



1 **Estimating extreme sea levels using a copula joint probability method**

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8 **Abstract.** Extreme sea levels pose significant risks to coastal communities and infrastructure. Joint probability methods are
9 widely used to estimate return levels of extreme sea levels by combining tidal and non-tidal components, but most
10 implementations assume independence between tide and surge. This assumption is not always valid, and neglecting correlation
11 can affect the estimation of return levels and their associated uncertainties. Here, we introduce a Copula Joint Probability
12 Method (CJPM) that explicitly accounts for non-linear correlation between peak tides and skew surges, generalising the Skew
13 Surge Joint Probability Method (SSJPM). Using long tide gauge records (≥ 100 years) from 23 locations, we assess how
14 incorporating correlation affects both central estimates and confidence intervals of estimated return levels. We find that
15 accounting for this correlation can shift estimated return levels by up to approximately 10 cm at some locations. Importantly,
16 uncertainty in the peak tide–skew surge correlation can be a statistically significant contributor to the width of confidence
17 intervals, in some cases exceeding the contribution from uncertainty in the extreme skew surge distribution. At other locations,
18 correlation has a negligible effect, and CJPM and SSJPM estimates are indistinguishable. These results demonstrate that
19 explicitly representing correlation and its uncertainty provides a more complete quantification of return levels and their
20 associated confidence intervals, and helps determine whether correlation materially affects return level estimation. The CJPM
21 provides a flexible framework that can be applied across a wide range of settings without requiring assumptions about the
22 strength or cause of any correlation.

23 **Short summary (500-character incl. spaces).** Extreme sea levels pose a risk to coastal communities. We present a joint
24 probability method that uses a copula to account for non-linear correlation between peak tides and skew surges. Accounting
25 for this correlation can shift central estimates of return levels by up to about 10 cm. We further quantify how uncertainty in
26 this correlation affects confidence intervals, allowing assessment of whether correlation plays a statistically significant role in
27 return level estimation.

28 **1 Introduction**

29 Extreme sea levels (ESLs) pose an increasing risk to coastal communities and ecosystems (Diaz, 2016; Hinkel et al., 2014). In
30 2018, about 748 million people (10 % of 2018 global population) lived within 5 km of the coast and 2.86 billion (38 %) within



31 100 km (Cosby et al., 2024). In 2020, 896 million people (11 % of 2020 global population) lived in areas less than 10 m above
32 sea level and hydrologically connected to the sea (Haasnoot et al., 2021). This number may increase beyond 1 billion by 2050
33 (Oppenheimer et al., 2019). Furthermore, infrastructure and assets worth 6.5–11 trillion USD are located at sites assessed to
34 flood with a probability of at least 1 % per year (Haasnoot et al., 2021; Kirezci et al., 2020; Kulp et al., 2019). Accurate
35 estimation and prediction of ESLs help ensure that flood defences are neither under- nor over-engineered, both of which have
36 social and financial cost implications (Batstone et al., 2013; Tebaldi et al., 2021).

37 ESLs refer to the occurrence of exceptionally high or low sea levels relative to mean sea levels (Gregory et al., 2019). These
38 deviations from the mean are caused by tides, storm surges, and waves. For many purposes, including this study, it is useful
39 to regard sea level as the sum of three components: the mean sea level, tides, and non-tidal residuals (Pugh and Woodworth,
40 2014a). Tide gauges are a common source of sea level data, typically providing observations at a temporal resolution of 15
41 minutes to 1 hour. This temporal resolution is insufficient to capture the instantaneous effects of waves. Consequently, the
42 term “sea level” commonly excludes these waves in studies using tide gauge data. Tides refer to the sea level component driven
43 by astronomical forces, most notably in relation to our Moon, followed by our Sun. Tides are largely periodic and can be
44 predicted with sufficient precision to be treated as deterministic (Pugh and Woodworth, 2014b). Non-tidal residuals are the
45 differences obtained after subtracting mean sea level and tides from the observed sea level. Non-tidal residuals are mainly
46 driven by meteorological effects such as storms but can also include tide–surge interaction, tide gauge errors, or tidal prediction
47 errors. Hence, non-tidal residuals are also referred to as storm surges, or simply surges, and are treated as a stochastic
48 component.

49 Various methods have been developed to estimate ESLs, based on extreme value theory (Coles, 2001), joint probability
50 methods (Pugh and Vassie, 1978), combinations of both (Tawn, 1992; Batstone et al., 2013; Baranes et al., 2020), or mixture
51 models (Caruso and Marani, 2022; D’Arcy et al., 2023; Palmer et al., 2024). A commonly used extreme value theory-based
52 approach involves fitting declustered peak-over-thresholds—hourly (or more frequent), mean-detrended sea level observations
53 exceeding a specified height threshold and occurring at least a given time interval apart—to the generalised Pareto distribution
54 (Buchanan et al., 2016; Kopp et al., 2023). However, the peak-over-threshold-generalised Pareto distribution approach does
55 not explicitly account for the deterministic tidal component of sea level. Ignoring this deterministic tidal component has been
56 shown to result in biased ESL estimates (Dixon and Tawn, 1999).

57 In contrast, joint probability methods (JPMs) separate the deterministic tides and the stochastic surges before computing their
58 combined distribution. In JPMs, tide and surge are treated as random variables. Their sum, the storm tide, is used as a measure
59 of ESLs. The distribution of annual maximum storm tide is obtained after integrating the joint probability density function
60 (PDF) of tide and surge. However, the first proposed JPM (Pugh and Vassie, 1978) has three weaknesses. First, it does not
61 account for the temporal dependence of hourly storm tides, which is generally significantly autocorrelated. Second, it uses the
62 empirical distributions of tide and surge for the integration, limiting the range of estimable storm tides, particularly at the
63 extremes which are of interest. Tawn (1992) introduced the Revised JPM, which addressed the first weakness using the



64 extremal index (Leadbetter, 1983) and the second by modelling the upper tail of the surge distribution using an extreme value
65 distribution.

66 The third weakness is the assumption that tide and surge are mutually independent random variables. However, in reality, tide
67 and surge can be correlated at certain coasts due to drivers such as tide–surge interaction (Dixon and Tawn, 1994; Olbert et
68 al., 2013) and confounding variables such as seasonality (Pirazzoli and Tomasin, 2007; Williams et al., 2016). Batstone et al.
69 (2013) partially addressed this with the Skew Surge JPM (SSJPM), which uses the distribution of peak tide and skew surge
70 instead of hourly tide and non-tidal residuals. Peak tide and skew surge are often approximately independent (Batstone et al.,
71 2013; Williams et al., 2016), partially supporting this assumption of independence. However, correlation between peak tide
72 and skew surge can still be present, particularly at sites where correlation is driven by seasonal relationships (Williams et al.,
73 2016). Furthermore, Santamaria-Aguilar and Vafeidis (2018) identified non-seasonal correlation between peak tide and skew
74 surge at locations within shallow continental shelves with mixed semidiurnal tidal regimes. D’Arcy et al. (2023), Palmer et al.
75 (2024), and Baranes et al. (2020) are some recent studies that propose variations of the SSJPM that account for seasonal
76 correlation.

77 In this paper, we propose an approach that is agnostic to the underlying causes of the correlation and is therefore more general
78 and potentially applicable under a wider range of circumstances. Correlation affects not only the central estimates of predicted
79 ESLs, but also contributes to uncertainty in these estimates. Accounting for uncertainty in correlation, which can lead to wider
80 confidence intervals in predicted ESLs, provides a more complete characterisation of the uncertainty in predicted ESLs. Our
81 results also show that even weak correlation between peak tide and skew surge can have a statistically significant impact on
82 estimates of extreme annual maximum storm tides.

83 To address the challenge that correlation between tide and surge poses to JPMs, we propose a Copula JPM (CJPM). The CJPM
84 builds on the SSJPM by introducing a copula to account for correlation between peak tide and skew surge. Copulas have been
85 used to estimate joint probabilities of events such as compound flooding (Lee et al., 2024; Li et al., 2021; Yavuzdoğan and
86 Tanır Kayıkçı, 2020; Galiatsatou et al., 2019; Masina et al., 2015) or compound weather extremes (Tavakol et al., 2020;
87 Mesbahzadeh et al., 2019). However, their use for capturing correlation between tide and surge within JPMs has not been
88 extensively explored. A copula is used to combine the marginal distributions of peak tide and skew surge, derived using the
89 SSJPM, to obtain their joint PDF (Sklar, 1959; van Vliet, 2023). Copulas can take many functional forms and provide a flexible
90 framework for representing a wide range of correlation structures, including but not limited to linear correlation. This includes
91 correlation in the peak tide–skew surge joint PDF, regardless of the underlying drivers of said correlation (e.g. seasonal
92 relationships and tide–surge interaction). In other words, the CJPM does not assume that peak tide and skew surge are mutually
93 independent.

