, and must be evaluated using expensive numerical simulations.

660 In terrestrial climate science, resolving this grid-cell vulnerability has historically relied on parameterizing the sub-grid cas-
cade as effective diffusion (Smagorinsky, 1963), and more recently, through stochastic parameterization that treats unresolved
convective scales as noise (Palmer, 2012; Berner et al., 2017). In ionospheric physics, turbulence parameterization through eddy
diffusion has likewise been frequently applied in global models (Roble et al., 1988; Wu et al., 2017; Vaishnav et al., 2021), and
stochastic parameterization has been applied in magnetospheric modeling (Watt et al., 2022). By formalizing Farley-Buneman
665 micro-fields as a stochastic variable, ~~our application of RG theory (the closure and RG analysis of~~ Section 2.1) effectively
imports this probabilistic Earth-system framework into high-latitude electrodynamics, ~~offering explicitly renormalized~~ the
RG establishes the dissipative universality class and the irrelevance of the micro-field noise, and the closure supplies transport
coefficients in three dimensions that ~~elucidate the transport~~ describe the movement of momentum and energy “sub-grid.” We
emphasize ~~this last point~~; the three dimensions ~~entertained by our diffusion tensor mean that, unlike scalar eddy diffusion, our~~
670 ~~transport coefficients retain~~ the diffusion tensor retains the anisotropic structure of the instability, and ~~it its value~~ is determined
by specific saturation physics rather than prescribed ~~according to empiri~~ empirically.

Regardless of whether one subscribes to (1) or (2) above, we maintain that ~~, in presenting these results, we have shed crucial~~
~~light on these results~~ identify the potential for a specific novel process (described by non-linear synchronization theory).

This prospect must be further elucidated, but, at present, we suffice to provide the foundation for a concrete and productive
675 application of RG theory for the purpose of space weather modeling. We thereby conclude with the abridged summary that
kinetic processes in auroral plasma turbulence may act in apparent concert to produce a distinct synchronization ~~phenomena~~
phenomenon in the auroral ionosphere, one that is only observable in statistical aggregates, and is otherwise confined to the
physics of microelectrical, superconductive devices.

680 Appendix A: Cross-correlation and multivariable regression methodology

This appendix details the statistical framework and lag analysis utilized to correlate magnetospheric wave power with the occurrence rate of ionospheric Farley-Buneman turbulence, supporting the multivariable linear regression model presented in Eq. (42).

To account for the broadband nature of the magnetospheric ‘driving signal’, the electric field wave spectra observed by the
685 Arase spacecraft are partitioned into discrete frequency bands. These bands are defined relative to the local electron cyclotron frequency, f_e , allowing for the isolation of specific wave modes, such as lower-band chorus waves ($0.1f_e < f < 0.5f_e$) and electrostatic cyclotron harmonic waves ($f_e < f < 2f_e$), where we follow the approach of Ivarsen et al. (2025a).

Because wave-particle interactions, pitch-angle scattering, and subsequent precipitation of electrons along magnetic field lines introduce complex, frequency-dependent delays that adhere to a host of local physical processes, a rigorous cross-
690 correlation analysis is necessary to map the driving signal to the resulting ionospheric response. A baseline macroscopic propagation lag is established between the inner magnetosphere and the E-region—determined in this event to be 3 m 20 s. To refine this mapping, a sliding-window cross-correlation is applied independently across the spectral bands (as detailed in Figure A1e) to identify specific temporal deviations, τ_i , from the baseline lag (see Eq. 42).

With the optimal frequency-dependent lags established, the radar echo detection rate, $\hat{n}(t)$, is modeled as a linear combina-
695 tion of the lagged, logarithmically scaled wave powers within each frequency regime. A constrained optimization algorithm minimizes the mean squared error to extract the domain-specific slopes (β_{2i}) and intercepts (β_{2i-1}). The parameter space is strictly bounded to enforce non-negativity across all coefficients, reflecting the assumption that the high-latitude Farley-Buneman instability acts as a strictly driven, non-hysteretic dissipative sink for the external free energy. Finally, this temporal analysis inherently relies on an assumption of spatial coherence across the longitudinal and latitudinal separation between the
700 radar’s observational volume and the mapped ionospheric footprint of the spacecraft (Figure A1g).

We note with interest how the four red signals in Figure A1a–d) show distinct and, in places, strong correlation with the black signal, echoing Ivarsen et al. (2025a).

