2026-05-20 3rd Review by Chris W. Hughes of

Revealing horizontal gravity force in geopotential coordinates via metric tensors

by

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Submitted to Ocean Science.

Specific Comments

The author argues that a consistent hydrostatic balance can be found using his two equations:

$$\nabla_G p = (\partial_x p + \partial_x N \partial_Z p)\hat{\boldsymbol{x}} + \left(\partial_y p + \partial_y N \partial_Z p\right)\hat{\boldsymbol{y}} + (\partial_Z p)\hat{\boldsymbol{Z}} \quad (35)$$

$$\nabla_G \Phi = (\partial_x \Phi + \partial_x N \partial_Z \Phi)\hat{\boldsymbol{x}} + \left(\partial_y \Phi + \partial_y N \partial_Z \Phi\right)\hat{\boldsymbol{y}} + (\partial_Z \Phi)\hat{\boldsymbol{Z}}. \quad \text{(R3)}$$

I agree that the two equations are consistent with the equation

$$-\frac{1}{\rho}\nabla_G p + \nabla_G \Phi = 0.$$

The problem is that each of them is wrong in a way which balances. This is made even more clear from the explicit consideration of (R3), which, as the author shows, becomes

$$\nabla_G \Phi = -g\left(\hat{\boldsymbol{x}}\partial_x N + \hat{\boldsymbol{y}}\partial_y N\right) - g\hat{\boldsymbol{Z}}. \quad \text{(R7)}$$

or, since $\nabla\Phi = \boldsymbol{g} = -g\hat{\boldsymbol{Z}}$,

$$\nabla_G \Phi = -g\left(\hat{\boldsymbol{x}}\partial_x N + \hat{\boldsymbol{y}}\partial_y N\right) + \nabla\Phi.$$

from which we see that $\nabla_G \Phi$ appears to be the gradient of the potential plus two "horizontal" terms (actually directed perpendicular to the Cartesian near-vertical coordinate rather than being truly horizontal). It is therefore an incorrect representation of the gravitational acceleration.

We keep coming back to the same issue. Since gravity is the gradient of a potential, its component tangent to a constant potential surface is zero by construction. It is futile to keep searching for a way to claim otherwise.