Response to Reviewer #1

Manuscript: *The Ice Sheet State and Parameter Estimator (ICESEE) Library (v1.0.0): Ensemble Kalman Filtering for Ice Sheet Models*

We thank the reviewer for the careful and thorough evaluation of our manuscript. We appreciate the detailed comments and constructive suggestions. The review has helped us improve both the scientific presentation and the documentation of ICESEE. Below we provide a point-by-point response to all comments. Reviewer comments are shown in blue and our responses are provided below each comment.

Reviewer Comment:

Generally speaking, I find that certain sections in the description of the equations lack sufficient details, particularly the implementation of non-linear observation operators, the ensemble initialization and modelling of the model error. The descriptions of the test cases are also very brief and missing important details/discussions on the exact choices of the model and assimilations set-up, so that it is difficult to understand if the experiments show good results for good or wrong reasons.

There are several points in particular that I'm concerned about for the applications:

- Choice of the state variables:
  - o for both applications the velocity components (u and v) are presented as state variables, however the momentum equations are time independent so that the velocity is a diagnostic variable. As such, they are not part of the state vector in GilletChaulet (2020) and Choi et al. (2025), so that the momentum equations act as a nonlinear observation operator for the ice thickness (state variable) and the boundary conditions (bed elevation and friction parameters).
  - o For the application with ISSM, both h, s and b are estimated. However, I assume that the application with ISSM solves the shallow-shelf equations with the floatation condition, so that the 3 variable are not independent; i.e. s=b+h for the grounded part and s and h are related by the floatation condition for ice-shelfs.
- Estimation of b and c under floating ice: by definition, surface observations are not sensitive to the bed conditions where the ice is floating. In in Gillet-Chaulet (2020) and Choi et al. (2025), the basal conditions are not estimated where ice is floating, so I would expect the basal conditions below the ice-shelf to remain the same with or without assimilation on Fig. 7.
- Use of model error: From Sec. 2.3, I understand that what the authors call "Noise modelling" is the modelling of model errors. Usually model errors represent errors due to the model assumptions (simplified physics, resolution, etc. . .). Here the applications are twin experiments where the synthetic observations have been generated with the model, so we are considering the case of a perfect model experiment, in this case the use of model errors is not justified?
- Initial ensembles: the description of how the initial states are constructed is sometimes a little imprecise, but I also wonder about the decision to use biased initial ensemble states, e.g. for the smb in the icepack experiment. EnKF as most of the other DA algorithms rely on the assumptions of unbiased observations and models. Although it might actually work,

as it seems to be the case, I'm wondering about this choice for synthetical applications used to illustrate the library performances.

Response:

We thank the reviewer for these important general comments. We agree that the original manuscript did not sufficiently clarify several aspects of the EnKF formulation and the experimental setups. In the revised manuscript, we have expanded Sections 2, 4.1, and 4.2 to improve the description of nonlinear observation operators, ensemble initialization, model-error treatment, and the exact experimental configurations. Below we respond to each concern in order.

- **Choice of the state variables**
  - o We thank the reviewer for this important observation. We clarify that in ICESEE, velocity is treated as a diagnostic variable obtained from the momentum solver and stored in the augmented vector for convenience, rather than as an independently prognosed state variable. In ICESEE, this is implemented by recomputing the diagnostic velocity field with the coupled ice-flow model during the forecast step and storing it in the augmented vector. The analysis observation operator then selects the corresponding model-equivalent velocity values. We have revised the manuscript to clarify that velocity is not an independently prognosed state variable, but a diagnostic quantity obtained from the momentum solver.
  - o We also agree that in the ISSM application, h, s, and b are not independent. In the revised manuscript, we now explicitly state that ice thickness h is the prognostic variable, while surface elevation s is a diagnostic quantity geometrically constrained by the flotation condition and grounded/floating mask. Bed topography b is treated as an estimated parameter. After each assimilation update, before calling the ISSM solver, ICESEE restores geometric consistency by recomputing the base and surface elevations from the updated thickness and bed fields according to the hydrostatic flotation relations. In grounded regions, the base is set equal to the bed and the surface satisfies $s = b + h$, while in floating regions the surface and base are recomputed to satisfy flotation balance. This clarification has been added in Section 4.2.