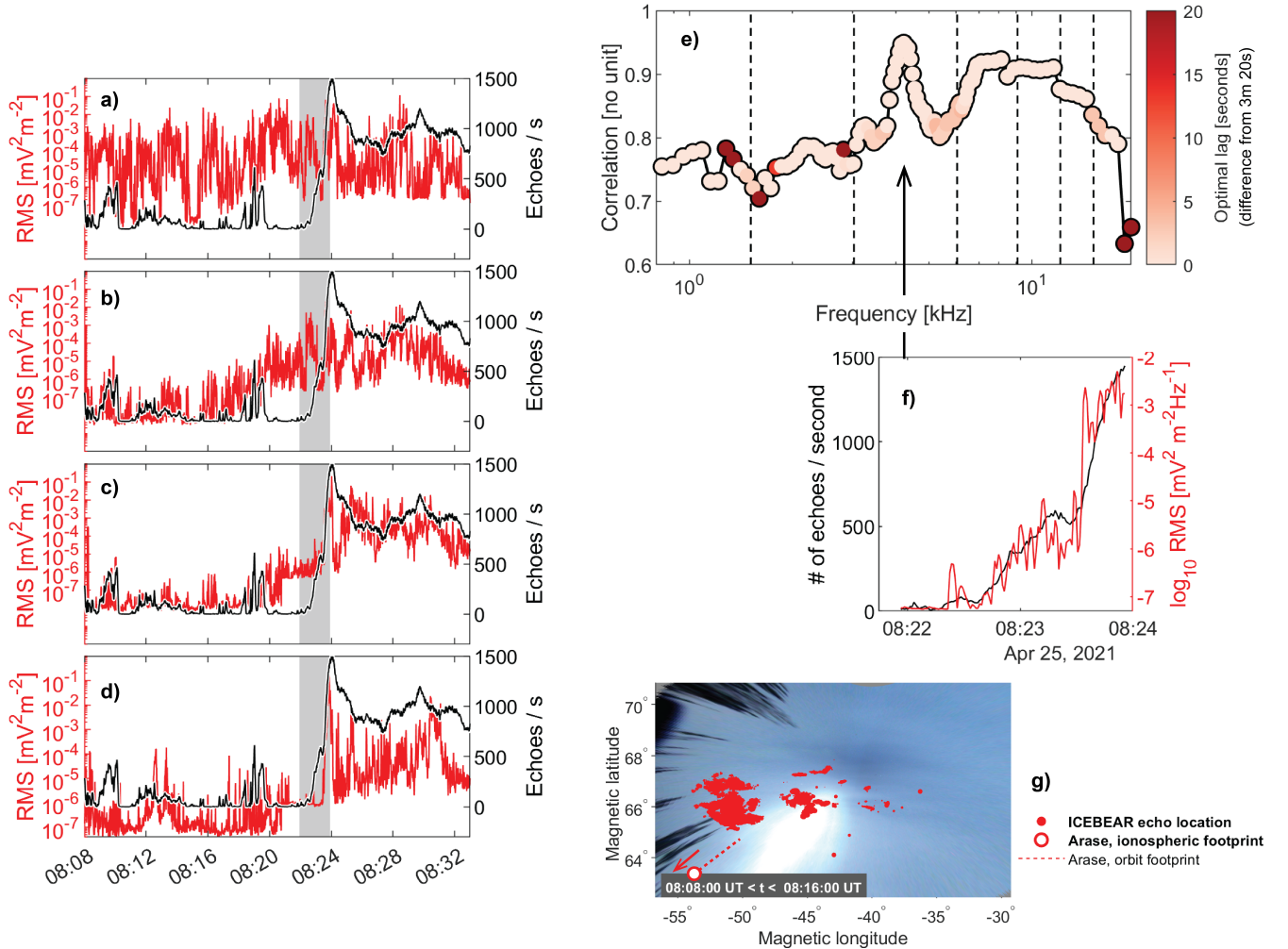


Figure A1. Additional information underlying the analysis in Figure 2 and Eq. (42). **Panels a–d** show, with solid red line, integrated power inside the four frequency bands shown in Figure 2c, with the VHF echo detection rate superposed (black). **Panel e** shows a cross-correlation analysis applied to each individual frequency, with a white-red colorscale denoting optimal lag (deviation from the stated 3 m 20 s). The point of highest correlation is the explosive onset of activity, which is shown in **Panel f**. **Panel g** shows the relative distances between observed VHF echo locations and the ionospheric footprint of Arase, superposed on a TREX RGB image that exhibits considerable light pollution from the moon (the white blob).

Appendix: ~~Appendix B: Phenomenological derivation of the linear scaling law~~

Appendix B: Phenomenological derivation of the linear scaling law

705 In this appendix, we shall present a qualitative explanation, as an alternative to our RG theory application, as to why the statistical aggregate of FB turbulence exhibits a linear scaling against magnetospheric wave power.

The mathematical substitution of the deterministic mode-coupling term ($\psi \nabla \psi$) with a stochastic noise term ($\delta \mathbf{v}$) constitutes random phase approximation, which formally lumps the turbulent cascade into a diffusion term. Methodologically, this ~~allows the macroscopic scale separation to be achieved entirely~~ closure achieves scale separation through the temporal coarse-graining of uncorrelated phase histories, ~~rather than relying on spatial momentum-shell integration~~; the spatial rescaling analysis of Section 2.1 then establishes the universality class, with no resummation required, since the noise coupling is irrelevant. By discarding the phase correlations required for three-wave interactions, this approach explicitly neglects the dynamic flux of energy across scales and treats the plasma as an ensemble of turbulent waves rather than a structured fluid capable of forming shocklets. This departure from a fluid description is validated by the two-stream origin of the instability: saturation is dominated by local electrostatic feedback (where polarization fields clamp the local drift to C_s), a notion that is firmly supported by Figure 3bd, as well as the state of electrojet turbulence research [see, e.g., ~~Foster and Erickson (2000); Oppenheim and Dimant (2013); Koustov et al. (2005)~~ Foster and Erickson (2000); Oppenheim and Dimant (2013); Koustov et al. (2005); Chau and St.-Maurice (2016); St.-Maurice et al. (2023)].

A phenomenological argument based on the foregoing recovers the central result (Eq. 37), by way of adiabatic elimination. On time scales longer than the initial growth time, the unstable structures will lose amplitude after zero growth is approached: Drexler et al. (2002) showed that the eigenfrequency is a weak function of space (altitude), which forces the introduction of steadily increasing parallel electric fields inside unstable structures. Such fields short-circuit the internal, perpendicular fields by passing parallel currents. (Oppenheim and Dimant, 2013) This, incidentally, gives the wave energy to electrons which in turn give it to the gas through inelastic collisions. (St-Maurice and Goodwin, 2021)

725 Assume, then, that inside an unstable volume, the faster the individual structures grow, the more of them there are. This introduces an equivalence between time and space: shorter growth times mean more structures. *The number of structures in an unstable volume is therefore proportional to the linear growth rate.* Using fluid theory approximation as a guide this means that the number of structures inside the unstable volume is given by,

$$n \propto a \frac{\Psi}{1 + \Psi} \frac{k^2}{\nu_i} (V_d^2 - C_s^2), \quad (\text{B1})$$

730 where a is a constant, ν_i the ion-neutral collision frequency, and V_d is the relative ion-electron drift, very close to E/B at 100 km altitude. Therefore, when V_d^2 is well above C_s^2 , the number of VHF radar backscatter targets becomes essentially proportional to the electric field squared, E^2 . With a dominance of the 500 to 600 m/s threshold speed observed at 100 km, E/B has to be exceeding 1500 m/s (see Figure 1 in St-Maurice et al. (2023), and the near proportionality to E^2 is justified.

Appendix C: The stochastic closure for the anomalous transport coefficient

735 This appendix derives the anomalous transport coefficient of the effective field theory as a stochastic closure: the ensemble of saturated Farley-Buneman polarization micro-fields is treated as a Gaussian advecting velocity field, and its effect on the coarse-grained turbulent density is the Taylor-Kubo (Bourret-Kraichnan) eddy diffusivity (e.g., Meunier et al., 2023), evaluated in the regime of strong-correlation (mixing-length, Bates et al., 2014). The construction is a second-cumulant (variance) closure of the averaged equation of motion. The closure fixes the magnitude of the transport coefficient, while the universality class, and the status of the noise coupling under coarse-graining, are established in Appendix D.