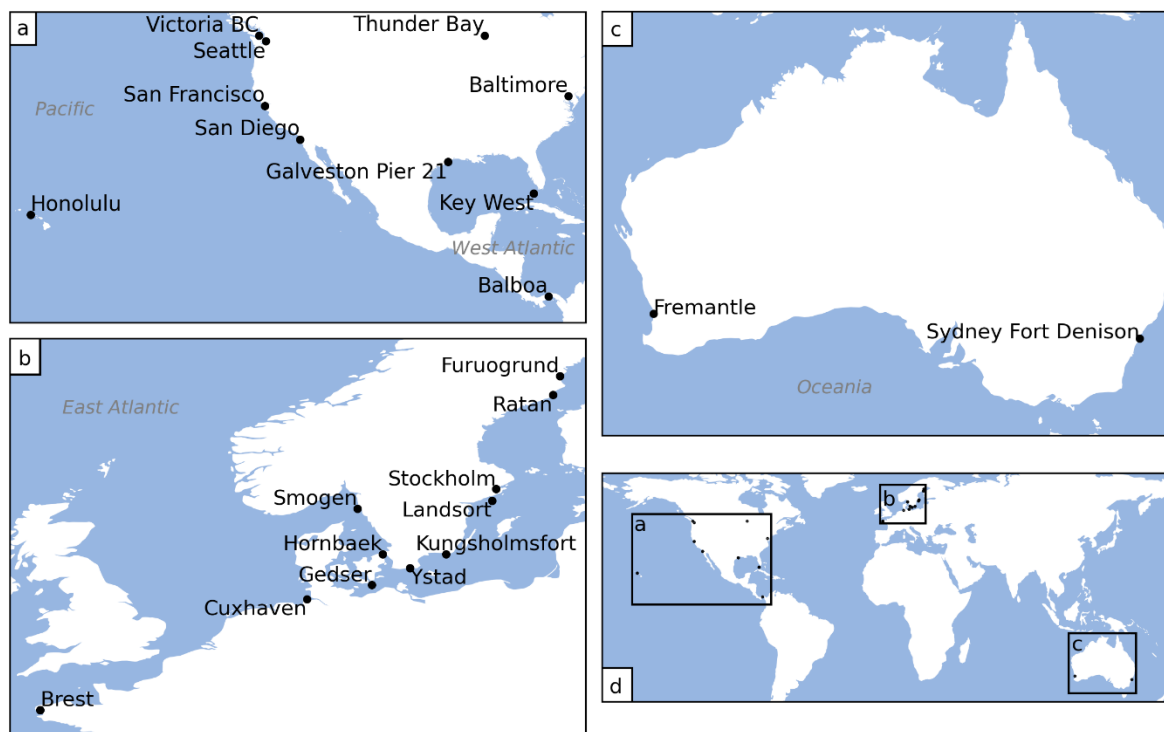
94 To test the robustness of the CJPM, we use long tide gauge records as they provide ample observations for validation. We
95 apply the CJPM to tide gauge records, from the GESLA 3.0 dataset, with at least a 90 % data completion rate over ≥ 100
96 calendar years. The CJPM and SSJPM are fitted to the qualifying tide gauge records, and the resulting return level estimates



97 are compared with empirical return levels. Cross-validation is also used to compare the performance of the CJPM and the
 98 SSJPM. In addition, we test the sensitivity of the CJPM and SSJPM to the generalised Pareto distribution threshold μ .

99 2 Data and Methodology

100 2.1 Data



101
 102 **Figure 1. Locations of the tide gauges used in this study. Coordinates and record lengths of each tide gauge are listed in Table 1.**

103 To validate the robustness of the Copula Joint Probability Method (CJPM), numerous long and high-resolution tide gauge
 104 records are required. These were obtained from the Global Extreme Sea Level Analysis (GESLA 3.0) dataset (Caldwell et al.,
 105 2015; Haigh et al., 2023; Woodworth et al., 2017). We note that the GESLA 4.0 dataset is now available and can be used for
 106 future studies. For consistency across tide gauges, records with sampling intervals shorter than hourly were averaged to produce
 107 hourly observations. No additional filtering was applied to the downsampled observations to avoid further data loss, which
 108 also aligns with the default option in the toolkit of Williams et al. (2019). To calculate unbiased annual means at each tide
 109 gauge record, observations in a calendar year were discarded if the calendar year had more than 876 missing hourly
 110 observations (i.e. below 90 % completion rate based on 8760 hourly observations per year). The 40 tide gauge records with
 111 observations remaining in at least 100 calendar years were shortlisted for this study (Table 1). These records originate from 23
 112 unique locations (Fig. 1). At 15 of the 23 locations, two or three alternative records from different contributors are available.



113 **Table 1. Tide gauge records used in this study, contributed by the Marine Environmental Data Section (MEDS), University of Hawaii**
 114 **Sea Level Center (UHSLC), National Oceanic and Atmospheric Administration (NOAA), Danish Meteorological Institute (DMI),**
 115 **Réseaux de référence des observations marégraphiques (REFMAR), Copernicus Marine Environment Monitoring Service**
 116 **(CMEMS), Swedish Meteorological and Hydrological Institute (SMHI), and the Bureau of Meteorology (BOM). A map of these**
 117 **coordinates is shown in Fig. 1. The length of usable records is given in years. The standard deviations of peak tides σ_x and skew**
 118 **surges σ_y and the maximum recorded peak tide X_{max} and skew surge Y_{max} are provided in metres (m).**

Continent	Country	Tide gauge name	Contributor	Lat.	Lon.	Length	σ_x	X_{max}	σ_y	Y_{max}
Americas	Canada	Thunder Bay Ontario	MEDS	48.4	-89.2	104	0.08	0.33	0.11	0.64
		Victoria BC	UHSLC	48.4	-123.4	107	0.27	1.42	0.13	0.86
	Panama	Balboa	UHSLC	9.0	-79.6	108	0.46	3.26	0.10	0.69
	USA	Baltimore	NOAA	39.3	-76.6	115	0.10	0.50	0.21	2.16
		Galveston Pier 21	UHSLC	29.3	-94.8	105	0.09	0.45	0.14	2.33
		Galveston Pier 21	NOAA	29.3	-94.8	108	0.09	0.45	0.14	2.33
		Honolulu	NOAA	21.3	-157.9	101	0.17	0.64	0.05	0.62
		Honolulu Hawaii	UHSLC	21.3	-157.9	109	0.17	0.64	0.05	0.62
		Key West	NOAA	24.6	-81.8	103	0.12	0.55	0.08	0.92
		Key West Fl	UHSLC	24.6	-81.8	102	0.12	0.55	0.08	0.92
		San Diego	NOAA	32.7	-117.2	110	0.31	1.47	0.07	0.45
		San Diego Ca	UHSLC	32.7	-117.2	108	0.31	1.47	0.07	0.51
		San Francisco	NOAA	37.8	-122.5	120	0.23	1.29	0.08	0.67
		San Francisco Ca	UHSLC	37.8	-122.5	120	0.23	1.31	0.08	0.67
Seattle		NOAA	47.6	-122.3	122	0.37	1.99	0.12	0.82	
Europe	Germany	Cuxhaven	UHSLC	53.9	8.7	101	0.21	2.02	0.34	3.65
		Gedser	UHSLC	54.6	11.9	108	0.07	0.32	0.21	1.80
	Denmark	Gedser	DMI	54.6	11.9	114	0.07	0.32	0.21	1.80
		Hornbaek	UHSLC	56.1	12.5	110	0.08	0.35	0.21	1.59
		Hornbaek	DMI	56.1	12.5	117	0.08	0.35	0.21	1.75
	France	Brest	UHSLC	48.4	-4.5	152	0.64	3.88	0.13	1.37
		Brest	REFMAR	48.4	-4.5	153	0.64	3.89	0.13	1.38
		Brest 60Minute	CMEMS	48.4	-4.5	104	0.64	3.89	0.13	1.37
	Sweden	Furuogrund	SMHI	64.9	21.2	105	0.12	0.48	0.22	1.23
		Furuogrund	CMEMS	64.9	21.2	105	0.11	0.48	0.22	1.23
		Kungsholmsfort	SMHI	56.1	15.6	131	0.08	0.35	0.17	1.20
		Kungsholmsfort	CMEMS	56.1	15.6	131	0.08	0.35	0.17	1.20
		Landsort	SMHI	58.8	17.9	118	0.10	0.38	0.16	0.85
		Landsort1	CMEMS	58.8	17.9	118	0.10	0.37	0.16	0.85
		Ratan	SMHI	64.0	20.9	129	0.11	0.46	0.21	1.24
		Ratan	CMEMS	64.0	20.9	129	0.11	0.46	0.21	1.24
		Smogen	SMHI	58.4	11.2	109	0.08	0.40	0.18	1.47
		Smogen	CMEMS	58.4	11.2	109	0.08	0.39	0.18	1.47
Stockholm		UHSLC	59.3	18.1	125	0.10	0.39	0.16	1.05	
Stockholm		SMHI	59.3	18.1	132	0.10	0.39	0.16	1.05	
Stockholm	CMEMS	59.3	18.1	132	0.10	0.39	0.16	1.05		
Ystad	SMHI	55.4	13.8	100	0.08	0.34	0.18	1.66		
Ystad	CMEMS	55.4	13.8	100	0.08	0.34	0.18	1.65		
Oceania	Australia	Fremantle	BOM	-32.1	115.7	106	0.14	0.67	0.14	1.01
		Sydney Fort Denison	BOM	-33.9	151.2	102	0.21	1.16	0.08	0.54



120
121 These records, despite originating from the same location, can vary in length. Our results at these locations were generally
122 similar across alternative tide gauge records. Therefore, for each location, we present results based on the record with the most
123 calendar years of observations. In case of ties, results based on records provided by the National Oceanic and Atmospheric
124 Administration (NOAA) and the Swedish Meteorological and Hydrological Institute (SMHI) are presented.
125 Hourly storm tide observations were obtained by subtracting the annual mean of the hourly tide gauge observations from the
126 tide gauge records (Rasmussen et al., 2018). Hourly tidal observations for each year were derived using the *MATLAB* tidal
127 harmonic-analysis package *UTide* (Codiga, 2011). To prevent double counting, a tidal cycle was defined as the interval
128 between one local minimum in the hourly tidal observations and the observation preceding the next local minimum (Koh et
129 al., 2024). Within each tidal cycle, the maximum tide and storm tide were identified, and their difference was defined as the
130 skew surge (Batstone et al., 2013). The standard deviations of peak tide and skew surge observations at each tide gauge record
131 are tabulated in Table 1, along with their maximum recorded values.