- **Estimation of b and c under floating ice**

  We agree with the reviewer that surface observations have little direct sensitivity to basal conditions beneath floating ice. We have clarified this point in the revised manuscript. In the ISSM inversion step, ICESEE first recomputes the grounded/floating mask using the updated thickness and bed fields. Wherever the ice is floating, the basal friction coefficient is explicitly set to zero before the stress-balance inversion is performed. Thus, basal friction is not estimated beneath floating ice.

  Bed topography is treated separately from basal friction. It remains part of the augmented EnKF parameter vector, but direct bed information is available only on the prescribed radar-flight-line mask. Ten sparse bed surveys are assimilated at years 2, 8, 14, 20, 24, 30, 36, 40, 46, and 50. At each survey, a 15-km radius selects the observations used in the grouped patch-based local analysis of the bed block. Bed increments are applied only at those survey times, and the bed is held fixed between them; geometry restoration itself does not update the bed. We keep the treatment of bed topography sufficiently general so that if, e.g., a user assimilated gravity or submersible data constraining the bed topography, the bed topography could be estimated even under floating ice. The revised manuscript now documents the observation mask, the grouped local bed analysis and its observation-selection radius, and the grounded/floating geometry-restoration step separately. Friction is not interpreted beneath floating ice; those regions are shaded gray in the revised friction figures.

- **Use of model error**

  We thank the reviewer for pointing out this ambiguity. We agree that the terminology "noise modelling" in the original manuscript was misleading. In these twin experiments, we do not introduce model-error terms in the strict sense of structural model uncertainty. Instead, the stochastic perturbations represent uncertainty in the initial or forecast ensemble and maintain the covariance support required by the filter. In the revised manuscript, Sections 2.3 and 2.4 distinguish initial-ensemble perturbations, optional forecast perturbations, and covariance inflation. The ISSM experiment uses no additive or multiplicative inflation; its spread comes from the initialized random fields, while sparse bed observations are handled by the grouped local bed analysis described separately in the manuscript (Section 4.2.2).

- **Initial ensembles**

  We agree that the original description of the initial ensemble construction was too imprecise and that the rationale for the biased initial conditions required clarification. In the revised manuscript, we now describe the ensemble initialization procedure in much greater detail, including the generation of spatially correlated perturbations for both state and parameter fields.

  Regarding the displaced initial conditions (e.g., SMB offsets in the Icepack experiment and bed/friction perturbations in the ISSM experiment), we agree that zero-mean anomalies about a displaced prior mean do not make the prior itself unbiased. We have therefore removed that argument. The revised manuscript instead reports the prior construction explicitly, including its nonzero background offsets, spatially heterogeneous anomalies, bounds, and consistency adjustments. EnKF does not require the prior mean to equal the truth, but correction is possible only where the ensemble-projected covariance provides support in observation space; this limitation is now stated.

  The synthetic observations are generated independently from the reference trajectory by adding zero-mean Gaussian measurement errors. The displaced prior is generated separately and defines the initial background-state mismatch; it is not part of the observation-error model. This distinction is now stated explicitly without describing the prior as unbiased.

Detailed remarks:

Reviewer Comment:

Acronyms:

- o EnKF is defined multiple times and used to refer to ensemble Kalman Filters in general and the version proposed by Evensen (2003). Use different acronyms.
- o The Ensemble Transform KF is usually referred to as ETKF not EnTKF.

Response:

- We agree and have revised the terminology throughout the manuscript. "EnKF methods" is now used to refer to the family of ensemble Kalman filtering algorithms, while "stochastic EnKF" refers specifically to the perturbed-observation formulation of Evensen (1994, 2003).
- We thank the reviewer for pointing this out. The acronym has been corrected throughout the manuscript.

Reviewer Comment:

Notations: In the equations for the KF, use upper case for matrices and lower case for vectors to avoid confusion.