In the overdamped limit ($z = 2$, Appendix D) and with the saturation substitution of Eq. (18), the deterministic part of the MSR action, written so that the convective and diffusive operators share a common γ_{eff} normalization, reads,

$$S_0 = \int dt d^d x \tilde{\psi} \gamma_{\text{eff}} [(\partial_t + \mathbf{V}_d \cdot \nabla) - \nabla \cdot \mathbf{D}_{\text{eff}} \cdot \nabla] \psi, \quad (\text{C1})$$

745 where d is the spatial dimension. The effective diffusion tensor is that of Eq. (2), so that the units of γ_{eff} and $\mathbf{D}_{\text{eff}}$ adhere to $[\gamma_{\text{eff}}] = \text{T}^{-1}$ and $[\mathbf{D}_{\text{eff}}] = \text{L}^2 \text{T}^{-1}$. The stochastic vertex produced by the random phase approximation over the saturated micro-fields is that of Eq. (13),

$$S_{\text{int}} = \frac{i\sigma_v}{2} \gamma_{\text{eff}}^2 \int dt d^d x \left(\tilde{\psi} \nabla \psi \right)^2, \quad (\text{C2})$$

750 where σ_v is the variance of the stochastic drift fluctuation $\delta \mathbf{v}$ (units $\text{L}^2 \text{T}^{-2}$), constrained by the power of the turbulent electric fields through Eq. (8). Throughout, σ_v is a velocity variance, and the symbol $\mathbf{D}^v$ (units $\text{L}^2 \text{T}^{-1}$), to be introduced, is reserved for the diffusivity it generates.

From S_0 , with the Fourier convention $\psi(\mathbf{x}, t) = \int_{\mathbf{k}, \omega} \psi(\mathbf{k}, \omega) e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)}$, the inverse bare response propagator is

$$G_0^{-1}(\mathbf{k}, \omega) = \gamma_{\text{eff}} (-i\omega + i\mathbf{V}_d \cdot \mathbf{k} + \mathbf{k} \cdot \mathbf{D}_{\text{eff}} \cdot \mathbf{k}), \quad (\text{C3})$$

and in the frame comoving with $\mathbf{V}_d$ the advection is absorbed into the shift $\omega \rightarrow \omega - \mathbf{V}_d \cdot \mathbf{k}$,

$$G_0(\mathbf{k}, \omega) = \frac{1}{\gamma_{\text{eff}} (-i\omega + \mathbf{k} \cdot \mathbf{D}_{\text{eff}} \cdot \mathbf{k})}. \quad (\text{C4})$$

755 At tree level (the non-interacting, zero-loop approximation) there is no additive-noise $\tilde{\psi}\tilde{\psi}$ term: the noise is multiplicative, so the bare correlation function vanishes, $C_0(\mathbf{k}, \omega) = 0$. This structural fact controls what follows. Expanding $(\tilde{\psi} \nabla \psi)^2$ in Fourier space, the four-point vertex with ψ -leg momenta $\mathbf{k}_3, \mathbf{k}_4$ carries

$$\mathcal{V}(\mathbf{k}_3, \mathbf{k}_4) = i\sigma_v \gamma_{\text{eff}}^2 k_{3,i} k_{4,i} \quad (\text{C5})$$

760 together with overall momentum and frequency conservation. We retain Eqs. (C2) and (C5) as the field-theoretic representation of the averaged micro-field ensemble. The transport coefficient is, however, fixed by the closure below, not by a diagrammatic resummation: with $C_0 = 0$, the only one-loop self-energy (the perturbative correction to the propagator) is the equal-time tadpole (a diagram in which a single loop closes on itself), which vanishes identically in the Itô discretization convention (the causal prescription in which the noise acts before the response; see, e.g., Gardiner, 2009) (Eq. C10 below), so no nonvanishing self-energy loop is available to carry the renormalization. The physical effect of the multiplicative noise resides in the noise-induced
 765 drift, equivalently the Itô-Stratonovich shift, and that is what the cumulant closure computes.

Stochastic equation of motion and noise statistics. Before averaging, the density fluctuation obeys an advection-diffusion equation in which the total advecting velocity is the mean drift plus the fluctuating micro-field velocity,

$$\partial_t \psi + (\mathbf{V}_d + \delta \mathbf{v}) \cdot \nabla \psi = \nabla \cdot (\mathbf{D}_{\text{eff}} \cdot \nabla \psi) + S, \quad (\text{C6})$$

770 with $\delta \mathbf{v}$ the zero-mean Gaussian field of Eq. (8) and S the external source of Eq. (25), which drops out of the averaging below. Because the saturated polarization fields oppose the driver, the fluctuation is predominantly longitudinal (aligned with $\hat{\mathbf{V}}_d$):

$$\langle \delta v_i(\mathbf{x}, t) \delta v_j(\mathbf{x}', t') \rangle = \sigma_v \hat{V}_{d,i} \hat{V}_{d,j} C(t - t') g(\mathbf{x} - \mathbf{x}'), \quad C(0) = 1, \quad (\text{C7})$$

where $\sigma_v = \langle \delta v_{\parallel}^2 \rangle$, C is the normalized temporal autocorrelation, and g has finite spatial range ℓ_c , the energy-containing structure scale (tens of meters), distinct from the 3-m radar Bragg scale that the instrument samples.

775 The decorrelation time of the micro-field noise is the observed lifetime of a Farley-Buneman structure (Prikryl et al., 1988, 1990)

$$\tau_c \equiv \int_0^{\infty} C(\tau) d\tau \sim 1 \text{ s}, \quad (\text{C8})$$

an empirical quantity. The physical mechanism limiting structure lifetimes is the monotonic growth of the aspect angle with altitude (Drexler et al., 2002): because the eigenfrequency is a weak function of altitude, the field-aligned group velocity grows to ~ 10 km/s after a small increase in aspect angle (Moorcroft, 1984), and with the unstable E-region spanning ~ 10 – 20 km in
 780 altitude, structures disperse in ~ 1 – 2 s. The lifetime τ_c is not the damping time $\gamma_{\text{eff}}^{-1} \sim \text{ms}$ of the dispersion relation; the two differ by two to three orders of magnitude and play distinct roles. Two separations follow and must be kept apart: (i) relative to the magnetospheric driver, $\tau_c \sim 1 \text{ s} \ll T_{\text{driver}} \sim \text{minutes}$, so the mean-field closure below is well justified and the turbulence tracks the driver without hysteresis (the tracer property); (ii) at the structure scale, the Kubo number,

$$\text{Ku} \equiv \frac{\delta v_{\parallel} \tau_c}{\ell_c} \sim O(1)\text{--}10, \quad (\text{C9})$$

785 (with $\delta v_{\parallel} \sim 0.1\text{--}0.5 C_s$ and ℓ_c of tens of meters) measures the ratio of the advective displacement during one correlation time to the structure scale. For $\text{Ku} \gtrsim 1$ the micro-field ensemble is not in the weak-correlation, white-noise limit, and the eddy diffusivity is set by strong-correlation (trapping) physics.