132 2.2 Background on Joint Probability Methods

133 In general, JPMs require the joint probability distribution f_{XY} of tide X and surge Y (Pugh and Vassie, 1978). In the Skew
134 Surge JPM (SSJPM), X and Y represent peak tide and skew surge respectively. Storm tide Z is treated as a random variable
135 and assumed to be the sum of random variables X and Y , i.e. $Z = X + Y$, with the cumulative distribution function (CDF) of
136 Z calculated as

$$137 F_Z(z) = \int_{-\infty}^{\infty} \int_{-\infty}^{z-x} f_{XY}(x, y) dy dx. \quad (1)$$

138 With sufficient observations of X , a good estimator of $F_Z(z)$ is (Tawn, 1992)

$$139 F_Z(z) = \left(\prod_{t=1}^N F_{Y|X}(z - X_t | x = X_t) \right)^{\frac{1}{N}}, \quad (2)$$

140 where X_t is the t th observation of X when ordered chronologically while the index t numbers tidal cycles from the earliest to
141 the most recent. N is the number of observations of X , and $F_{Y|X}$ is the conditional CDF of Y given X . However, due to the
142 difficulties associated with obtaining f_{XY} and $F_{Y|X}$, X and Y are often assumed to be independent random variables. Their
143 assumed independence implies that $F_{Y|X}$ is equal to F_Y , the marginal CDF of Y , thereby replacing the challenging task of
144 obtaining $F_{Y|X}$ with the simpler task of determining the univariate marginal distribution F_Y .

145 In the earliest JPM (Pugh and Vassie, 1978), one year of observations was considered sufficient to estimate the marginal
146 distributions of X and Y . Since Tawn (1992), the preferred estimator of F_Y is the piecewise function

$$147 F_Y(y) = \begin{cases} \hat{F}_Y(y) & \text{for } y \leq \mu, \\ G(y) (1 - \hat{F}_Y(\mu)) + \hat{F}_Y(\mu) & \text{for } y \geq \mu, \end{cases} \quad (3)$$



148 where the empirical CDF of surge \hat{F}_Y is used below a threshold μ , while the upper tail (i.e. above the threshold μ) is modelled
 149 using an extreme value distribution G . Since Batstone et al. (2013), the preferred parametric distribution for G is the generalised
 150 Pareto distribution (GPD), defined by the CDF

$$151 \quad G(y) = 1 - \left(1 + \xi \frac{y - \mu}{\sigma}\right)^{-\frac{1}{\xi}}, \quad (4)$$

152 where ξ is a shape parameter and σ is a scale parameter. In our implementation, μ was selected to be the level between the
 153 97.5th (Batstone et al., 2013) and the 99.7th (Baranes et al., 2020) percentiles of skew surge observations—common thresholds
 154 in studies where ≥ 100 years of records are available—for which the GPD provided the best fit (according to the Kolmogorov-
 155 Smirnov test). A sensitivity test showed that the SSJPM and CJPM exhibit similar sensitivity to μ within this percentile range.
 156 Bootstrapping was used to obtain the confidence intervals of ξ and σ , which represent uncertainty in the height of extreme
 157 skew surges and their distribution. While only the empirical CDF of skew surge, \hat{F}_Y , is required for the SSJPM, we also define
 158 the empirical CDFs of peak tide, \hat{F}_X , and storm tide, \hat{F}_Z , as

$$159 \quad \hat{F}_X(X_i) = \frac{i}{N + 1}, \quad \hat{F}_Y(Y_j) = \frac{j}{M + 1}, \quad \hat{F}_Z(Z_k) = \frac{k}{L + 1}, \quad (5)$$

160 where X_i is the i th observation of X when ordered in ascending order, i is the rank of X_i , and N is the number of observations
 161 of X . Y_j , j , and M are defined analogously for skew surge Y , and Z_k , k , and L for storm tide Z . Note that the X_i 's referenced
 162 here and the X_t 's in Eq. (2) are the same collection of peak tides. Their difference is that X_i 's are ordered by magnitude and
 163 X_t 's by time. Equations (2)—(5) are combined to obtain F_Z . In practical applications, it is useful to convert F_Z into a function
 164 of return level z and return period RP (in years) via the transformation

$$165 \quad RP(z) = \frac{1}{1 - F_Z^{T\theta(z)}(z)}, \quad (6)$$

166 where T is the average number of observations of Z per year and θ is the extremal index. The extremal index, θ , accounts for
 167 autocorrelation by reducing T to the effective average number of *independent* observations of Z per year (Leadbetter, 1983).

168 Similarly, the empirical return level of storm tides can be estimated as $\widehat{RP}(Z_k) = \left[1 - \hat{F}_Z^{T\theta(Z_k)}(Z_k)\right]^{-1}$. Given L observations
 169 of Z over D years, T can be estimated using $T = L/D$. $1/\theta(z)$ can be interpreted as the mean cluster size—the average length
 170 of time that Z remains above a threshold z before subsiding below z . We used the estimator (Ferro and Segers, 2003; Baranes
 171 et al., 2020)

$$172 \quad \frac{1}{\hat{\theta}(z)} = \frac{2 \left[\sum_{k=1}^{E(z)-1} (I_k - 1) \right]^2}{(E(z) - 1) \sum_{k=1}^{E(z)-1} [(I_k - 1)(I_k - 2)]}, \quad (7)$$

173 where $E(z)$ is the number of observations above z and I_k is the interexceedance time. The estimator $1/\hat{\theta}(z)$ was then fit to
 174 the functional form

$$175 \quad \frac{1}{\hat{\theta}(z)} = 1 + ae^{-bz}, \quad (8)$$



176 with $a, b \geq 0$.

177 2.3 Copula Joint Probability Method (CJPM)

178 Assuming statistical independence between X and Y may not be reasonable at certain locations (Santamaria-Aguilar and
 179 Vafeidis, 2018; Williams et al., 2016), potentially causing the SSJPM to be unsuitable at those locations. If the joint PDF f_{XY}
 180 can be obtained, then the assumption of independence between X and Y need not be made. To model f_{XY} , we propose using a
 181 copula. Introduced by Sklar (1959) (translated by van Vliet, 2023), Sklar’s theorem states that, in the bivariate case, every joint
 182 CDF F_{XY} can be expressed in terms of its marginals and a copula C such that

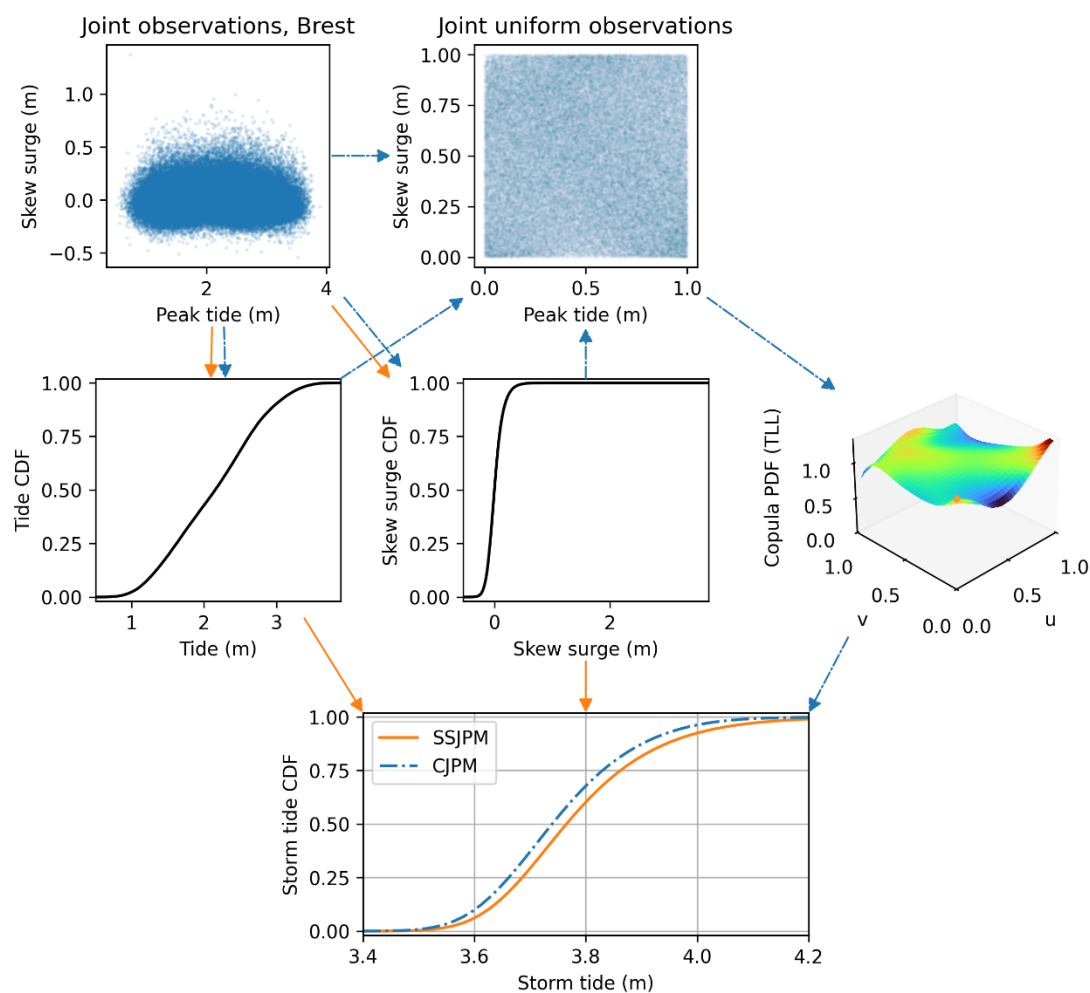
$$183 \quad F_{XY}(x, y) = C(F_X(x), F_Y(y)). \quad (9)$$