Response:

We thank the reviewer for this suggestion and agree that the original notation could lead to ambiguity between ensemble members, ensemble matrices, and covariance matrices. In the revised manuscript, we have adopted a consistent notation throughout the Kalman filter formulation. Lowercase symbols are now used for vectors, while uppercase symbols are reserved for matrices.

Reviewer Comment:

Introduction: Introduce and cite other existing open source libraries that implement ensemble KF; e.g. PDAF used by Gillet-Chaulet (2020) and DART used by Choi et al. (2025).

Response:

We thank the reviewer for this important suggestion. In the revised Introduction, we have expanded the discussion of existing open-source data assimilation frameworks to better situate ICESEE within the broader ensemble data assimilation ecosystem. Specifically, we now introduce and cite the Parallel Data Assimilation Framework (PDAF) and the Data Assimilation Research Testbed (DART).

Reviewer Comment:

Abstract "while physics-based inverse . . . : I don't think that the inverse methods used here are more — physics-based — than the ensemble DA. Basically the inverses methods can be interpreted in terms of Bayesian estimation where the regularisation acts as a prior. Here assuming that the friction coefficient has smooth variations does not include more physics than an informed prior initial state.

Response:

We thank the reviewer for this important clarification and agree with the underlying conceptual point. The original wording was imprecise and could incorrectly suggest that traditional inverse methods are more physics-based than ensemble-based data assimilation. As the reviewer correctly notes, both approaches can be interpreted within a Bayesian estimation framework, where regularization terms in inverse methods act as prior constraints, analogous to prior assumptions embedded in ensemble initialization and covariance structures in EnKF-based methods. We have therefore revised the Abstract to remove this distinction and replaced the phrase "physics-based inverse methods" with "traditional inverse methods" to better reflect the intended comparison.

Reviewer Comment:

Sec 2 in general: Explain how you deal with nonlinear observation operators H in the different KF equations.

Response:

We thank the reviewer for raising this important point. In the original manuscript, the treatment of the observation operator (H) was not sufficiently detailed, particularly for nonlinear observation mappings. In the revised manuscript, we now explicitly clarify that ICESEE supports both linear and nonlinear observation operators. For nonlinear operators, the projection into observation space is performed independently for each forecast ensemble member, i.e., $(H(\mathbf{x}_i^f))$, allowing the forecast ensemble to be mapped directly into observation space without requiring tangent-linear approximations or Jacobian evaluations. The corresponding observation-space error statistics are then estimated directly from the transformed ensemble, consistent with the standard EnKF treatment of nonlinear systems. We have incorporated this clarification into Section 2 and revised the associated notation and covariance definitions accordingly.

Reviewer Comment:

Equation 4: $R_e = \frac{1}{N-1}DD^T$ in Evensen (2003), Eq. 51.

Response:

We thank the reviewer for catching this. The reviewer is correct that the sample covariance of the observation perturbation matrix requires the normalization factor $\frac{1}{N_e-1}$. This has now been corrected in the revised manuscript. And we now use $R = (1/(N_e - 1))\Upsilon\Upsilon^T$ instead for more clarity.

Reviewer Comment:

Section 2.3:

- o make two different sections to describe initialisation procedures and model error.
- o Give more details on the random field generations and explain the differences between the possible choice so that the user could take informed decision on the initialisation procedures.
- o I understand that noise modelling refers to "model error"; see my main comment; why using model error in perfect model twin experiments. Improve the description of the methods. I don't understand where the decorrelation length appears.
- o Discussion on the innovation: why this discussion here in a section about ensemble initialisation and model error? The innovation should be the difference between the observations and their model equivalent. (d_i) is defined in Eq. 3 as the perturbed observation vectors; not the innovation which would be $(d_i - H(x_i^f))$?

Response:

We thank the reviewer for these detailed and constructive comments. Following these suggestions, we have substantially revised and reorganized the original Section 2.3.