The naive one-loop self-energy contracts the internal $\tilde{\psi}$ and ψ legs of Eq. (C5) at equal time, giving $\langle \tilde{\psi}(\mathbf{x}, t) \psi(\mathbf{x}, t) \rangle_0 = G_0(\mathbf{0}, t = 0^-)$. In the Itô convention the response propagator is strictly causal, $G_0(\mathbf{r}, t) = 0$ for $t \leq 0$, so

$$790 \quad \int \frac{d\Omega}{2\pi} e^{+i\Omega 0^+} \frac{1}{\gamma_{\text{eff}}(-i\Omega + D_{\text{eff},mn} q_m q_n)} = 0, \quad (\text{C10})$$

the contour closing in the upper half-plane, where the integrand has no pole. The self-energy therefore vanishes in Itô; the eddy diffusivity is not a self-energy correction but the noise-induced drift, which in Itô is displaced into the deterministic operator. Equivalently, in Stratonovich the same quantity appears as the finite equal-time contraction; the two conventions differ precisely by the tensor $\mathbf{D}^v$ below.

795 Next, we shall derive the second-cumulant closure. Averaging Eq. (C6) over the Gaussian ensemble of Eq. (C7) and retaining the second cumulant, the mean field obeys a renormalized advection-diffusion equation,

$$\partial_t \langle \psi \rangle + \mathbf{V}_d \cdot \nabla \langle \psi \rangle = \nabla \cdot [(\mathbf{D}_{\text{eff}} + \mathbf{D}^v) \cdot \nabla \langle \psi \rangle] + S, \quad (\text{C11})$$

with the Taylor-Kubo eddy-diffusivity tensor

$$D_{ij}^v = \int_0^\infty d\tau \langle \delta v_i(t) \delta v_j(t - \tau) \rangle. \quad (\text{C12})$$

800 The closure is the second-cumulant truncation of the averaged equation; its validity is controlled by the separation (i) above, not by white-noise statistics at the structure scale. Inserting Eq. (C7) factorizes Eq. (C12) into the equal-time covariance and the time integral of C ; the value depends on the correlation regime set by Ku :

$$D_{ij}^v = \hat{V}_{d,i} \hat{V}_{d,j} \times \begin{cases} \sigma_v \tau_c, & \text{Ku} \ll 1 \quad (\text{weak correlation; Taylor-Kubo}), \\ \delta v_{\parallel} \ell_c, & \text{Ku} \gtrsim 1 \quad (\text{strong correlation; mixing length}). \end{cases} \quad (\text{C13})$$

805 In the weak limit the diffusivity grows linearly with the correlation time. In the strong limit a fluid element is trapped within a coherent structure for many turnover times, the displacement saturates, and the diffusivity reduces to the mixing-length form $\delta v_{\parallel} \ell_c$, independent of τ_c . Because the Farley-Buneman ensemble realizes $\text{Ku} \sim O(1)$, the mixing-length branch is the physical one, and the transport coefficient does not depend on the wave lifetime. Three features hold on either branch: the result is ultraviolet finite (set by ℓ_c , not by a momentum cutoff); it holds in general dimension d ; and its anisotropy is inherited directly from the longitudinal covariance, reproducing the drift-aligned structure $\hat{\mathbf{V}}_d \otimes \hat{\mathbf{V}}_d$ without an angular phase-space

810 integral. The renormalized transport tensor entering Eq. (C11) is, in the physical regime,

$$\underline{D_{ij}^{\text{ren}} = D_{\text{eff},ij} + D_{ij}^v = \gamma_{\text{eff}}^{-1} (C_s^2 \delta_{ij} - V_{d,i} V_{d,j}) + \delta v_{\parallel} \ell_c \hat{V}_{d,i} \hat{V}_{d,j}, \quad \text{Ku} \gtrsim 1.} \quad (\text{C14})$$

We are now in a position to derive three important results.

1. Bohm-order magnitude. With $\delta v_{\parallel} \sim 0.1\text{--}0.5 C_s \approx 40\text{--}200$ m/s (the polarization excess up to the marginal-stability amplitude below) and $\ell_c \sim 3\text{--}50$ m (Bragg to structure scale), the longitudinal anomalous diffusivity is

$$815 \quad \underline{D_{\parallel}^v \sim \delta v_{\parallel} \ell_c \sim 10^2\text{--}10^4 \text{ m}^2 \text{ s}^{-1}.} \quad (\text{C15})$$

This is Bohm order, and the correspondence is structural rather than a numerical coincidence: Bohm diffusion $T_e/eB = v_{\text{th}}\rho$ is itself a mixing length (thermal speed times gyroradius), and $\delta v_{\parallel} \ell_c$ is the same product formed from the saturated micro-field velocity and the structure scale. With the conventions $\frac{1}{16} T_e/eB \approx 125 \text{ m}^2 \text{ s}^{-1}$ (classical) and $T_e/eB \approx 2000 \text{ m}^2 \text{ s}^{-1}$ (Eq. 46; $T_e \sim 0.1$ eV, $B \sim 5 \times 10^{-5}$ T), the estimate brackets both. The mixing-length value does not depend on the lifetime τ_c ,

820 which removes the dominant source of spread carried by the weak-correlation form $\sigma_v \tau_c$. The closure thus supports the Bohm parameterization of the main text as an order-of-magnitude, structural result rather than a kinetic identity; it stands beside the Einstein-relation estimate of the Discussion (Eq. 46) as an independent route to the same transport, neither being a first-principles kinetic derivation.

2. Marginal stability as a self-consistency condition. Along $\hat{V}_d$, Eq. (C14) gives

$$825 \quad \underline{D_{\parallel}^{\text{ren}} = \gamma_{\text{eff}}^{-1} (C_s^2 - V_d^2) + D_{\parallel}^v,} \quad (\text{C16})$$

the bare term negative for $V_d > C_s$ (recall that the instability appears as negative diffusion). Marginal stability, $D_{\parallel}^{\text{ren}} = 0$, is the closure-agnostic balance in which the positive eddy diffusivity cancels the negative bare diffusion,

$$\underline{D_{\parallel}^v = \gamma_{\text{eff}}^{-1} (V_d^2 - C_s^2),} \quad (\text{C17})$$

which in the mixing-length regime reads $\delta v_{\parallel} \ell_c = \gamma_{\text{eff}}^{-1} (V_d^2 - C_s^2)$. Since the micro-field velocity scales with the fluctuation amplitude, $\delta v_{\parallel} \propto \langle |\psi|^2 \rangle^{1/2}$, Eq. (C17) is a self-consistency condition that fixes the saturated amplitude: the anomalous drag returns the parallel transport to zero at the point where the excess drift is balanced by mixing-length diffusion. The substitution of Eq. (18) is thereby recovered, with $\tilde{\alpha}$ determined by the proportionality between $\delta v_{\parallel}$ and $\langle |\psi|^2 \rangle^{1/2}$ and by ℓ_c , rather than fitted.