184 The copula C itself is a valid multivariate CDF that supports marginals uniform in the interval $[0,1]$. Taking the second-order
 185 mixed partial derivative of Eq. (9) yields

$$186 \quad f_{XY}(x, y) = c(F_X(x), F_Y(y))f_X(x)f_Y(y), \quad (10)$$

187 where $c = \frac{\partial^2 c}{\partial F_X \partial F_Y}$ is called the copula density of C . One interpretation is that the copula density $c(F_X(x), F_Y(y))$ combines the
 188 marginal PDFs f_X and f_Y to form the joint PDF f_{XY} via the product of the three terms. The CJPM consists of three overall
 189 steps: fitting the marginals following the SSJPM using Eq. (3)–(5), fitting the joint peak tide–skew surge observations to a
 190 copula as described in the following paragraph, and finally combining the copula with Eq. (1) to obtain Eq. (11)–(12) and
 191 consequently $F_Z(z)$. These steps and those of the SSJPM are shown using a schematic diagram, where their differences can be
 192 visualised (Fig. 2).

193 To fit our observations to a copula, we require *joint observations* of peak tide and skew surge (X_t, Y_t) —observations of pairs
 194 of X and Y measured during the same tidal cycle t . After computing the marginals \hat{F}_X and F_Y , we transformed the joint
 195 observations to pseudo observations—observations where marginals are uniformly distributed but may not be jointly
 196 uniform—using the probability integral transform $(U_t, V_t) = (\hat{F}_X(X_t), F_Y(Y_t))$. Using the Python library *pyvinecopulib*
 197 (Nagler and Vatter, 2025), we fitted the Transformation local likelihood (TLL) copula (Geenens et al., 2017) to the pseudo
 198 observations (U_t, V_t) using maximum likelihood estimation. Bootstrapping was used to obtain the confidence intervals of the
 199 TLL copula’s parameters, which represent uncertainty in peak tide–skew surge correlation. The TLL copula is a non-
 200 parametric local likelihood density estimator, adapting locally to the empirically observed distribution, making it suitable for
 201 representing both central and tail dependence structures without assuming a parametric form. Mathematical details of the TLL
 202 copula are provided in Appendix A. These properties make the TLL copula suitable for the CJPM for two reasons.
 203 First, just like observations of Z , observations of (X_t, Y_t) and, consequently, (U_t, V_t) are generally significantly autocorrelated.
 204 In JPMs, we preserve the autocorrelation in (X_t, Y_t) and make corrections to Z using the extremal index θ . Local likelihood
 205 density estimators have shown some success at handling autocorrelated observations (Geenens and Wang, 2018).



206

207 **Figure 2. Schematic of the SSJPM (orange solid) and CJPM (blue dash-dotted), illustrated using tide gauge records and intermediate**
 208 **results from Brest, France. In the SSJPM, the marginal distributions of peak tide and skew surge are fitted to observations, then**
 209 **convoluted to obtain the storm tide distribution. In the CJPM, the marginal distributions are fitted using the same procedure as the**
 210 **SSJPM. Using the marginal distributions, the joint observations are transformed to joint uniform observations to which the TLL**
 211 **copula is fitted. The storm tide distribution is subsequently obtained using Eq. (12).**

212 Second, the methodology of JPMs requires the full PDF f_{XY} , although we are most interested in the extremes of Z (e.g. return
 213 periods of at least 10 years). Certain studies into the correlation between tides and surges address this by conditioning on large
 214 values of Z (Arns et al., 2020; Ragno et al., 2024). However, we suspect this introduces collider bias into analyses. We further
 215 explain collider bias and discuss its implications in the discussion section. Hence, we require a copula such as the TLL that
 216 can represent the bulk of the observations while also capturing the characteristics in the upper tails. Local likelihood density
 217 estimators have also been shown to outperform conventional kernel density methods at the copula boundaries and the tails of
 218 the joint distributions (Geenens, 2014; Geenens et al., 2017; Geenens and Wang, 2018).



219 The requirement for a copula to fit the full PDF of f_{XY} despite our primary interest being in the extremes of Z also complicates
220 the use of traditional measures to evaluate the copula's goodness-of-fit. This is because those measures are dominated by the
221 bulk of the observations in the central tendency, while extremes make up a small proportion of these observations. Even in
222 cases where the TLL was deemed a worse overall fit to observations than one- and two-parameter parametric copulas, our
223 testing showed that the TLL outperformed parametric copulas at estimating return levels (see the discussion in Sect. 4 relating
224 to Fig. 4 and Table 2).

225 Combining Eq. (1) and (10) and making the change of variables $u = F_X(x)$ and $v = F_Y(y)$, we obtain

226
$$F_Z(z) = \int_0^1 \int_0^{F_Y(z - F_X^{-1}(u))} c(u, v) dv du. \quad (11)$$

227 With sufficient observations of X , a good estimator of Eq. (11) is

228
$$F_Z(z) = \left(\prod_{t=1}^N C_{V|U} \left(F_Y(z - X_t) | u = \hat{F}_X(X_t) \right) \right)^{\frac{1}{N}}, \quad (12)$$

229 where $C_{V|U}$ is the conditional CDF of V given U , and N is the number of observations of X . Note the similarities between
230 Eq. (2) and (12), where $F_{Y|X}$ is replaced by $C_{V|U}$, Y by $F_Y(Y)$, and X by $\hat{F}_X(X)$. Equation (12) can be expressed in terms of
231 return levels and return periods by combining Eq. (12) with Eq. (6)–(8).

232 2.4 Cross-validation

233 As the CJPM has more parameters than the SSJPM, it is important to assess whether the CJPM exhibits overfitting. Cross-
234 validation evaluates the performance of the CJPM and SSJPM on their ability to describe the data on which they were trained
235 and their ability to predict unseen data. In this study, we employed 4-fold cross-validation.

236 In the 4-fold cross-validation, each tide gauge record was shuffled and divided into four equal subsets. In each iteration, three
237 subsets were used for training (i.e. the CJPM and SSJPM were fitted to these subsets), and the remaining subset was used for
238 validation. During training, the parameters a and b in Eq. (8) were fixed at specified values because shuffling the tide gauge
239 data destroys the autocorrelation represented by θ . Hence, a and b were set to the values obtained when fitting the JPMs to
240 the full tide gauge records.

241 The training error for each JPM was calculated by first determining the empirical return periods of the 20 largest observations
242 in the training set. Model-predicted return levels for these return periods were then obtained. The mean absolute difference
243 between the 20 observations and the model-predicted return levels were defined as the training error. The validation error was
244 calculated analogously using the 20 largest observations from the validation set instead of the training set.

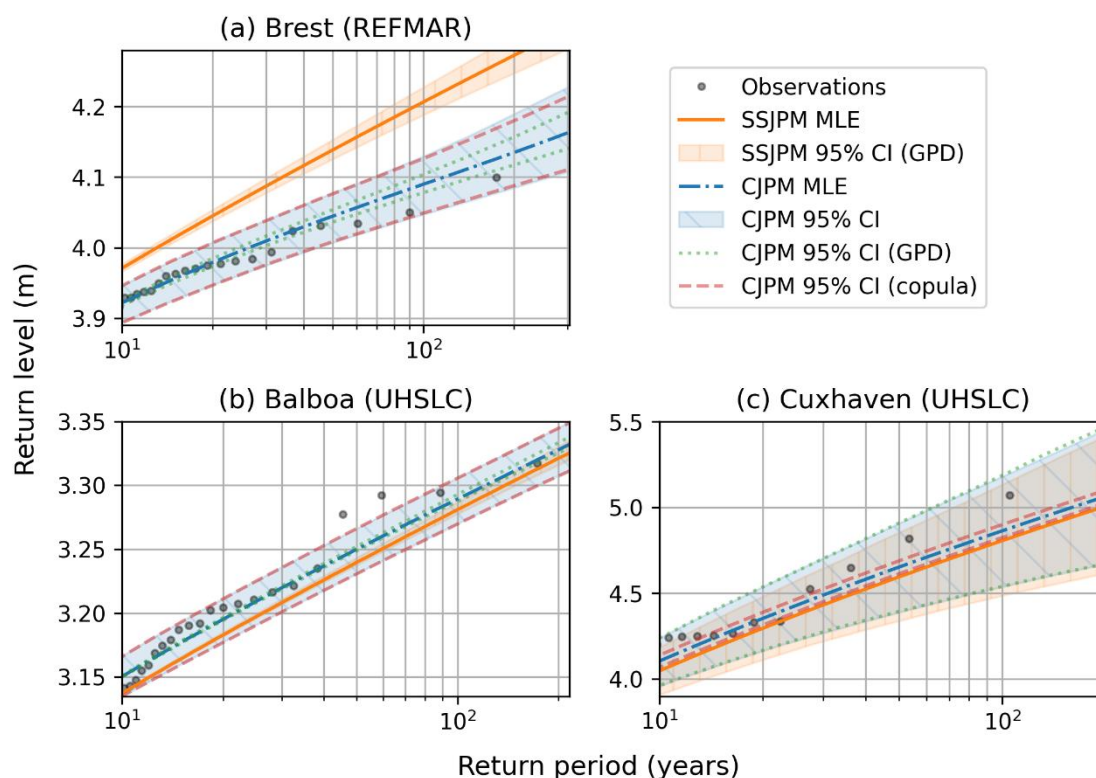
245 This procedure was repeated for four iterations such that each subset served as the validation set once. The performance of
246 each JPM was evaluated by aggregating the training and validation errors across all folds.