- o The original section has been split into two separate subsections. Section 2.3 now focuses exclusively on ensemble initialization, while a new Section 2.4 describes forecast perturbations and model-error representation. This separation clarifies the distinction between initial uncertainty representation and forecast-time ensemble spread maintenance.
- o We expanded the description of the random-field generation methods. The revised manuscript now explains the two supported initialization strategies: (i) a pseudo-random spectral field

generator following Evensen [2003], which is computationally efficient and suitable for structured grids and large ensemble experiments, and (ii) geostatistical random-field generation using `gstools`, which allows explicit covariance modeling and cross-variable correlations through Cholesky factorization. We also clarify the practical trade-offs between these choices so that users can make informed decisions based on model geometry, prior information, and computational cost.

o We clarified the role of forecast perturbations in the synthetic twin experiments. Since the same numerical model is used to generate the reference trajectory, synthetic observations, and forecast ensemble, the experiments follow a perfect-model framework. In this setting, the optional forecast perturbations are used only to maintain ensemble spread and prevent premature collapse over long assimilation windows, rather than to represent structural model error.

o We clarified the role of the decorrelation scale. The revised manuscript now explicitly states that a user-specified scale r_h enters only the FFT random-field generation step and does not appear directly in the Kalman update equations. We retained $L_d = \sqrt{L_x L_y}/(n_x n_y + 1)$ as an order-of-magnitude tuning guide for structured-grid FFT applications, while clarifying that it neither defines nor automatically sets r_h.

o We agree that the innovation discussion was misplaced in the original manuscript. We removed this material from the ensemble-initialization and model-error discussion and moved it to a dedicated subsection of the stochastic EnKF formulation (Section 2.1.1, *ICESEE Observation-Error and Analysis-Factor Modes*). The revised subsection distinguishes the standard independently perturbed-observation formulation, the spatially generated observation-error mode, and the legacy forecast-anomaly analysis-factor mode used in the reported experiments. We also revised the notation to distinguish the observation ensemble D, the observation-error matrix Υ, the forecast-observation anomalies Y', the combined analysis factor $A = Y' + \Upsilon$, and the innovation $D' = D - Y^f$. In the legacy mode, Y' enters the analysis factor but is not added to the observation vectors; consequently, the innovation remains $D' = \bar{D} - Y^f$.

Reviewer Comment:

Section 2.4:

o Better explain what inflation and localisation are, for non-specialist users.

o Improve the description of the available implemented choices for localisation o There is not too much discussion on the inflation methods ; which are more introduced in section 2.5

o "For the serial implementation of the stochastic EnKF in ICESEE": I understood that the stochastic EnKF from Evensen was parallel and that the deterministic filters were serial ?

Response:

We thank the reviewer for these helpful suggestions.

o In the revised manuscript, we expanded Section 2.4 to provide a clearer conceptual introduction to covariance inflation and localization for non-specialist readers.

- o We also expanded the description of the covariance-localization choices implemented in ICESEE, as requested. The serial filter suite includes Gaspari-Cohn covariance tapering with prescribed or adaptively estimated length scales.

- o We clarified the available inflation strategies by distinguishing between additive and multiplicative inflation in Section 2.4, while retaining the more detailed discussion of how inflation is applied in the parameter-estimation framework in Section 2.5. This improves the logical flow between the general filter-stabilization concepts and their specific implementation for joint state-parameter estimation.

- o We agree that our original wording regarding the "serial implementation of the stochastic EnKF" was imprecise. Stochastic versus deterministic refers to the analysis formulation, not to whether the code is serial or parallel. ICESEE provides serial and MPI-parallel stochastic-EnKF analysis paths.

  In addition to the reviewer's request for a clearer account of covariance localization, we introduced a grouped patch-based local-analysis option and revised the ISSM experiment so that sparse bed observations act locally on the bed-parameter block. Following the local-analysis principle in Evensen [2003], each node is updated using observations inside its prescribed support. ICESEE uses SciPy's `cKDTree` [Bentley, 1975, Virtanen et al., 2020] to find the observations that support it; the tree accelerates neighborhood searches and is not itself a localization or analysis method. ICESEE does not repeat a separate transform at every grid cell: nodes sharing the same active-observation subset are grouped, one ensemble transform is computed for that group, and it is applied to all corresponding rows across the MPI ranks. This grouping makes local parameter analysis practical for the distributed augmented stochastic EnKF. The Icepack case uses neither explicit covariance localization nor local analysis. In the revised ISSM experiment, only the bed block receives the grouped local transform, using a 15-km observation-selection radius; the remaining variables retain the global transform. We have explicitly described this additional analysis option and the revised ISSM configuration in the manuscript, rather than presenting it as a covariance-localization method requested by the reviewer.