830

3. Galilean covariance. The construction respects the boost $\psi(\mathbf{x}, t) \rightarrow \psi(\mathbf{x} - \mathbf{u}t, t)$, $\mathbf{V}_d \rightarrow \mathbf{V}_d + \mathbf{u}$, under which the convective derivative $\partial_t + \mathbf{V}_d \cdot \nabla$ is invariant. The eddy diffusivity of Eq. (C13) is an intrinsic property of the micro-field statistics and is a k^2 (diffusive) object; the closure therefore renormalizes only the diffusion tensor and generates no k^0 (mass) or k^1 (velocity)

835

term. In the isotropic case (Forster-Nelson-Stephen; Forster et al., 1977) the diffusion tensor is drift independent; here $\mathbf{D}_{\text{eff}}$ contains the term $-\mathbf{V}_d \otimes \mathbf{V}_d$, so the boost also acts on the tensor, whose anisotropic part transforms covariantly with the drift while the advection velocity itself is unshifted. This is the precise sense in which $\mathbf{V}_d$ is not renormalized by the closure; the corresponding statement under coarse-graining is the Ward identity (the exact constraint that a continuous symmetry imposes on the renormalized couplings) of Appendix D.

Appendix D: Renormalization-group classification of the effective field theory

This appendix carries out the renormalization-group (RG) step that the closure of Appendix C deliberately omits. It formalizes the rescaling analysis of Section 2.1, which presents the physical interpretation; here the emphasis is on the complete operator classification and the formal flow. Whereas Appendix C sought to compute the transport coefficient, this appendix seeks to establish the macroscopic universality class, to formally classify every operator in the action of Eq. (14) as relevant, marginal, or irrelevant, and to determine the status of the micro-field noise coupling under coarse-graining. The result is precise and deliberately modest: the system flows to a $z = 2$ diffusive (dissipative) fixed point; the noise coupling is irrelevant, scaling as b^{-d} ; and its sole and important effect is a finite, non-universal renormalization of the diffusion tensor, whose value is the mixing-length coefficient of Appendix C.

The quantities to track in Eq. (14) are the fields $(\psi, \tilde{\psi})$, the damping γ_{eff} , the stiffness $\mathbf{D}_{\text{eff}}$, the advection velocity $\mathbf{V}_{d0}$, and the noise coupling σ_v , whose covariance (Eq. C7) has finite spatial range ℓ_c . There is no bare $\tilde{\psi}\psi$ vertex: the noise is multiplicative and the bare correlation function vanishes. Recall that we coarse-grain by rescaling

$$\mathbf{x} \rightarrow b\mathbf{x}, \quad t \rightarrow b^z t, \quad \psi \rightarrow b^x \psi, \quad \tilde{\psi} \rightarrow b^{\tilde{x}} \tilde{\psi}, \quad \int dt d^d x \rightarrow b^{z+d} \int dt d^d x. \quad (\text{D1})$$

The field exponents are fixed by demanding that the stiffness (diffusion) operator be scale invariant, since it is the kinetic term that defines the fixed point:

$$\chi + \tilde{\chi} = 2 - z - d. \quad (\text{D2})$$

All other operators are measured against this choice; an operator whose integrated term scales as b^Δ is relevant for $\Delta > 0$, marginal for $\Delta = 0$, and irrelevant for $\Delta < 0$.

Operator classification. Applying Eq. (D2) to each term of Eq. (14) gives the scaling exponents:

operator	term	Δ (general z)	at $z = 2$
inertia	$\tilde{\psi} \partial_t^2 \psi$	$2 - 2z$	b^{-2} (irrelevant)
cross	$\tilde{\psi} (\mathbf{V}_{d0} \cdot \nabla) \partial_t \psi$	$1 - z$	b^{-1} (irrelevant)
damping	$\tilde{\psi} \gamma_{\text{eff}} \partial_t \psi$	$2 - z$	b^0 (marginal)
stiffness	$\tilde{\psi} C_s^2 \nabla^2 \psi$	0	b^0 (marginal)
advection	$\tilde{\psi} \gamma_{\text{eff}} (\mathbf{V}_{d0} \cdot \nabla) \psi$	1	b^{+1} (relevant, redundant)
noise	$\sigma_v \gamma_{\text{eff}}^2 (\tilde{\psi} \nabla \psi)^2$	$2 - z - d$	b^{-d} (irrelevant)

The damping exponent $2 - z$ vanishes at $z = 2$; requiring the first-order time derivative to be marginal selects the dynamic exponent $z = 2$, at which the diffusive relaxation $\omega \sim k^2$ balances. The inertial and cross terms are then irrelevant: the second-order MSR dynamics flows to the overdamped, first-order theory, so the overdamped reduction used in the main text is derived rather than assumed. The advection term is relevant but redundant, as shown below.

The $z = 2$ dissipative fixed point. At $z = 2$ the surviving marginal operators are the damping and the stiffness; the fixed-point action is the linear advection-diffusion theory

$$S^* = \int dt d^d x \tilde{\psi} \gamma_{\text{eff}} [(\partial_t + \mathbf{V}_d \cdot \nabla) - \nabla \cdot \mathbf{D}_{\text{eff}} \cdot \nabla] \psi. \quad (\text{D3})$$

870 This is a Gaussian fixed point: the noise coupling, the only non-quadratic term, is irrelevant, so it does not modify the fixed-point action, and the dynamic exponent is $z = 2$ exactly, with no fluctuation correction, because the fluctuation that would generate one is irrelevant. The universality is that of purely dissipative relaxational dynamics; in the Hohenberg-Halperin taxonomy (Hohenberg and Halperin, 1977) of dynamic critical phenomena, the nearest label is Model A (non-conserved relaxational dynamics with $z = 2$), with the caveat that ψ is a conserved density relaxing away from any critical point, so the assignment is an analogy rather than a strict classification.

875 Next, we shall describe the flow of the noise coupling. The dimensionless coupling measuring the noise strength relative to diffusion at the structure scale is the Péclet number, of order the Kubo number of Appendix C,

$$g(\ell_c) = \frac{\delta v_{\parallel} \ell_c}{D_{\text{eff}}} \equiv \text{Pe} \sim \text{Ku} \sim O(1). \quad (\text{D4})$$

Its scaling dimension is $\Delta_{\text{noise}} = -d$ at $z = 2$, so under coarse-graining $b = e^{\ell}$,

$$\frac{dg}{d\ell} = -dg + O(g^2), \quad g(\ell) = g(\ell_c) e^{-d\ell}, \quad g^* = 0. \quad (\text{D5})$$

880 The Gaussian fixed point $g^* = 0$ is infrared stable. Physically, a velocity field correlated only over ℓ_c contributes nothing to advection at scales $L \gg \ell_c$; the micro-field ensemble dilutes away under coarse-graining. This is the precise content of the now oft-repeated statement that the noise is irrelevant: it is strong at the structure scale ($\text{Ku} \sim 1$) yet RG-irrelevant in the macroscopic limit, with no contradiction between the two statements.

