

Reply to Referee #3

We thank Referee #3 for the careful and constructive review, particularly for the suggestions regarding figure presentation and readability. We address each comment below and describe the corresponding clarifications and proposed revisions, as well as our rationale where we prefer to retain the current presentation.

5 Some comments

1. *A lot of the existing work on working with Tweedie distributions with rainfall is not mentioned, and this is required to situate this manuscript in the literature. For instance:*

- *Dunn (2004): the first paper I know of that uses Tweedie distributions with rainfall (at least in any substantive way).*
- *Other papers by Hasan et al. (2010, 2012, 2015) are also probably relevant, or at least worth mentioning, as they cover different scales too.*

Hasan and Dunn (2010) and Hasan et al. (2019) are already included, but we will expand the Introduction to include the earlier work of Dunn (2004) and the relevant studies of Hasan and Dunn (2012, 2015). Although their scopes differ from the present study, these studies still provide additional context for the development and application of Tweedie models to precipitation and are therefore worth mentioning.

2. *The method for estimating p (e.g., by regressing $\log \text{Var}(y)$ on $\log \mu$) is presented as new work, but it is **not new work**. The first mention I know of using this method to estimate p is found in Dunn & Smyth (2005) (and it is also mentioned in their textbook: Dunn & Smyth (2018)).*

We agree that the regression-based estimation of p is an established method and did not intend to present this estimation procedure as new work. As discussed in the Introduction, the contribution of the present study lies in applying this framework jointly across temporal aggregation, season, spatial location, and observing systems using radar composites and gauge observations. We will expand the literature review on existing Tweedie work as discussed and make the distinction clearer.

3. *The authors should also mentioned that finding the MLEs of p is possible, as described in Dunn & Smyth (2018) and Dunn & Smyth (2005)). The authors should mentioned **why** the presumably-superior MLE method was **not** used even though it is available (presumably, because it is much slower than the method presented, and/or the authors did not use the R package: <https://cran.r-project.org/web/packages/tweedie/index.html>).*

As described by Dunn and Smyth (2005, 2018), maximum-likelihood estimator of p is an alternative, particularly when a Tweedie model and its systematic mean structure are specified. In the present study, we estimate p as the empirical exponent of the block-wise mean–variance relationship across multiple dimensions. The regression-based approach therefore provides a direct and computationally tractable estimate of the scaling relationship examined here. Applying profile likelihood throughout the analysis would require repeated numerical likelihood evaluations and model fitting across radar grid cells and the different subdivisions of the data, which could be substantially more computationally demanding and may also encounter the numerical issues discussed by the Dunn and Smyth (2005, 2018) and noted by Referee #1. However, we agree that the MLE approach should be mentioned as an alternative, and we will clarify why the regression-based approach was adopted here.

4. The authors do not mention what software they used, which they definitely should.

We agree and will specify the software used in the Methods. The analysis was implemented in Python with p calculated directly using NumPy rather than a specialized Tweedie package. This clarification is also included in our response to Referee #1.

5. Many of the figures do not present well in grey-scale (e.g., in Fig. 4 the two lines are indistinguishable; the two shading used in Fig. 5). They authors should consider this when drafting their manuscript.

We agree that greyscale readability should be checked, although the colour versions provide clear visual separation. We will revise the colour choices where necessary and use different figure styling to ensure that the plotted elements remain distinguishable in greyscale.

6. The subtle range of colours in Fig. 7 can be hard to distinguish; would the authors consider adding some contour lines, even a small number of contour lines?

We appreciate the suggestion. We considered adding contour lines, but some spatial fields in Fig. 7 contain substantial fine-scale heterogeneity. Adding contours could obscure the figures rather than clarify the broader spatial patterns. We therefore prefer to retain the maps without contour lines. However, we will examine alternative colour scales to determine whether the distinction between different ranges of p can be further improved.

- 65 7. Some of the figures are very busy, containing a lot of information that is hard to differentiate (e.g., Fig. 2). I don't know if it is possible or sensible, but perhaps consider two side-by-side plots that may make the information easier to read.

70 Thank you for the comment. We prefer to retain the radar and gauge estimates together, since their direct comparison across accumulation lengths is a central purpose of Fig. 2. However, we agree that the additional extrapolated gauge segments and their associated uncertainty increase the visual complexity. We will therefore simplify Fig. 2 by moving the short-scale extrapolation and associated uncertainty to the Appendix.

- 75 8. Is the use of three decimal places of p actually suitable (e.g., Tables 2 and 3)? Consider two decimal places. The differences between, say, $p = 1.41$ and $p = 1.42$ is almost indistinguishable, so I doubt any would notice or care about the third decimal places.

We agree that two decimal places provide sufficient precision for the report values. We will revise Tables 2 and 3 accordingly and check the remaining manuscript for consistency.

80 **Minor issues**

- *Line 35: "...sequence; for example, a first-order..."*
- *Line 40: "...framework, originating from..."*
- *Line 43: Rather than "On the amount side", consider "For rainfall amounts".*
- *Line 174: I'm not sure what is meant by "representative of products"*
- 85 • *Line 207: x-axis rather than x-axis.*

We thank the referee for pointing these wording and formatting issues. We agree with the suggested revisions and will make the corresponding changes throughout the manuscript.