Reviewer Comment:

Section 2.5:

- o "ICESEE adopts the dynamical model for parameters introduced . . . " : Explain what it is.

- o "ICESEE applies a complementary approach by deliberately inflating the parameter subspace more than the state subspace " : The inflation procedure has not been described ; How the ratio of inflation between state and parameters is chosen ? Is it a user-free parameter ? I don't see much description of this in the experiments.

- o discussion on the "noise modelling" : See above, I don't understand what the noise modelling represent ?

Response:

We thank the reviewer for these important comments.

- o In the revised manuscript, we clarified the description of the dynamical parameter model adopted from Ruckstuhl and Janjić [2018]. We now explain that this approach treats model parameters as latent bounded stochastic variables whose evolution is described by a Beta-distributed process. This allows parameter variability to be maintained across assimilation cycles while enforcing physically admissible bounds.

- We also revised the description of parameter inflation to distinguish library capability from experimental choice. ICESEE permits separate user-defined inflation factors for state and parameter blocks, but stronger parameter inflation is an optional stabilization tool rather than a default rule. The revised manuscript now states the factors used in each experiment; in particular, the ISSM experiment uses neither additive nor multiplicative inflation.

- We revised the terminology surrounding "noise modeling" throughout this section to improve consistency with Sections 2.3 and 2.4. In particular, we avoid interpreting these perturbations as structural model error in the perfect-model twin experiments presented here. Instead, they are now described as ensemble spread-maintenance perturbations used to preserve sufficient variability in the parameter subspace and prevent premature ensemble collapse.

Reviewer Comment:

Section 3.2.6: "and loading observations and transforming them to the model grid " : How is this done ? Requires interpolation/extrapolation ? Is the library restricted to assimilate observations located on the model grid ?

Response:

We thank the reviewer for raising this point. The original phrase was conceptually backwards. The observation operator maps the model state or model output to observation locations and observation space; the innovation is then formed in observation space. For off-grid data, this forward mapping may use interpolation, projection, or a model-specific sampling routine. The Kalman cross-covariance maps the observation-space innovation back to a state increment. ICESEE is therefore not restricted to observations on the native model grid, provided that the application wrapper supplies the appropriate forward operator.

Reviewer Comment:

Section 3.4.2: "During the initialization phase, the observation operator H " : This is for a linear observation operator. What kind of operator are implemented if the observations needs to be projected on the model grid ? How to deal with non-linear observation operators ?

Response:

We thank the reviewer for pointing out this ambiguity. We agree that the original wording in Section 3.4.2 could be interpreted as implying that the observation operator is always linear and explicitly constructed.

In the revised manuscript, we now clarify that for linear observation operators, the operator matrix ($\mathbf{H}$) is constructed explicitly during initialization and reused throughout the assimilation cycle. When observations are not located directly on the model grid, this operator maps or interpolates model fields to the observation locations.

For nonlinear observation operators, ICESEE does not construct a global matrix representation. Instead, the nonlinear mapping $H(\mathbf{x}_i^f)$ is evaluated independently for each ensemble member during the forecast and analysis steps. This allows ICESEE to support nonlinear observation operators without requiring tangent-linear approximations or Jacobian evaluations. A detailed remark has been added to the manuscript.