885 An irrelevant coupling does not flow to a new fixed point, but it shifts the marginal couplings by a finite amount before it dies. The diffusion tensor picks up the eddy contribution as the noise is integrated out,

$$\frac{dD_{\text{eff}}}{d\ell} = a g(\ell) D_{\text{eff}} = a g(\ell_c) e^{-d\ell} D_{\text{eff}}, \quad (\text{D6})$$

with $a = O(1)$ a geometric constant. The total renormalization is the convergent integral

$$\delta D_{\text{eff}} = \int_0^{\infty} d\ell a g(\ell_c) e^{-d\ell} D_{\text{eff}} = \frac{a g(\ell_c)}{d} D_{\text{eff}} \sim \delta v_{\parallel} \ell_c \hat{V}_{d,i} \hat{V}_{d,j}, \quad (\text{D7})$$

the mixing-length value of Eq. (C14). Two points are structural. First, the integral converges precisely because g is irrelevant; the renormalization is finite and saturates, with no logarithm and no running of $\mathbf{D}$ at large scales. Second, its magnitude relative to the bare diffusivity is $\delta D/D_{\text{eff}} = a g(\ell_c)/d \sim O(1)$, consistent with the eddy diffusivity being of the same (Bohm) order as the bare transport. The renormalization is active below the crossover scale L_c of Eq. (43), where g has decayed to $O(1) \times (\ell_c/L_c)^d$; above L_c the diffusion tensor has saturated at $\mathbf{D}_{\text{eff}} + \delta \mathbf{D}$ and the Gaussian fixed point of Eq. (D3) governs.

The advection operator is the one relevant term (b^{+1}), which would naively destabilize the fixed point. It does not: the action is invariant under the Galilean boost $\psi(\mathbf{x}, t) \rightarrow \psi(\mathbf{x} - \mathbf{u}t, t)$, $\mathbf{V}_{d0} \rightarrow \mathbf{V}_{d0} + \mathbf{u}$, so the advection can be removed by passing to the comoving frame. The invariance holds for the full bare action, not only at the fixed point, because the inertia, the cross term, and $(\mathbf{V}_{d0} \cdot \nabla)^2$ assemble into the squared comoving derivative $(\partial_t + \mathbf{V}_{d0} \cdot \nabla)^2$, while the damping and advection form its first power. The relevant advection is therefore a redundant operator, a choice of reference frame rather than a physical instability, and the associated Ward identity (e.g., Forster et al., 1977) forbids any renormalization of $\mathbf{V}_{d0}$, confining the noise correction to the k^2 (diffusive) sector, in agreement with the closure statement of Appendix C. The macroscopic content of the fixed point is comoving-frame $z = 2$ diffusion with the renormalized, drift-aligned tensor.

The calculation separates what the RG fixes from what it does not. Universal: the dynamic exponent $z = 2$ (exact); the dissipative relaxational class; the form of the macroscopic law (advection-diffusion); and the drift-aligned structure of the transport tensor, dictated by the boost covariance and the instability axis. Non-universal, supplied by the closure: the magnitude $\mathbf{D}_{\text{eff}} + \delta \mathbf{D}$ (the mixing-length, Bohm-order coefficient), together with γ_{eff} , V_d , C_s , and L_c , all set by microscopic plasma parameters. The RG is thus not a competitor to the closure: it proves that the closure coefficient is the complete macroscopic transport law, with corrections decaying as b^{-d} .

The fixed point is Gaussian and $z = 2$ is exact because two features of the problem forbid anomalous scaling. First, the advection of ψ is linear (a passively advected scalar), so there is no mode-mode interaction of the KPZ type. Second, the advecting velocity is short-range correlated (finite ℓ_c), so the random-advection coupling is short ranged and irrelevant. Anomalous dynamic scaling ($z \neq 2$) would require a scale-free velocity spectrum extending to large scales, the regime of the Kraichnan model. The finite ℓ_c is therefore an empirical premise of this classification: kinetic simulations show the saturated spectra cascading to longer wavelengths up to the domain size (Oppenheim et al., 2008; Oppenheim and Dimant, 2013), and whether the cascade halts at a characteristic scale in the open ionosphere is an observational question, taken up in a related study. No non-trivial fixed point is claimed here.

Appendix E: Confrontation of the theory with kinetic simulations, and with internal, reduced Monte-Carlo tests

The effective theory of this paper is a macroscopic transport law, and it must answer to the two bodies of work that constrain the microphysics it coarse-grains: the nonlinear saturation theory developed by St-Maurice et al., in which finite structures generate polarization fields that oppose the driver and clamp the local drift (St-Maurice and Hamza, 2001; St-Maurice et al., 2023), and the fully kinetic simulation program of Dimant, Oppenheim, et al., collectively as close to ground truth as is likely to be found at present, represented here by the three-dimensional particle-in-cell (PIC) simulations of Oppenheim and Dimant (2013) and the recent flux-tube-spanning simulations of Oppenheim et al. (2026). The two lines of work do not agree in every respect, most prominently on the magnitude and interpretation of the nonlinear currents (Dimant et al., 2021; St-Maurice and Goodwin, 2021); our effective theory inherits its core mechanism, the wave saturation mechanism, from the former school, and tests its coarse-grained outputs against the latter, and its anomalous drag emerges at Bohm order, subject to scrutiny to either account. Table E1 sets the simulation results beside the corresponding elements of the effective theory, row by row; the paragraphs that follow take up the quantities that no existing simulation constrains.

One caution applies to the transport row of the table: conductance changes and the eddy diffusivity D^v are different observables, connected only through a height-resolved conductivity model. The simulated nonlinear currents confirm the heuristic predictions of the Dimant-Oppenheim line of work, while the magnitude and interpretation of such currents remain debated between the two schools named above; our effective theory's anomalous drag, of Bohm order locally and modest when height-integrated, does not depend on how that debate resolves.

Three predictions of the effective theory are open to direct numerical falsification. First, the Gaussianity of the coarse-grained micro-field velocity: binning existing PIC output over volumes containing $N \gtrsim 30$ saturated structures and computing the excess kurtosis of the summed drift fluctuation tests the random-phase step directly. Second, the response scaling: a scan of otherwise identical simulation runs across driving field strengths tests whether the count of saturated structures, rather than their amplitude, carries the response, with $\beta = 1$ against $\beta \approx 2$ as the discriminating outcome (see Figure E1 below). Third, the structure lifetime: simulations sustaining turbulence for seconds, or structure-tracking diagnostics in flux-tube runs, would bound τ_c and with it the Kubo number that selects the mixing-length branch in Appendix C.