247 However, shuffling a tide gauge record breaks any seasonality that may be present in the record. To preserve the effect that
 248 seasonality may have on the performance of the CJPM and the SSJPM, we also employed an *unshuffled 4-fold cross-validation*
 249 where each tide gauge record was divided into four equal contiguous subsets without shuffling.

250 3 Results

251 We illustrate the impact of incorporating a copula into the SSJPM by highlighting results from three locations (Fig. 3). We
 252 plot return levels as a function of return period, i.e. the inverse of Eq. (6), which represents the average number of years
 253 between two instances of storm tide Z exceeding a given height. For instance, the CJPM curve passes through the coordinates
 254 (50 years, 4.05 m) at Brest (Fig. 3a), indicating that the CJPM estimates Z to exceed 4.05 m on average once every 50 years.
 255 As the estimator $1/\theta(z)$ is only valid at large values of z , we limit our discussion to return periods above 10 years.



256

257 **Figure 3. Return levels as a function of return period at three tide gauge locations. Grey points denote empirical return levels. The**
 258 **SSJPM's central estimate and its 95 % confidence interval are represented by the orange solid line and the vertically hatched shaded**
 259 **region respectively. The corresponding CJPM results are shown using the blue dash-dotted line and the diagonally hatched shaded**
 260 **region. The green dotted lines indicate the CJPM's confidence interval due to uncertainty in GPD parameters (with copula**
 261 **parameters held constant), while the red dashed lines indicate the CJPM's confidence interval due to uncertainty in copula**
 262 **parameters (with GPD parameters held constant).**



263 The longest available tide gauge record is from Brest, France, with a duration of 153 years, provided by the Réseaux de
264 référence des observations marégraphiques (REFMAR). Here, there is a clear difference between the estimates of the CJPM
265 and the SSJPM (Fig. 3a). When compared to the empirical return levels, the SSJPM appears to overestimate ESLs at Brest.
266 Pirazzoli and Tomasin (2007) also found that tide–surge interaction and seasonality caused the JPM to overestimate return
267 levels at Brest, though their results were based on tide and non-tidal residuals instead of peak tide and skew surge. The SSJPM’s
268 central estimate is 10 cm higher than that of the CJPM’s at a return period of roughly 60 years, further diverging at larger
269 return periods. A change of the same order of magnitude was also reported by D’Arcy et al. (2023) when incorporating peak
270 tide–skew surge interaction into the SSJPM and applying it to tide gauge records of Sheerness, England. Compared to the
271 SSJPM, the CJPM shows closer visual and numerical agreement with the observed historical ESLs of Brest, with the CJPM’s
272 confidence intervals encompassing Brest’s empirical return levels. In the CJPM, the uncertainty in peak tide–skew surge
273 correlation and the uncertainty in the height of extreme skew surges contribute to uncertainty in estimated return levels. The
274 contributions from each source of uncertainty are shown by the red dashed and green dotted lines respectively, while the blue
275 shaded region represents their combined effect. Of the two sources of uncertainty, correlation appears to be the larger
276 contributor to the uncertainty in Brest’s estimated return levels, as indicated by the wider confidence interval. The uncertainty
277 associated with correlation alone, and the resulting confidence interval, is sufficient to encompass the empirical return levels
278 and thus capture the observed spread in empirical return levels. On the other hand, uncertainty in the height of extreme skew
279 surges alone does not appear to fully account for the observed variability in Brest’s empirical return levels. At a return period
280 of 100 years, the confidence interval of the CJPM has a width of 8 cm when correlation uncertainty is included, and 3 cm
281 otherwise. By accounting for the uncertainty in peak tide–skew surge correlation, the CJPM produces a more wider confidence
282 interval which better agrees with empirical return levels. We found similar results at Honolulu, San Diego, San Francisco, and
283 Thunder Bay, where correlation contributes more to uncertainty than the height of extreme skew surges when estimating return
284 levels and the CJPM shows closer agreement with empirical return levels than the SSJPM.

285 Our results at Balboa, Panama, illustrate a scenario where the central estimates of the CJPM and the SSJPM do not diverge
286 drastically, but the CJPM shows somewhat closer agreement with the empirical return levels by accounting for uncertainty in
287 peak tide–skew surge correlation (Fig. 3b). The difference in their central estimates is less than 1.4 cm between return periods
288 of 10 and 100 years. This difference is small when compared to the CJPM’s confidence interval, which has a width of more
289 than 3 cm and fully encompasses the SSJPM’s estimates. This indicates that the SSJPM’s estimates fall within the range of
290 plausible outcomes represented by the CJPM. Similar to Brest, uncertainty in the CJPM’s return level estimates is largely
291 driven by uncertainty in peak tide–skew surge correlation, with relatively small contribution from uncertainty in the height of
292 extreme skew surges. With the exception of two outliers, the uncertainty associated with correlation encompasses the empirical
293 return levels, which the uncertainty associated with extreme skew surge distribution does not capture. Other locations where
294 the CJPM and SSJPM had similar central estimates but the uncertainty in correlation was found to be at least as impactful as
295 the uncertainty in extreme skew surge heights are Landsort, Seattle, Sydney Fort, and Victoria BC.



296 At Cuxhaven, Germany, uncertainty in the height of extreme skew surges is the dominant contributor to uncertainty in return
297 level estimates, while uncertainty in peak tide–skew surge correlation has a smaller contribution (Fig. 3c). Here, the uncertainty
298 associated with extreme skew surge distribution is sufficient to capture most of the variability in empirical return levels.
299 However, with a central estimate approximately 6 cm lower than that of the CJPM, the SSJPM appears to slightly
300 underestimate ESLs when compared against empirical return levels. Meanwhile, the empirical return levels lie fully within the
301 CJPM’s confidence intervals. Nonetheless, this difference between the CJPM and SSJPM is small relative to their confidence
302 intervals of about 65 cm wide at the 100-year return period mark. Fremantle and Furuogrund are the other locations where
303 peak tide–skew surge correlation does not contribute substantially to uncertainty in estimated return levels, but leads to small
304 differences in central estimates when accounted for.

305 At the remaining 10 locations, the CJPM produced estimates that do not meaningfully differ from those of the SSJPM. The
306 central estimates and confidence intervals of both the CJPM and the SSJPM were similar, and the uncertainty in peak tide–
307 skew surge correlation had negligible influence on the uncertainty of estimated return levels.

308 In the 4-fold cross-validation, the CJPM and SSJPM had an overall training error of 3.4 cm and 5.1 cm respectively. This
309 indicates that the CJPM fits the observed data better, consistent with the results shown in Fig. 3 and other tide gauge locations.
310 The CJPM also performed better than the SSJPM in out-of-sample evaluation, with an overall testing error of 4.6 cm compared
311 to the SSJPM’s 5.7 cm. The increase in error between the training and testing datasets of 1.3 cm for the CJPM is larger than
312 the SSJPM’s 0.6 cm, suggesting a slightly greater tendency for overfitting in the CJPM compared to the SSJPM. This is
313 expected as the CJPM is a generalisation of the SSJPM and has more parameters, with the SSJPM being a special case of the
314 CJPM where $c(u, v) = 1$. Nonetheless, the CJPM’s lower testing error, together with existing evidence of peak tide–skew
315 surge correlation (D’Arcy et al., 2023; Santamaria-Aguilar and Vafeidis, 2018; Williams et al., 2016), suggests that the CJPM’s
316 increased complexity could be warranted. Furthermore, the cross-validation errors were computed using only the central
317 estimates of return levels without considering that the CJPM distinguishes correlation-associated uncertainty. Hence, the cross-
318 validation results do not fully reflect the additional insights the CJPM provides relative to the SSJPM.

319 Results of the unshuffled 4-fold cross-validation are consistent with these findings. In the unshuffled cross-validation, the
320 CJPM and SSJPM had an overall training error of 3.5 cm and 5.3 cm respectively, close to those of the shuffled cross-
321 validation. The overall testing error is 7.8 cm and 8.8 cm for the CJPM and SSJPM respectively. With an increase of 3.2 cm
322 and 3.1 cm respectively over the CJPM’s and SSJPM’s testing error in the shuffled cross-validation, these results indicate that
323 seasonality affects the performance of both the CJPM and SSJPM to a similar degree.

324 **4 Discussion**

325 The CJPM generalises the SSJPM by relaxing the SSJPM’s assumption of independence between peak tide and skew surge,
326 i.e. the SSJPM is a special case of the CJPM with the constraint $c(u, v) = 1$. Our results highlight two potential advantages of
327 the CJPM over the SSJPM. First, by accounting for the correlation between peak tide and skew surge, the CJPM’s central



328 estimates of return levels were found to be in closer visual and numerical agreement with empirical return levels than the
329 SSJPM. Second, the CJPM accounts for the uncertainty in peak tide–skew surge correlation. The isolated impact of peak tide–
330 skew surge correlation on the uncertainty in estimated return levels can be used to determine whether peak tide–skew surge
331 correlation causes the central estimates of the CJPM and SSJPM to deviate significantly. Combining correlation uncertainty
332 with other sources of uncertainty can yield a wider, more conservative confidence interval in return level estimates which
333 could more fully reflect the total uncertainty in ESL estimates.

