

Response to Reviewer Comments

We thank the reviewer for these thoughtful comments. Our responses are in blue. Line numbers are based on the revised manuscript.

Reviewer #2

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The authors have considered all my comments and all relevant points were addressed adequately. However, two issues arise from the revised manuscript and supplementary data:

1. Line 134 and 135: The numbers for the yearly prior emissions apparently refer to 2019. This should be stated in the text.

Accepted.

2. The revised supplementary table S1 contains uncertainties as requested. But for some countries, the (absolute) posterior uncertainties are larger than the prior uncertainties. Such an increase in uncertainties is also visible in the revised Figures 7a, 9 and 10. The method section suggests that posterior and prior uncertainties are defined by $\hat{S}$ and S_a , which are related by Eq. (3). The projection to countries is described in Eqs. (5) and (6). Following these equation and the assumption that the matrices S_0 and S_a are real-symmetric and positive definite, the posterior uncertainties must always be smaller or equal to the prior uncertainties. Thus, there must be some inconsistency that should be clarified.

We thank the reviewer for identifying this inconsistency. The reviewer is correct that the posterior uncertainty should not exceed the corresponding prior uncertainty when both are propagated consistently.

We found that the inconsistency arose from an error in our post-processing of the annual prior uncertainties. In our multi-year inversion, the scaling factor for each year is represented as the sum of a multi-year mean component and a year-specific anomaly, each assigned a 50% prior uncertainty. The inversion itself correctly used this full prior covariance structure. However, in post-processing the national annual prior uncertainties, we inadvertently propagated only one of these components, whereas the posterior uncertainties were derived from the combined annual covariance. We have corrected this inconsistency by applying the same annual transformation and country aggregation to both the prior and posterior covariance matrices. The revised posterior uncertainties are now smaller than or equal to the corresponding prior uncertainties, as expected from Eqs. (3) and (6). Table S1 and Figures 7a, 9, and 10 have been updated accordingly. This correction concerns the reported uncertainty estimates only and does not affect the optimized emission estimates or the conclusions of the study.

The revised Figures 7b and 12 highlight an issue that I overlooked in the first round of review. Since this is not a new issue of the revised manuscript, I do not request any changes. But I want to inform the authors why I think the uncertainty analysis is no optimal:

The prior and posterior uncertainties on the trends are apparently based on the deviation of the emission time series from a linear fit. This method is adequate if fluctuations in a (long) time series provide sufficient information on the uncertainties. But the prior emissions in this work contain very little temporal fluctuations by construction. This does not mean that the trends are perfectly known a priori. For a more realistic uncertainty estimate, one could consider the uncertainties on prior and posterior emissions. A linear fit that takes into account those uncertainties will lead to considerably larger uncertainties on the trend, especially for the prior.

We thank the reviewer for this helpful observation. In the current analysis, the uncertainty of the linear trend is estimated from the deviations of the annual emission estimates from the fitted linear trend. We agree that this residual-based approach does not explicitly account for the uncertainties associated with the individual annual emission estimates. This issue is particularly relevant for the prior emissions, which exhibit relatively little interannual variability by construction and can therefore yield relatively small trend uncertainties when the uncertainty is inferred solely from the regression residuals.

Following the reviewer’s suggestion, we performed an additional sensitivity analysis in which the covariance of the annual emission estimates was propagated to the uncertainty of the linear trend. Because a linear trend can itself be expressed as a linear combination of the annual emission estimates, its uncertainty can be propagated from the annual emission covariance matrix in the same manner as other aggregated quantities. The comparison is shown in the figure below. Panel (a) shows the trend estimates and uncertainties obtained using the residual-based approach adopted in the manuscript, whereas panel (b) shows the same linear trend estimates, but with their uncertainties calculated by propagating the annual emission covariance. The resulting trend uncertainties differ between the two approaches, with particularly noticeable differences for several prior trends. The comparison also illustrates the substantial uncertainty associated with trends derived from the short five-year record (2019–2023).

Given the limited length of the present time series and the reviewer’s clarification that no revision is requested on this point, we retain the current trend analysis in the manuscript. We appreciate the reviewer for highlighting this methodological consideration and will take the uncertainties of the annual emission estimates and their temporal covariances into account in future analyses of longer-term emission trends.