The reduced model rests on four internal assertions that Monte-Carlo sampling can test directly, without reference to plasma dynamics: that the summed micro-field of finitely many saturated structures approaches the Gaussian statistics the closure assumes; that the linear macroscopic response emerges from the amplitude-phase dynamics rather than from the slow-manifold approximation used to derive it; that count-carried and amplitude-carried responses are observationally distinguishable; and that the eddy diffusivity possesses the two-branch Kubo structure, with a strong-correlation limit independent of the structure lifetime. We test each assertion on seeded synthetic ensembles, adopting $C_s = 500$ m/s (moderately heated), $\ell_c = 30$ m, $\delta v \sim 100$ m/s, and $\tau_c = 1$ s throughout; the complete Monte-Carlo and its documentation, sufficient to reproduce every number quoted below, are archived on Zenodo ([10.5281/zenodo.21323500](https://zenodo.org/record/21323500)).

The Gaussian closure assumption (the random phase approximation of Sect. 2.1 and Appendix C) is tested on independent snapshots of N saturated structures, each carrying an amplitude drawn from $|\mathcal{N}(0, \sigma)|$, a phase drawn uniformly from $[0, 2\pi)$,

Table E1. Effective theory against three-dimensional kinetic simulations (Oppenheim and Dimant, 2013; Oppenheim et al., 2026).

Quantity	Kinetic simulations	This theory
Phase speed of dominant saturated modes	Near the (heated) ion-acoustic speed; ~ 1300 m/s in 3-D versus ~ 1800 m/s in 2-D at $E_0 = 105$ mV/m (Oppenheim and Dimant, 2013)	Drift clamped to $(1.0-1.1)C_s$ by marginal stability, Eq. (18), Appendix C
Saturation mechanism	Growth arrested by wave-driven heating, mode coupling, and parallel-field dissipation; saturated $(\delta E^2)^{1/2} \approx 160$ mV/m for $E_0 = 105$ mV/m (Oppenheim and Dimant, 2013)	Positive eddy diffusivity cancels the negative bare diffusion, $D_{\parallel}^{\text{en}} \rightarrow 0$ (Eq. C17); the kinetic routes are coarse-grained into one anomalous drag
Energy-containing perpendicular scale	3-m waves dominate growth; saturated spectra cascade to longer wavelengths and spread in k -space (Oppenheim and Dimant, 2013; Oppenheim et al., 2026)	Finite ℓ_c of tens of meters, an empirical input from the observed structure scale; short-range velocity correlations underpin the b^{-d} irrelevance (Appendix D)
Anisotropy and parallel structure	Field-aligned waves initially coherent over kilometers along $\mathbf{B}$, energy spreading in k -space as turbulence saturates; flow-angle cosine spectra; dominant flow angles increase with altitude (Oppenheim and Dimant, 2013; Oppenheim et al., 2026)	Drift-aligned tensor $\hat{\mathbf{V}}_d \otimes \hat{\mathbf{V}}_d$; quasi-two-dimensional perpendicular dynamics, local in altitude; the $z = 2$ class holds in general d
Anomalous transport magnitude	Electron Pedersen drift exceeds the ion drift; nonlinear currents up to 80% above the laminar ion Pedersen current (Oppenheim and Dimant, 2013); height-integrated Pedersen conductance +21% above laminar (Oppenheim et al., 2026)	$D_{\parallel}^v \sim \delta v_{\parallel} \ell_c \sim 10^2-10^4$ m ² /s, Bohm order (Appendix C); order-unity local modification, modest when height-integrated
Electron heating	Simulated T_e climbs from a few hundred K to a sustained 1500–2500 K at $E_0 = 105$ mV/m (artificially heavy electrons understate real heating); heating coefficient an order of magnitude above the early heuristic (Oppenheim and Dimant, 2013); observed storm-time T_e reaches 4000 K (Schlegel and St.-Maurice, 1981)	Heating not modeled; enters through T_e in the Bohm value T_e/eB , and the heated T_e stays within the claimed order-of-magnitude window
Structure lifetime τ_c	Runs span $\lesssim 0.1$ s of turbulent evolution; second-scale lifetimes not addressable	$\tau_c \sim 1$ s from radar statistics (Prikryl et al., 1988, 1990)
Micro-field ensemble statistics	Not reported	Gaussian under the random phase approximation; reduced Monte-Carlo gives kurtosis $\propto N^{-1}$, Gaussian beyond $N \sim 30$ structures
Macroscopic response scaling $n(\Delta E)$	Runs concentrated at $E_0 = 105$ mV/m; no systematic scan of the response across driving strength	Thresholded linear response, $\beta = 1$ for the saturated count-carried response, against $\beta \approx 2$ unclamped (Fig. E1); measured $\beta = 0.98 \pm 0.09$ (Figure 3c)

950 and a Gaussian spatial kernel weight with position uniform over $\pm 5\ell_c$; the observable is the summed micro-field velocity at a point. With 2×10^4 snapshots per ensemble size, the excess kurtosis of the sum decays as $N^{-0.99}$ (fitted) across $N = 3$ to 300,

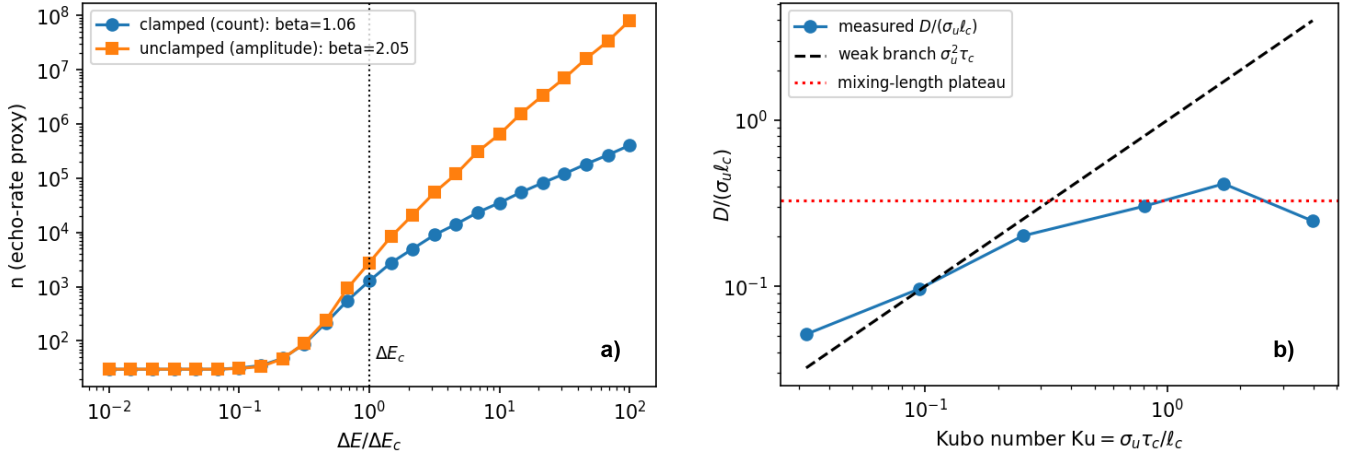


Figure E1. Panel a): Reduced Monte-Carlo discriminator for the macroscopic response. A lognormal ensemble of driven patches (independent scattering volume, drawn from the lognormal distribution of local drive power) with a detection floor produces the observed elbow in both models, but the response above the critical power separates them: when the per-structure amplitude is clamped by saturation, the structure count carries the response and $\beta = 1.01$; when amplitudes grow with the excess drive, $\beta = 2.08$. The measured $\beta = 0.98 \pm 0.09$ (Figure 3c) selects the saturated, count-carried model. **Panel b):** Reduced Monte-Carlo test of the two-branch Kubo diffusivity (Appendix C). Three hundred tracers are advected through a one-dimensional micro-field of 60 Gaussian structures (width ℓ_c , amplitudes from $\mathcal{N}(0, \delta v)$, exponentially distributed lifetimes), and the diffusivity D is measured from displacement variance as the structure lifetime τ_c is scanned from 0.01 to 3 s (Kubo numbers 0.03 to 4). At low Ku the measured diffusivity follows the weak-correlation branch $D = \sigma_u^2 \tau_c$ (dashed); at $Ku \gtrsim 1$ it saturates onto the mixing-length plateau $D/(\sigma_u \ell_c) \approx 0.3\text{--}0.4$ (dotted), where D is independent of τ_c . The physical regime ($\tau_c \sim 1$ s, $\ell_c \sim 30$ m) sits on the plateau, confirming within the reduced model that the mixing-length branch of Appendix C governs the macroscopic transport. Single seed; scan endpoints carry $\sim 40\%$ sampling uncertainty.

matching the N^{-1} decay required by the central limit theorem. At $N = 300$ we compare 5×10^3 standardized samples of the sum against the unit normal distribution with a two-sided Kolmogorov-Smirnov test, which measures the largest deviation between the empirical and normal cumulative distributions; the resulting $p = 0.42$ states that deviations of the observed magnitude arise by chance in 42% of samples drawn from an exact normal distribution, meaning that normality is not rejected at any conventional significance level. A companion ensemble in which structure amplitudes are redrawn at renewal instants with exponentially distributed lifetimes reproduces the assumed noise model of Appendix C: the measured autocorrelation follows $e^{-\tau/\tau_c}$ (fitted decay time 1.11 s for a 1.0 s input; Kubo integral $1.18\tau_c$; single-seed accuracy 10–15%).

Whether the linear response emerges from the dynamics, rather than from the slow-manifold algebra, is tested by integrating the amplitude-phase pair of Sect. 2.1.3 explicitly (Euler, $dt = 2$ ms, 300 s, 16-member ensembles) with $\lambda_0 = 50 \text{ s}^{-1}$, $k = 2.09 \text{ m}^{-1}$, the feedback clamp targeting $1.15C_s$ (via the quadratic feedback coefficient $\tilde{\alpha}$ in $V = V_0(1 - \tilde{\alpha}A^2)$), and a driver whose envelope is an Ornstein-Uhlenbeck process (Maller et al., 2009) with 120 s correlation time and 30% relative fluctuation. Below threshold ($\mathcal{E}_0 < E_c$), $\langle A^2 \rangle$ follows the linear damping response; above threshold, the feedback clamp suppresses the growth (partial amplitude saturation). The 1-s-averaged response thereby tracks the driver envelope with correlation coefficient 0.98 (Figure E1a).

The discrimination between count-carried and amplitude-carried responses (Figure E1) uses an ensemble of 4000 independent local drive levels, lognormally distributed ($\sigma_{\ln} = 0.8$) about that level, with an activation threshold for each Monte Carlo-draw and a meteor-echo detection floor. When the per-structure amplitude is clamped, the structure count carries the absorbed power and the fitted exponent over the two decades above the elbow is $\beta = 1.06$; when amplitudes instead grow with the excess drive, $\beta = 2.05$. The measured $\beta = 0.98 \pm 0.09$ (Figure 3c) selects the clamped, saturated model.

The two-branch Kubo structure is tested by direct tracer dispersion. A one-dimensional longitudinal velocity field is built from 60 structures with Gaussian kernels of width ℓ_c on a periodic domain of $100\ell_c$, amplitudes drawn from $\mathcal{N}(0, \delta v)$ and redrawn at exponentially distributed renewal times; 300 tracers are advected and the diffusivity is measured from the displacement variance. The root-mean-square micro-field velocity at the tracer positions, σ_u , provides the natural normalization; it is the simulation counterpart of $\delta v_{\parallel}$ in Appendix C. Scanning the lifetime from 0.01 to 3 s (Kubo numbers 0.03 to 4) recovers the weak-correlation branch, $D = 1.02\sigma_u^2\tau_c$ at $Ku = 0.09$, and the mixing-length plateau $D/(\sigma_u\ell_c) = 0.3\text{--}0.4$ at $Ku \gtrsim 1$; decisively, D changes by less than 15% while τ_c triples across the plateau, the τ_c -independence that Appendix C asserts for the physical regime. Three caveats bound these results: all numbers derive from a single realization of the synthetic ensembles, with the scan endpoints carrying roughly 40% sampling uncertainty; the discriminator's per-draw statistics are a model choice, though the fitted exponents are insensitive to the lognormal width; and all four tests address the reduced model, not the plasma, which is the province of Table E1 and the diagnostics above.

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Competing interests. At least one of the (co-)authors is a member of the editorial board of Annales Geophysicae.

Data availability Science data of the ERG (Arase) satellite were obtained from the ERG Science Center operated by ISAS/JAXA and ISEE/Nagoya University (<https://ergsc.isee.nagoya-u.ac.jp/index.shtml.en>). This includes Lv.3 HEP (DOI 10.34515/DATA.ERG-01002) version 01_01, Lv.3 MEP-e (DOI 10.34515/DATA.ERG-02003) version 01_01, Lv.2 LEP-e (DOI 10.34515/DATA.ERG-05000) version 04_01, Lv.2 PWE/OFA (DOI 10.34515/DATA.ERG-08000), and Lv.2 MGF (DOI 10.34515/DATA.ERG-06001) version 04_04. SuperMAG data can be accessed at <https://supermag.jhuapl.edu/mag/>. ICEBEAR 3D echo data for 2020, 2021 is published with DOI 10.5281/zenodo.7509022. [The reduced Monte-Carlo code of Appendix E and its documentation are archived on Zenodo \(10.5281/zenodo.21323500\).](#)

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