Reviewer Comment:

Section 4.1:

- o "to estimate key ice sheet state variables, including ice velocity, ice thickness, and surface mass balance (SMB). " See my main comment, the velocity is not a state variable. Is the smb solution of an equation or constant in time ? In this case it is a parameter and not a state variable ?
- o The description of the experiment is not sufficient. We don't know what the reference solution represent. It seems that there is a small transient in the reference during the first 40 years but we don't have information about the boundary conditions, basal friction ?, bed elevation ? is there a floating part? the "true" smb, is it constant? Uniform?
- o Why 425 degrees of freedom for each variable? Is the grid regular?
- o Are the observations at every grid point?
- o There is several methods presented in section 2.3 for the initial ensembles and I don't know what is used, pseudo random can mean many things. Again I don't see the link between the initialisation and the "noise" modelling with a decorrelation length scale.
- o I'm surprised by the standard deviation values given for the initial states. The spread of the initial ensemble should reflect the uncertainty in the initial state. Here as shown in Fig. 5 the initial spread is very small, one order of magnitude small that the initial difference between the mean state and "truth".
- o I don't know what a "smooth linear bump" is.
- o " The ensemble spread around the mean remains nearly constant throughout the simulation, indicating stable uncertainty representation and limited ensemble collapse " : The objective of DA is not only to improve the mean but also reduce the initial spread. See above, here the initial spread is very low and does not include the truth. Looking at the thickness in Fig. 5 it seems that the spread is increasing, I assume due to inflation and the " noise " modelling. So I think it is for wrong reasons.
- o Fig 5. From the individual members (grey lines EnKF) it seems that there is short term variability (annual ? or following the time step ?) I don't understand where it comes from.
- o " true value, showing that ICESEE can recover slowly evolving or weakly constrained parameters that are not directly observed. " According to 4.1.1 smb is observed.

Response:

We thank the reviewer for these careful comments and have substantially revised Section 4.1 to improve both the physical description of the Icepack experiment and the interpretation of the assimilation results.

- o We clarified the role of the assimilated quantities. In the revised manuscript, ice thickness is now explicitly described as the prognostic state variable, velocity as a diagnostic quantity obtained from the momentum balance, and surface mass balance (SMB) as a time-independent forcing parameter included in the augmented assimilation vector. This addresses the distinction between state variables, diagnostic variables, and forcing parameters.

- o We expanded the description of the reference experiment. The revised text now clarifies that the setup is based on Icepack's synthetic ice-stream benchmark, which contains both grounded and floating sections, and that the bed topography, basal friction law, and boundary conditions follow the original benchmark configuration. We also explicitly define the "true" SMB as a spatially varying but time-independent forcing field.

- o The revised experiment no longer uses the 425-degree-of-freedom configuration reported in the original manuscript. It uses a 24×16 structured rectangular mesh split into triangles and continuous quadratic (CG2) finite elements, giving 1617 scalar degrees of freedom for each field. We now report the mesh, element family, and degree-of-freedom count explicitly.

- o We clarified that in this idealized twin experiment, synthetic observations are generated on the same finite-element mesh as the model state and are available at all model degrees of freedom. Therefore, no interpolation or projection is required.

- o We clarified the ensemble initialization strategy used in this experiment. The revised manuscript now explicitly states that pseudo-random spatially correlated perturbations are used, following the procedure introduced in Section 2.3, together with the corresponding decorrelation length scales used for each variable.

- o We revised the initial ensemble standard deviations to better reflect the uncertainty around the biased initial condition. The updated values now produce a broader and more physically consistent ensemble spread during the initial assimilation cycles.

- o We clarified the meaning of the "smooth linear bump" by explicitly defining it as a linearly tapered thickness anomaly over the first 25% of the inflow region, decreasing from -350 m to 0 m.

- o We revised the interpretation of the ensemble spread. We agree that the goal of data assimilation is not to preserve constant spread but rather to reduce uncertainty while converging toward the reference trajectory. The revised manuscript now reflects this corrected interpretation.

- o Regarding the short-term variability previously visible in Figure 5, we identified and corrected an implementation issue in the parallel forecast routine that introduced artificial oscillations in some ensemble trajectories. All Icepack experiments have been re-run with the corrected implementation, and the updated figures now show smoother trajectories and physically consistent spread evolution.

- o We corrected the statement regarding SMB recovery. Since SMB is included in the augmented assimilation vector and directly observed in this twin experiment, the revised manuscript now describes its recovery as joint estimation of an observed forcing parameter, rather than recovery of an unobserved quantity.

Reviewer Comment:

Section 4.2:

- o See main comments. The velocity is not a state variable , h, s and b are not independent. Please explain.

- o Description of the initial ensemble is somehow unprecise and contradictory :

  - " glacier in 3D " : Choi et al. (2025) use a vertically integrated model ; so the setup is 2D.
  - I don't understand the base and surface parametrisations : In Choi et al., the bed is defined and the model is spin-up by evolving the ice thickness so that the base and surface elevation are defined by the floatation condition and not " parameterized"
  - What is "exponential" noise ? Noise with a spatial correlation defined by an exponential variogram ?
  - If an offset is added to the bed elevation why not directly proving the correct equations for the bed elevation ?
  - What are a "Gaussian random field " and an " exponential kriging field" ? Same as above ?
  - Why adapting the variogram for the initial friction field? This should reflect the uncertainty on the initial friction field; as it is independent of the bed elevation in Choi et al., why the variogram should better reflect the adjusted bed topography?
  - What is "nudging the mean friction field"? You just mentioned that you generated an initial ensemble using random fields.
  - Initial conditions for the bed, base and surface: clarify, the base is equal to the bed elevation where grounded, while it is defined by the by the floatation condition and the ice thickness on the floating part. Would be better to define the initial ensemble for the ice thickness which is the state variable.
  - "observations are assumed to be available everywhere": does that mean at every mesh node?
  - "bed topography is observed once": Maybe this deserves discussion are the observations have been used to generate the initial bed ensemble; they are used twice if they are assimilated. As a general principle the initial ensemble should be independent of the assimilated observations.
  - Inversion of the friction coefficient: I think this could deserve discussion and at least it would be interesting to compare the performance of the experiment using the inversion and an experiment where the friction is updated by the EnKF. More or less both methods try to achieve the same results, but the EnKF use the prior information defined by the ensemble why the inversion in fact introduce another "prior" from the regularisation term. I assume that the inversions are performed using the perturbed observations as for the EnKF? So that the results reflects the uncertainty on the surface observations?
  - "Effective pressure": is the friction law dependent on the effective pressure?
  - I don't understand the "adaptive under relaxation strategy" : explain when it is applied and why is it used as Choi et al. (2025) show good results without this "strategy".

Response:

- o We thank the reviewer for these detailed comments. We have substantially revised Section 4.2 to clarify the ISSM synthetic twin setup, the augmented assimilation vector, the initial ensemble construction, and the friction inversion strategy.

- We clarified the definition of the augmented assimilation vector. In the revised manuscript, ice thickness (h) is explicitly defined as the prognostic state variable, surface elevation (s) as a diagnostic geometric quantity, u and v as diagnostic velocity components recomputed by the stress-balance solver, and b and c as the bed-topography and basal-friction parameters. We also clarify that h, s, and b are not dynamically independent, but are linked through ISSM's geometry update and hydrostatic flotation condition.
- We corrected the dimensionality description. The revised manuscript now states that the ISSM configuration is a vertically integrated two-dimensional (plan-view) model, consistent with Choi et al. [2025].
- We revised this description to avoid implying that base and surface are independently parameterized. The manuscript now clarifies that bed perturbations are applied directly, and the base, thickness, surface elevation, and grounded/floating mask are recomputed consistently using ISSM's geometry update and hydrostatic flotation relations.
- We clarified the terminology. The revised text now explicitly states that the bed perturbations are generated as Gaussian random fields with an exponential covariance model, rather than using the ambiguous term "exponential noise."
- We now give the construction explicitly. The historical uniform 284-m displacement is not used in the revised experiment. The prior bed combines a -80-m background displacement with conditional spatial realizations generated using an exponential covariance model; the heterogeneous anomaly has a nominal amplitude of 120 m and is bounded between -250 and 150 m. Its upstream texture is extended smoothly downstream with controlled amplitude beneath floating ice. Thickness is perturbed independently, after which base, surface, and the grounded/floating mask are recomputed to satisfy geometric and flotation consistency.
- We simplified and unified the terminology. The revised manuscript now consistently describes the bed perturbations as Gaussian random fields with an exponential covariance structure, removing the previous ambiguous distinction.
- We clarified that the friction variogram parameters define the assumed prior uncertainty structure of the basal friction coefficient. The reduced sill and range produce a lower-amplitude prior ensemble with a shorter spatial correlation range, and do not imply a deterministic dependence between friction and bed topography.
- We revised this language for clarity. The manuscript now explicitly states that the initial friction ensemble is generated by adding the random friction perturbation fields to a spatially uniform reference friction field.
- We expanded the description of the initial ensemble construction. The revised text now clarifies that in grounded regions the base is set equal to the bed elevation, while in floating regions it is recomputed using the hydrostatic flotation condition and the current thickness. This ensures geometric consistency for all ensemble members.
- We clarified that this means synthetic surface elevation and velocity observations are available at all mesh vertices over the computational domain.
- We agree this required clarification. In the revised configuration, bed observations are assimilated at ten sparse snapshot times (years 2, 8, 14, 20, 24, 30, 36, 40, 46, and 50), rather than once. The conditional prior uses a separate synthetic baseline radar realization on the same flight-line geometry, whereas the sequential analyses use independently perturbed realizations. The imposed displaced mean and heterogeneous anomalies prevent the prior from reproducing the reference bed.

- We added the requested controlled comparison. The revised manuscript now compares (i) a fixed wrong basal-friction field (WBF), (ii) friction updated only through EnKF cross-covariances (EBF), and (iii) member-wise ISSM inversion embedded in the EnKF cycle (IBF). This is a separate 40-year comparison experiment because direct WBF/EBF initialization at the 0.2-yr step used by the principal 100-year IBF experiment produced a sharp transient collapse of the diagnostic velocity; in EBF, analyzed friction changes consequently failed to propagate reliably into the following forecast. We therefore apply the same four-year, 0.1-yr pre-assimilation stabilization and retain $\Delta t = 0.1$ yr for all three comparison cases. A coupling preflight verified EBF forecast propagation under this configuration. The truth, initial ensemble, forcing, observation realization, stabilization, time step, and assimilation schedule are identical across WBF, EBF, and IBF, so only the friction treatment differs within the controlled comparison. Over the 16-year free forecast, IBF reduces mean velocity RMSE by about 23% relative to WBF and 18% relative to EBF, and reduces mean surface-elevation RMSE by about 22% relative to both. The comparison also makes explicit that IBF introduces a Tikhonov regularization prior, whereas EBF uses the finite-ensemble prior covariance.

  Yes. We now clarify that the ISSM inversion uses the same perturbed velocity observations supplied to the EnKF update, ensuring consistency between the ensemble uncertainty representation and the inferred friction fields.

- ISSM-inversion represents basal drag as $\boldsymbol{\tau}_b = -k^2 N^r \|\boldsymbol{v_b}\|^{s-1} \boldsymbol{v}_b$, where k (denoted by c in our augmented state vector) is the basal-friction coefficient used as the inversion control, N is effective pressure, $\boldsymbol{v}_b$ is basal velocity, $r = q/p$, and $s = 1/p$. So prior to the corrections we had used $p = q = 1$, so $r = s = 1$ and the law reduced to $\boldsymbol{\tau}_b = -k^2 N \boldsymbol{v}_b$. This made the basal traction to depend explicitly and linearly on effective pressure.

  In the new experiment runs, we used $p = 3$ and $q = 0$, so the law reduces to $\tau_b = -k^2 \|\mathbf{v_b}\|^{-2/3} \mathbf{v}_b$, hence now it doesn't depend on effective pressure.

- We agree that the original description introduced unnecessary complexity. During revision, we identified and corrected an issue in the parallel forecast routine that had motivated this stabilizing step. After correcting the implementation and re-running the ISSM experiments, the adaptive under-relaxation strategy was no longer required and has therefore been removed from the revised manuscript.

Summary

We thank the reviewer again for the constructive comments. The manuscript has been substantially revised to improve clarity, provide additional methodological details, better describe the numerical experiments, and address all concerns raised during the review.

References

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