334 Sources of correlation between peak tide and skew surge may include tide–surge interaction (Santamaria-Aguilar and Vafeidis,
335 2018; Olbert et al., 2013; Dixon and Tawn, 1994) and confounding variables such as seasonality (D’Arcy et al., 2023; Williams
336 et al., 2016). Tide–surge interaction has been observed to occur in shallow (Idier et al., 2019; Mawdsley and Haigh, 2016) and
337 enclosed (Costa et al., 2023; Yang et al., 2023) water bodies. These geographical characteristics are found where the Brest and
338 Thunder Bay tide gauges are located (GEBCO Compilation Group, 2024; Kastrisios et al., 2023)—two locations where the
339 CJPM and the SSJPM differed by more than 10 cm in their central estimates. However, the CJPM neither identifies the causes
340 of correlation nor distinguishes between different drivers of correlation. The CJPM only leverages observed correlation to
341 produce estimates of return levels that account for correlation. Further research will be needed to identify the presence of tide–
342 surge or peak tide–skew surge interaction at these locations, understand the underlying dynamics, and quantify their effects.
343 Pirazzoli and Tomasin (2007) also found indications that tide–surge interaction and seasonal correlation between tides and
344 non-tidal residuals are present at Brest. Artefacts may also manifest in observations of tide due to errors in harmonic analysis
345 (Horsburgh and Wilson, 2007), which can potentially induce correlation. In cases where a correlation structure between peak
346 tide and skew surge is absent or negligible, the copula density $c \approx 1$ and the CJPM converges to the SSJPM. Additionally, if
347 hourly tides and non-tidal residuals are independent, we expect the SSJPM and CJPM to converge to the Revised JPM of Tawn
348 (1992). Conversely, copulas could also be applied to the Revised JPM to account for correlation between hourly tides and non-
349 tidal residuals.

350 We also note that the specific value of μ , restricted to values between the 97.5th and 99.7th percentile of skew surges, did not
351 have a statistically significant impact on return levels estimated by either the CJPM and the SSJPM. We chose this percentile
352 range as it is typical of the thresholds used in similar studies. However, lower percentiles can also be tested and, if viable,
353 chosen, potentially reducing the uncertainty in the distribution of extreme skew surges. Identifying an appropriate value for μ
354 is challenging, and exploratory analyses of the underlying sea level records remain the most reliable method to do so. To our
355 knowledge, there is no generalised or global “one-size-fits-all” algorithm for selecting μ that can be regarded as a preferred
356 default across variants of SSJPM. This remains an important question under active investigation and several approaches have
357 been proposed (Collings et al., 2025; Murphy et al., 2025; Belzile et al., 2023). Incorporating these methods into the CJPM, or
358 the SSJPM in general, could be explored in future studies. When using the CJPM for local-scale analyses, we recommend
359 choosing μ based on the aforementioned exploratory analysis. However, this approach is most likely impractical for global-
360 scale analyses.



361 Our cross-validation results indicate that the CJPM, despite having more parameters than the SSJPM, does not substantially
362 overfit to training data; relative to the SSJPM, the CJPM maintains comparable generalisation performance despite its
363 increased model complexity. In the unshuffled cross-validation, results indicate that seasonality affects the performance of the
364 CJPM and the SSJPM to a similar extent. Other studies have accounted for seasonality in JPMs by isolating the winter and
365 summer seasons (Baranes et al., 2020), adding a seasonal covariate to the GPD parameters (D’Arcy et al., 2023), or using a
366 mixture distribution (Palmer et al., 2024). Similar approaches could be considered for future iterations of the CJPM.
367 Unfortunately, our methodology and analyses require ample observations for validation. Our criterion for tide gauge records
368 to be at least 100 years long precludes many regions from our analysis, especially those with shorter records that may be the
369 most vulnerable to ESLs. Many more tide gauge records could have been included in this study if we reduced our requirements
370 to, for example, 50 years. However, even our threshold of 100 years leaves us with few observations for cross-validation. With
371 the shortest possible testing sets of 25 years, only two observations have empirical return periods above 10 years. The third to
372 twentieth largest observations have empirical return periods between 1.25 and 8.33 years. Hence, long sea level records are
373 needed for robust cross-validation. However, the use of century-scale tide gauge records also raises potential issues related to
374 non-stationarity. Although annual de-meaning is applied to remove changes in mean sea level, cases where extremes are non-
375 stationary over such timescales have been observed (Calafat et al., 2021; D’Arcy et al., 2023; Palmer et al., 2024). In addition,
376 stationarity of the correlation between peak tide and skew surge could be of particular interest. Extending the CJPM to account
377 for non-stationarity via approaches such as time-varying GPD and copula parameters could be an interesting direction for
378 future research.

379 Quantifying the differences between the results of the SSJPM and the CJPM may be of scientific interest. The discovery of
380 substantial and statistically significant differences could motivate further investigation into the underlying causes of the
381 correlation between peak tide and skew surge. Practitioners can evaluate the copula-associated uncertainty and the differences
382 in return level estimates at a location to determine whether the additional complexity of the CJPM is warranted, or if the SSJPM
383 is adequate for their application.

384 The benefits of using a copula to model the correlation structure between peak tide and skew surge can be challenging to
385 quantify. Ragno et al. (2024) also utilised copulas when estimating quantiles of storm tide in the Adriatic and Tyrrhenian Seas.
386 However, following Arns et al. (2020), they selected joint peak tide–skew surge observations by conditioning on a storm tide
387 threshold. Due to collider bias, also known as Berkson’s paradox, such conditioning can create a false correlation between
388 peak tide and skew surge (McElreath, 2020). Collider bias occurs when a response variable (the collider) is causally influenced
389 by two or more explanatory variables that may not necessarily be mutually dependent, as is the case of tide, surge, and their
390 sum, i.e. storm tide. Conditioning on a collider, such as storm tide, causes a false correlation between the explanatory variables.
391 This false correlation can cause the benefits of using a copula to be overstated if comparisons are not done fairly and carefully.
392 When conditioning on large storm tides, the conditioned observations are likely to have a strong negative linear correlation



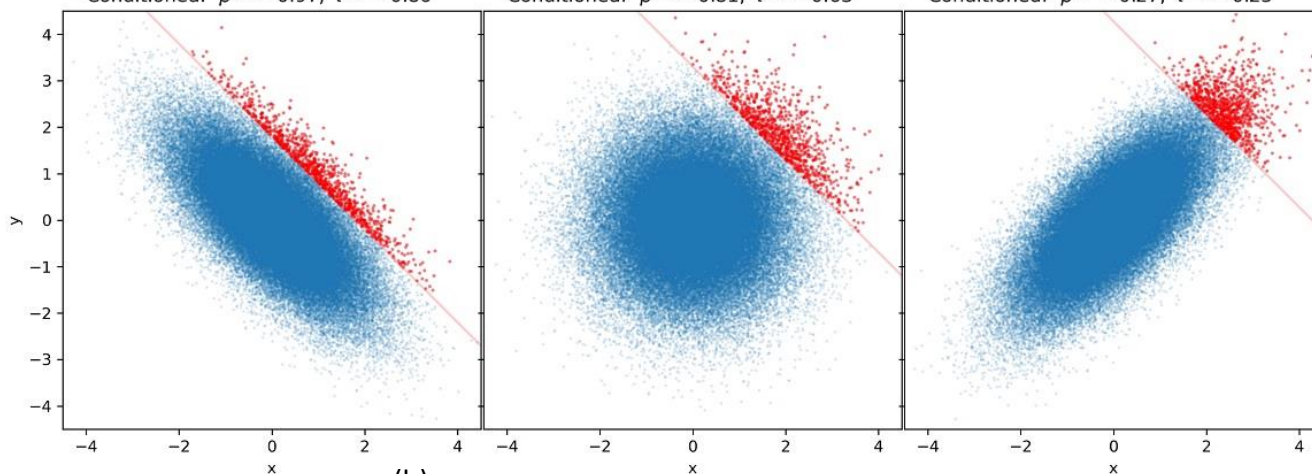
393

(a)

True: $\rho = -0.70, \tau = -0.49$
 Sample: $\rho = -0.70, \tau = -0.50$
 Conditioned: $\rho = -0.97, \tau = -0.86$

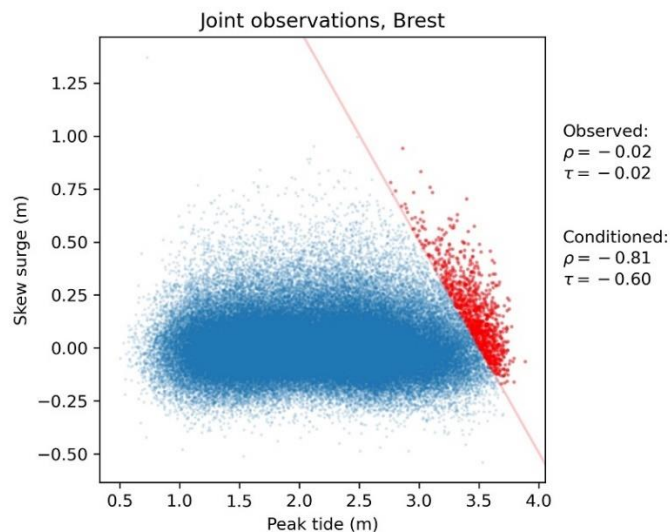
True: $\rho = 0.00, \tau = 0.00$
 Sample: $\rho = -0.00, \tau = -0.00$
 Conditioned: $\rho = -0.81, \tau = -0.63$

