June 8, 2026

Comment on: “Revealing horizontal gravity force in geopotential coordinates via metric tensors” by Peter C. Chu (2026, submitted to Ocean Science; hereafter C26).

I have examined the C26 preprint, the paper by McWilliams (2024, PNAS; hereafter M24) and, to some degree, the predecessor papers by McWilliams and colleagues on the subject of the existence and relevance of the horizontal component of gravity force in Geophysical Fluid Dynamics. