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Responses to Reviewer 2

We would like to thank the Reviewer for the careful reading of the manuscript and for the constructive comments and suggestions. We have revised the manuscript accordingly and provide point-by-point responses below. Our responses are shown in blue, the corresponding changes made in the revised manuscript are shown in red, and the Reviewer's comments are shown in black.

Brief Summary:

The Reviewer regards the deep-atmosphere GMCORE formulation as useful to document in the open literature and considers the presented baroclinic-wave and tropical-cyclone tests reasonable. The Reviewer also notes that the multiple tropical-cyclone initial conditions help establish that the track signal is systematic, while offering a set of mainly minor suggestions aimed at improving notation, numerical-method clarity, terminology, and presentation.

Response: Thank you very much for the careful and constructive assessment. We particularly appreciate the emphasis on clarity, because several of the points identified places where our notation or wording could give an impression different from what is actually implemented. In the revision, we have clarified the relation between the deep-atmosphere geometry and gravity, distinguished the generalized and terrain-following vertical coordinates, documented the Lorenz staggering and SWIFT/PPM treatment, clarified the temporal and vertical increments, expanded the polar-filter description, added model resolution information, and revised the relevant terminology and figure captions. Where an original statement was not sufficiently supported by the present experiments, we chose to remove or soften it rather than defend a stronger claim. We respond to each point below.

Line numbers cited below refer to the revised manuscript supplied with this response after accepting the tracked changes.

Specific points

1. Line 36. This is not strictly incorrect, but a more natural viewpoint is that the deep atmosphere equations are the more fundamental equation set and the shallow-atmosphere equations are an approximation.

Response: Thank you for this helpful suggestion. We agree that this is the more natural hierarchy of the equation sets. We have revised the sentence accordingly so that the deep-atmosphere equations are presented as the more fundamental set and the shallow-atmosphere equations as an approximation obtained by additional assumptions.

Revised manuscript text [Lines 39–47]: "The deep-atmosphere equations are generally considered to be a more fundamental set than the traditional shallow-atmosphere equations ... Standard shallow-atmosphere equations project the planetary rotation vector onto the local vertical and neglect the horizontal component ..."

2. Line 55. while most nonhydrostatic models use a height-based vertical coordinate. (Hydrostatic models commonly use a pressure- or mass-based coordinate.)

Response: Thank you for pointing this out. We have revised "most models" to "most nonhydrostatic models" as suggested.

Revised manuscript text [Lines 59–61]: "An important feature of the present formulations is that the deep-atmosphere extension is represented within the dry terrain-following mass-based coordinate while most nonhydrostatic models use height-based vertical coordinate (Wood and Staniforth, 2003)."

3. Lines 72–74. The sentence is a bit awkward because Wood and Staniforth (2003) did not use the vector-invariant form. Can it be rephrased?

Response: Thank you for noting this distinction. We have reorganized the sentence so that the source of the vector-invariant/Lamb-transformed form is separated from the deep-atmosphere formulations. Gassmann and Herzog (2008) is now cited for the vector-invariant formulation, while Wood and Staniforth (2003), Borchert et al. (2019), and Skamarock et al. (2021) are cited for the deep-atmosphere formulation and implementation context.

Revised manuscript text [Lines 77–80]: "Following the vector-invariant formulation based on the Lamb transformation (Gassmann and Herzog, 2008), together with the deep-atmosphere formulations of Wood and Staniforth (2003), Borchert et al. (2019), and Skamarock et al. (2021), GMCORE solves the fully compressible, nonhydrostatic deep-atmosphere equations in a dry terrain-following mass-based coordinate in vector-invariant form."

4. Some things are not clear about equations (1)–(7) and the subsequent discussion. Is the constant G related to g(r) in some way, or is it an arbitrary constant? Is G the same as g0 introduced later in the definition of π (equation (14))? What is s (line 124), and how does it relate to η? Is the $(r/a)^2$ factor in (2) a geometrical factor (as in (14)) rather than describing the vertical variation of g(r)?

Response: Thank you for this careful comment. The original notation did not distinguish these roles clearly enough. In the revised manuscript, g_0 denotes the reference gravitational acceleration at the planetary surface $r = a$, while the physical gravitational acceleration varies with radius as $g(r) = g_0(a/r)^2$. We also define s explicitly as the generalized vertical coordinate used in the Wood and Staniforth (2003) derivation, whereas η is the terrain-following hybrid coordinate used in the GMCORE implementation. Finally, the factor $(r/a)^2$ that appears in the deep mass-coordinate formulation is a geometrical area/Jacobian factor associated with the radial expansion of spherical control surfaces; it is distinct from the physical inverse-square variation of $g(r)$. We have revised the surrounding explanation so that these quantities are not conflated.

Revised manuscript text [Lines 80–82 and 130–149]: "Compared to the shallow-atmosphere equations, the deep extension introduces three important changes: the constant planetary radius a is replaced by the local radial distance $r = a + z$, the constant gravity g_0 becomes altitude-dependent $g(r) = g_0(a/r)^2$ ..." "They denoted s as the generalized vertical coordinate ... We use the dry mass π to substitute the generalized coordinate s and introduce terrain-following coordinate η ..." The subsequent hydrostatic relation in Eqs. (14)–(16) retains the geometrical $(r/a)^2$ factor in the mass-coordinate formulation.

5. Line 143. dictates the dry atmospheric mass?

Response: Thank you for pointing this out. We agree that "dry" should be specified here and corrected "atmospheric mass" to "dry atmospheric mass" during the revision. The relevant paragraph was subsequently removed in response to Comment 6.

Revised manuscript text [Lines 130–158]: The paragraph containing the phrase discussed in this comment is no longer present in the final revised manuscript, because that comparison paragraph was subsequently removed in response to Comment 6. The final Sect. 2.2 proceeds directly through the mass-coordinate definition, radial-distance diagnosis, and the spherical-geometry assumptions.

6. Lines 144–145. I don't understand this remark. Height-based coordinate models typically predict density, or density can easily be computed from the predicted variables. Computing the mass in an atmospheric layer is then simply a case of doing a (discrete) vertical integral, no more complicated than (17).

Response: Thank you for this correction. We agree that the original paragraph overstated the contrast with height-based coordinates. Our original wording was motivated by the possible practical convenience of a mass coordinate for some layer-integrated physical quantities, but this possible advantage has not been demonstrated by the experiments in the present manuscript. After reconsidering the point, we decided that the most careful response is to remove the paragraph rather than retain a claim that would require further dedicated evidence.

Revised manuscript text [Lines 130–158]: The original paragraph comparing the complexity of mass- and height-coordinate coupling to physical parameterizations has been removed. The final Sect. 2.2 now focuses on the definition of the mass-based coordinate, the radial-distance diagnosis, and the spherical-geometry assumptions.

7. Near line 156, perhaps mention that GMCORE uses a Lorenz vertical grid.

Response: Thank you for this suggestion. We have added the Lorenz staggering explicitly. GMCORE uses Arakawa C-grid staggering in the horizontal and Lorenz staggering in the vertical, with dry mass and thermodynamic variables on full levels and vertical velocity and vertical coordinate velocity on half levels.

Revised manuscript text [Lines 166–172]: "GMCORE uses an Arakawa C-grid staggering in the horizontal and a Lorenz staggering in the vertical ... Dry mass and thermodynamic variables are defined at the full levels ... The vertical velocity w and vertical coordinate velocity $\dot{\eta}$ are defined at the half levels."

8. Lines 158–160. Do you really use a third-order polynomial reconstruction? Bendall and Kent used the quadratic PPM reconstruction. Also, Bendall and Kent were targeting cubed-sphere grids; are there any issues in applying the scheme near the poles of a longitude-latitude grid?

Response: Thank you for pointing this out. We have corrected the reconstruction description to PPM, consistent with Bendall and Kent (2025). Regarding the application of SWIFT on the latitude–longitude grid, SWIFT mainly addresses the multidimensional inconsistency introduced by cross-directional transport, for which strict monotonicity cannot be maintained as directly as in one-dimensional transport. This improvement is made at the algorithmic level and is not closely tied to the specific grid discretization. We therefore use the same SWIFT treatment on the latitude–longitude grid.

Revised manuscript text [Lines 257–260]: "Tracer advection is handled by the SWIFT scheme (Bendall and Kent, 2025), which is derived from the FFSL formulation (Lin and Rood, 1996) with specific modifications to preserve monotonicity in two-dimensional transport and relies on PPM for the spatial reconstruction of tracers."

9. Line 178. Here I think you mean the temporal pressure increment (see equation (20)), but in equation (18) δp appears to be a vertical pressure increment. Please be explicit, or you will quickly lose readers. Also, what is gamma in equation (20)?

Response: Thank you for identifying this ambiguity. In the revised text, δ is used for the vertical increment across a model layer, whereas the temporal pressure increment is written explicitly as the difference between the updated and previous-stage pressure. We have also introduced γ as the ratio of the specific heats at constant pressure and constant volume, $\gamma = c_p/c_v$. This makes the two different types of increments explicit in the time-integration discussion.

Revised manuscript text [Lines 253–255 and 294–299]: "The symbol δ denotes the vertical increment across a model layer." "At full levels, the temporal increment of pressure $p^{\tau} - p^{*}$ is diagnostically linked to vertical increment of $\delta\varphi$ and the thermodynamic properties through the linearized equation of state ... where $\gamma = c_p/c_v$ is the ratio of the specific heats at constant pressure and constant volume ..."

10. Equations (22) and (23). How is W_j related to H_j?

Response: Thank you for asking us to clarify this relation. W_j is the continuous width parameter diagnosed from the local CFL-like constraint. It is converted to the smallest admissible odd stencil size N_j, and the actual convolution half-width is $H_j = (N_j - 1)/2$, equivalently $N_j = 2H_j + 1$.

Revised manuscript text [Lines 330–334]: "The diagnosed continuous width W_j is converted to the discrete stencil size by taking the smallest odd integer not smaller than W_j, with a minimum stencil size of three grid points: $N_j = max[3, oddceil(W_j)]$. ... For a stencil of width $N_j = 2H_j + 1$ ..."

11. Lines 217–219. How large does N_j become near the pole? The communication cost must become significant at some point as resolution is refined.

Response: Thank you for raising this practical question. At 1° horizontal resolution, the maximum half-width in the polar processes is typically about $H_j \approx 90$ grid points, corresponding to a total stencil width $N_j \approx 181$. The wider halo is not used uniformly by all processes: only the high-latitude processes require the largest width, and only the arrays to which the filter is applied use the expanded communication halo. For the resolutions considered in this manuscript, we find the additional communication and memory cost manageable. We nevertheless agree that the cost becomes increasingly important as resolution is refined; optimization of this communication pattern is ongoing, and our group is also exploring quasi-uniform/SCVT-type grid approaches for future developments.

Revised manuscript text [Lines 330–343]: "For a stencil of width $N_j = 2H_j + 1$ ... It should be noted that the wider communication halo is required only for the variables to which the filter is applied. In the parallel implementation, processes covering higher latitudes therefore use wider halos for those variables. In practice with a horizontal resolution of 1°, this half width halo usually reaches 90 points at the two poles ... Nevertheless, the additional communication cost remains manageable for the resolutions considered here.”

12. Lines 250–256. The word ‘configuration’ is sometimes used to distinguish the full radius and small radius cases, and sometimes to distinguish the shallow-atmosphere and deep atmosphere cases, with the potential to confuse the reader. Can you rephrase? Check also for other occurrences.

Response: Thank you for noting this ambiguity. We have revised the terminology so that the standard-Earth-radius and reduced-radius X20 cases are referred to as experiments, while NHD and NHS are used to distinguish the deep- and shallow-atmosphere equation-set configurations. We also checked the surrounding text for the same usage.

Revised manuscript text [Lines 404–410 and 445–446]: The baroclinic-wave discussion uses “standard Earth-radius experiment” and “X20 experiment” for the planetary-radius cases. The tropical-cyclone section states that the experiment is conducted with both the “nonhydrostatic deep-atmosphere (NHD) and nonhydrostatic shallow-atmosphere (NHS) configurations.”

13. What resolution, horizontal and vertical, was used for the baroclinic wave test? What vertical resolution was used for the tropical cyclone test?

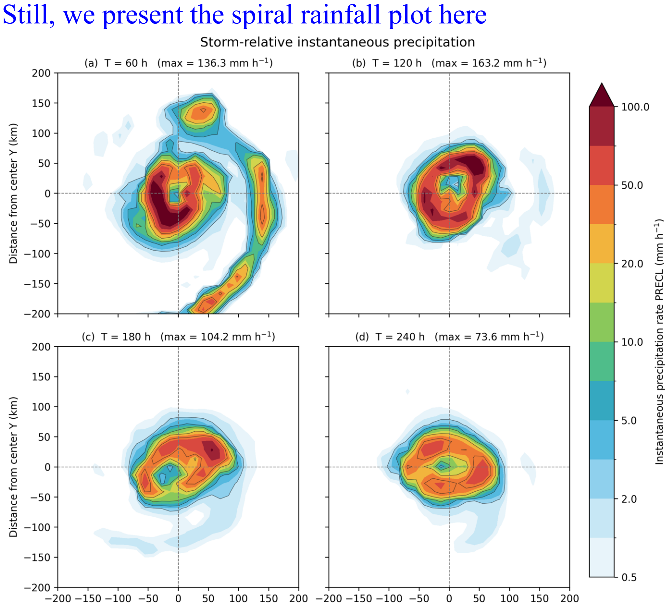
Response: Thank you for requesting these model-specific resolution details. The baroclinic-wave experiment uses a 1° (360×180) horizontal grid and the 30-level CAM vertical grid with a model top of 2.255 hPa (approximately 30 km). The tropical-cyclone experiments use 0.25° and 0.125° horizontal resolutions and the same 30-level vertical grid.

Revised manuscript text [Lines 366–368 and 439–444]: “The simulations use the 30-level vertical grid adopted from CAM, with the model top located at 2.255 hPa, corresponding to an approximate geometric height of 30 km. The horizontal resolution is 1°, corresponding to a 360×180 regular latitude–longitude grid.” The tropical-cyclone section notes the “low model top (about 30 km)” and states: “We follow the standard setup of Reed and Jablonowski (2011) and run this case at a horizontal resolution of 0.25° and 0.125° for 10 days with the same vertical grid as the baroclinic-wave.”

14. Figure 4 caption and line 325. The figure does not show rainfall, so the reader cannot see any spiral rainband structures.

Response: Thank you for pointing this out. We agree that the figure does not show rainfall. Because the present manuscript does not analyze the formation or mechanism of the rainbands, we decided not to add a separate rainfall figure simply to support that descriptive statement. Instead, we removed the rainband wording and restricted the discussion to the vertical-velocity structure that is actually shown in the figure.

Still, we present the spiral rainfall plot here



Revised manuscript text [Lines 473–488]: The revised Fig. 6 caption states that the mean vertical-velocity field illustrates “a core of intense upward motion centered at the origin.” The corresponding text states: “In Fig. 6a, the mean vertical velocity field exhibits a compact core of strong ascent near the storm center. This indicates that the mature vortex maintains a well-defined eyewall-like updraft region.” No spiral-rainband statement is made.

Typos etc.

Equations should be punctuated just like the rest of the text. Line 85. rate of planetary? Line 148. matches → match. Line 314. the present supplementary?

Response: Thank you for pointing out these presentation issues. We have checked the displayed equations and added punctuation according to their grammatical role in the surrounding sentences. We have also reviewed the wording describing the planetary rotation, corrected the relevant subject–verb agreement, and replaced the earlier wording with “the Supplementary Material” in the tropical-cyclone discussion.

Revised manuscript text [Lines 90–94, 147–149, and 469]: The governing-equation definitions now use the wording “Ω is the planetary angular rotation velocity”; the “matches->match” issue was removed due to content adjustment; and the tropical-cyclone discussion now states: “As already indicated in the Supplementary Material ...” Equation punctuation has also been checked throughout the revised manuscript.