True: $\rho = 0.70, \tau = 0.49$
 Sample: $\rho = 0.70, \tau = 0.50$
 Conditioned: $\rho = -0.27, \tau = -0.25$



394

(b)



395

396 **Figure 4. (a) Illustration of collider bias or Berkson’s paradox using randomly generated Gaussian samples with specified correlation**
 397 **coefficients. Three samples, each of size 100,000, are generated from bivariate Normal distributions with standard Normal marginals**
 398 **and correlation coefficients of -0.7 , 0 , and 0.7 . For each (x, y) pair, we compute $z = x + y$ and identify the 99th percentile of z**
 399 **within each sample. (x, y) pairs for which z is greater than or equal to the 99th percentile (i.e. conditioned data) are highlighted in**
 400 **red. The conditioned data can exhibit a strong negative correlation even when the underlying data do not. When working with**
 401 **conditioned data in isolation, care should be taken when making conclusions about the correlation between x and y . Conversely, the**
 402 **correlation measures of the full sample do not necessarily reflect the correlation structure at extreme values of z . (b) Illustration of**
 403 **collider bias or Berkson’s paradox using tide gauge records from Brest, France.**

404 regardless of the correlation (or lack thereof) in the original unconditioned observations (Fig. 4). Hence, if the SSJPM is applied
 405 to both the conditioned and unconditioned observations, return levels based on the conditioned observations are likely to be



406 overstated compared to results based on unconditioned observations. The SSJPM is not equipped to handle correlation and not
407 designed to be applied on observations conditioned this way. For this reason, comparisons to the SSJPM should be done against
408 results obtained from applying the SSJPM to unconditioned observations, as we have done in this paper. On the other hand,
409 applying the CJPM to either the unconditioned or conditioned observations should yield similar results as the CJPM can
410 account for the correlation (linear and non-linear) that may be present in either set of observations.

411 Collider bias may also cause measures of linear correlation between large storm tide-conditioned peak tide and skew surge to
412 be misunderstood. Such measures, which indicate that peak tide and skew surge (or tides and surges in general) are negatively
413 correlated at large storm tides, may be misunderstood to imply that extreme surges are more common during lower high tides
414 (or lower tide levels) and less common during higher high tides (or higher tide levels), even if this is not necessarily the case
415 (Fig. 4). To date, the most comprehensive set of peak tide–skew surge correlation measures is presented in Arns et al. (2020).
416 Supplementary Figure 2 of Arns et al. (2020) also helps illustrate how collider bias occurs due to conditioning on storm tides:
417 at the six locations shown, the Kendall rank correlation coefficient between peak tide and skew surge is close to 0 without any
418 conditioning but ranges from -0.72 to -0.55 when conditioned on large storm tides.

419 Linear correlation between peak tide and skew surge in the full length of tide gauge observations tends to be close to 0 as most
420 observations (e.g. the central tendency) are not correlated. Nevertheless, significant and genuine (not arising from collider
421 bias) correlation structure, both linear or non-linear, may exist in some subset of the observations' domain. The existence of
422 such correlation structure may statistically significantly influence the SSJPM, as evidenced by our results at Brest, or
423 potentially JPMs in general. For this reason, we found that measures of linear correlation may be insufficient for assessing the
424 importance of introducing a copula to the SSJPM for a given location or sea level record. Instead, a popular statistical method
425 for identifying and quantifying tide–surge interaction uses timing-based measures (Costa et al., 2023; Horsburgh and Wilson,
426 2007), although they are not useful for modelling peak tide–skew surge correlation in JPMs. This also reflects the difficulty of
427 defining useful measures of correlation between the magnitudes of tides and surges in the context of JPMs.

428 To illustrate the limitations of linear correlation measures, we provide a comparison of the TLL copula to parametric copulas
429 using the tide gauge records from Brest. The parametric copulas used are tabulated in Table 2, where various goodness-of-fit
430 scores and the Kendall's τ of each copula are also tabulated. The return levels and return periods of each CJPM and the SSJPM
431 are plotted in Fig. 5. Relative to empirical return levels with return periods of at least 10 years, the TLL-based CJPM achieved
432 a mean absolute error of 0.9 cm while the parametric copula-based CJPM had an average mean absolute error of 6.3 cm
433 (Fig. 5). This is despite the TLL copula not necessarily being the best fit to the observations (Table 2). This may be due to
434 uncorrelated central tendency having significant weight in the copulas' goodness-of-fit measures. This indicates that a copula
435 deemed to fit the data better may not produce the best estimates of return levels, hence our decision to use the TLL copula. We
436 also find that the Kendall's τ of all fitted copulas are similar and small in magnitude (Table 2) despite producing very different
437 return levels and return periods (Fig. 5). This indicates that the Kendall's τ of peak tide and skew surge would likely be a poor
438 indicator of the presence of any non-linear correlation structure and whether the copula-based CJPM should be adopted over
439 the SSJPM. A more helpful assessment of whether a copula is important for a given location could be to apply both the CJPM



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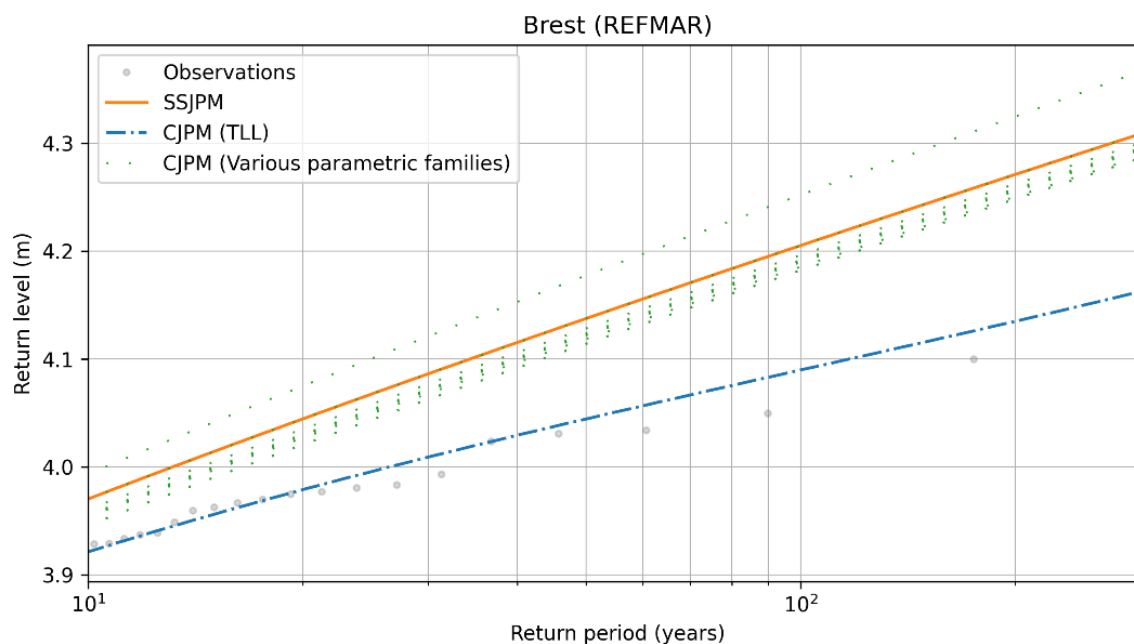
441

442 **Table 2. Goodness-of-fit scores and Kendall's τ of copula families fitted to the Brest (REFMAR) tide gauge record. AIC, BIC, and**
 443 **mBIC refer to the Akaike, Bayesian, and modified Bayesian information criterion respectively. The best-fitting copula based on each**
 444 **criterion is emphasised in bold.**

Copula	τ	mBIC	BIC	AIC	Log likelihood
Independence	0	4.61	0	0	0
Gaussian	-0.01	-42.60	-42.81	-52.39	27.20
Student's t	-0.02	-68.54	-68.76	-87.92	45.96
Clayton	-0.02	-218.39	-218.60	-228.18	115.09
Gumbel	-0.02	-156.77	-156.98	-166.57	84.28
Frank	-0.02	-91.05	-91.26	-100.85	51.42
Joe	-0.02	-177.38	-177.59	-187.17	94.58
Clayton-Gumbel (BB1)	-0.02	-206.80	-207.01	-226.17	115.08
Joe-Gumbel (BB6)	-0.02	-165.80	-166.01	-185.17	94.58
Joe-Clayton (BB7)	-0.02	-206.80	-207.01	-226.17	115.08
Joe-Frank (BB8)	-0.03	-259.38	-259.59	-278.75	141.38
TLL	-0.02	-118.03	-118.24	-803.44	473.24

445

446 and the SSJPM to a sea level record and determine whether the estimates of the SSJPM lie within the CJPM's confidence
 447 interval associated with peak tide-skew surge correlation. A statistically significant difference in estimates implies that the
 448 original observations were correlated, at least to a degree that can statistically significantly influence the SSJPM's results. The
 449 mean absolute error of TLL- and parametric-based CJPMs were also computed for the remaining 22 locations and compared
 450 in the same manner as at Brest. We found that the TLL-based CJPM performs similarly to the parametric-based CJPMs, with
 451 noticeable improvements at some locations including San Francisco, Seattle, Cuxhaven, and Fremantle. This suggests that
 452 while the flexibility and additional complexity of the TLL copula may not be necessary in all cases, it provides a robust and
 453 conservative option in the absence of prior knowledge of the underlying dependence structure, consistent with its qualitative
 454 properties discussed in Sect. 2.3. One potential downside is that the TLL copula can be more data-hungry than its parametric
 455 counterparts. If this is a concern, strategies such as defaulting to independence (i.e. the SSJPM) in the absence of clear evidence
 456 of dependence, selecting copula families via cross-validation with tail-focused scoring, or comparing a small set of standard
 457 parametric copulas (e.g. Gaussian, Student's t , Clayton, Gumbel, or Frank) can be considered.



458

459 **Figure 5. Return level estimates from the parametric copula-based CJPMs (green dotted), the TLL copula-based CJPM (blue dash-**
 460 **dotted), and the SSJPM (orange solid) at Brest. The parametric copulas used are listed in Table 2.**

461 The CJPM relaxes the assumption that the peak tide and skew surge distributions are independent, making it more general than
 462 the SSJPM and suitable at more locations. We showed that using a copula to account for the correlation structure between peak
 463 tide and skew surge can lead to central estimates different from the SSJPM’s while enabling quantification of correlation-
 464 associated uncertainty. Furthermore, the CJPM is agnostic to the underlying causes of the correlation, and fits a copula purely
 465 based on the empirical correlation structure of the underlying observations. For instance, the CJPM showed closer agreement
 466 with observations at Brest relative to the SSJPM without prior knowledge of whether the correlation between peak tide and
 467 skew surge is due to tide–surge interaction or seasonality. The agnostic nature of the CJPM makes it a good option for sites
 468 with insufficient information on the drivers of potential correlation between peak tide–skew surge and the correlation structure
 469 of their joint distribution. One such region is Southeast Asia, where tide–surge interaction is likely to be present at many coastal
 470 locations, but relatively few studies have been done to investigate its causes and how it may impact the height of storm tides
 471 and ESLs in this region (Koh et al., 2024). However, the sparse spatial distribution of tide gauges and the limited length of
 472 their records pose an additional challenge when estimating ESLs in this region. The performance of the CJPM under such
 473 conditions could be investigated in future studies. The CJPM contrasts with methods such as those proposed by Baranes et al.
 474 (2020), D’Arcy et al. (2023), and Palmer et al. (2024) that account for correlation due to seasonal relationships. On the other
 475 hand, this makes the CJPM unsuitable for attribution studies. Nevertheless, given the acceptable increase in computational
 476 cost of the CJPM over the SSJPM, the practical downsides of choosing the CJPM over the SSJPM appear limited when the
 477 objective is the estimation of return levels and return periods.



478 **5 Conclusions**

479 Correlation between peak tide and skew surge can measurably affect the estimation of extreme sea levels using joint probability
 480 methods, but quantifying the correlation structure and attributing correlation to its underlying physical causes is not trivial. We
 481 propose the Copula Joint Probability Method (CJPM), which builds on the Skew Surge Joint Probability Method (SSJPM) by
 482 introducing a copula to quantify the correlation without requiring prior understanding of its physical causes. Differing results
 483 between the SSJPM and CJPM may indicate the presence of non-negligible correlation and can motivate further investigation
 484 into underlying causes or drivers of the correlation, although the CJPM does not directly identify these drivers. In addition, the
 485 CJPM allows the contribution of correlation-associated uncertainty to estimated extreme sea levels to be quantified, providing
 486 additional insight beyond that available from the SSJPM. By not assuming independence between peak tide and skew surge,
 487 the CJPM is more general and may be applicable across a wider range of locations than the SSJPM. Further studies could
 488 evaluate the performance of the CJPM when applied to shorter tide gauge records (e.g., < 30 years) to assess its suitability at
 489 locations with limited data availability, such as Southeast Asia. Incorporating newer and improved methods for threshold
 490 selection of extreme skew surges could also be explored and may further improve the robustness of the CJPM.

491 **Appendix A: Transformation local likelihood (TLL) copula**

492 The TLL copula is obtained by first transforming joint observations (U_t, V_t) from the *copula space* to a *Gaussian layer*. Then,
 493 a local likelihood density estimator is used to estimate the PDF in the Gaussian layer before transforming the PDF back to the
 494 copula space to obtain the copula density. The TLL copula is defined as follows. We obtain joint observations in the Gaussian
 495 layer via $(A_t, B_t) = (\Phi^{-1}(U_t), \Phi^{-1}(V_t))$, where Φ is the CDF of the standard normal distribution. The marginals of the joint
 496 observations (A_t, B_t) are standard normally distributed, i.e. $A \sim N(0,1)$ and $B \sim N(0,1)$, and we denote their joint PDF as
 497 f_{AB} . According to Eq. (10), the copula density can be written as

498
$$c(u, v) = \frac{f_{AB}(\Phi^{-1}(u), \Phi^{-1}(v))}{\phi(\Phi^{-1}(u))\phi(\Phi^{-1}(v))}, \quad (A1)$$

499 where ϕ is the PDF of the standard normal distribution. Hence, c can be determined from f_{AB} . $\log f_{AB}$ is approximated using
 500 a second-order polynomial

502
$$\log f_{AB}(a', b') \approx P(a' - a, b' - b \mid \alpha_{a,b})$$

 503
$$= \alpha_{0;(a,b)} + \alpha_{1,0;(a,b)}(a' - a) + \alpha_{0,1;(a,b)}(b' - b)$$

 504
$$+ \alpha_{2,0;(a,b)}(a' - a)^2 + \alpha_{1,1;(a,b)}(a' - a)(b' - b) + \alpha_{0,2;(a,b)}(b' - b)^2, \quad (A2)$$

501 where the polynomial coefficients

505
$$\alpha_{a,b} = [\alpha_{0;(a,b)} \quad \alpha_{1,0;(a,b)} \quad \alpha_{0,1;(a,b)} \quad \alpha_{2,0;(a,b)} \quad \alpha_{1,1;(a,b)} \quad \alpha_{0,2;(a,b)}]^T \quad (A3)$$

506



507 are functions of a and b and are fitted to (A_t, B_t) . The best fitting polynomial coefficients are obtained by solving the weighted
508 maximum likelihood problem

$$\begin{aligned} 509 \quad \hat{\alpha}_{a,b} = \arg \max_{\alpha_{a,b}} & \left\{ \sum_{t=1}^N K \left(\begin{bmatrix} a \\ b \end{bmatrix} \mid \begin{bmatrix} A_t \\ B_t \end{bmatrix}, \mathbf{B}\mathbf{B}^T \right) P(A_t - a, B_t - b \mid \alpha_{a,b}) \right. \\ 510 & \left. - N \iint_{\mathbb{R}^2} K \left(\begin{bmatrix} a \\ b \end{bmatrix} \mid \begin{bmatrix} a' \\ b' \end{bmatrix}, \mathbf{B}\mathbf{B}^T \right) \exp \left(P(a' - a, b' - b \mid \alpha_{a,b}) \right) da' db' \right\}, \quad (\text{A4}) \end{aligned}$$

511 where

$$512 \quad K(\mathbf{x} \mid \boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{1}{2\pi\sqrt{\det(\boldsymbol{\Sigma})}} \exp \left(-\frac{1}{2} (\mathbf{x} - \boldsymbol{\mu}) \boldsymbol{\Sigma}^{-1} (\mathbf{x} - \boldsymbol{\mu}) \right) \quad (\text{A5})$$

513 is the Gaussian kernel (i.e. the PDF of the multivariate normal distribution) with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. \mathbf{B} is
514 a bandwidth matrix where $\mathbf{B}\mathbf{B}^T = \boldsymbol{\Sigma}$. $\mathbf{B} = 3N^{-1/10} \hat{\boldsymbol{\Sigma}}_{AB}^{1/2}$, where $\hat{\boldsymbol{\Sigma}}_{AB}$ is the empirical covariance matrix of (A_t, B_t) , has been
515 found to produce good results and is used in *pyvinecopulib*'s implementation (Nagler and Vatter, 2025; Nagler, 2018). Finally,
516 the approximation $f_{AB}(a, b) \approx \exp(\alpha_{0;(a,b)})$ is made, and the TLL copula density is given by

$$517 \quad c(u, v) = \frac{\exp \left(\alpha_{0;(\Phi^{-1}(u), \Phi^{-1}(v))} \right)}{\phi(\Phi^{-1}(u)) \phi(\Phi^{-1}(v))}. \quad (\text{A6})$$

518 Renormalisation is applied to Eq. (A6) to ensure that it is a valid probability density that integrates to 1 (Geenens, 2017; Nagler,
519 2018; Nagler et al., 2017).

520 Code and data availability

521 The analysis code used to produce the figures and tables can be downloaded from <https://doi.org/10.5281/zenodo.19312731>
522 (Koh, 2026). The GESLA 3 sea level data set can be downloaded from <https://gesla787883612.wordpress.com> (Caldwell et
523 al., 2015; Haigh et al., 2023; Woodworth et al., 2017). The UTide MATLAB functions can be downloaded from
524 <https://www.mathworks.com/matlabcentral/fileexchange/46523> (Codiga, 2011). The *pyvinecopulib* Python library can be
525 downloaded from <https://doi.org/10.5281/zenodo.14841456> (Nagler and Vatter, 2025).

526

527



528 **Author contributions**

529 ZYK: conceptualisation, data curation, formal analysis, investigation, methodology, software, validation, visualisation, writing
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531 (review and editing). JD: conceptualisation, funding acquisition, methodology, and writing (review and editing). BPH:
532 conceptualisation, funding acquisition, and writing (review and editing). LYC: conceptualisation, funding acquisition,
533 methodology, project administration, resources, supervision, and writing (review and editing).

534 **Competing interests**

535 The contact author has declared that none of the authors has any competing interests.

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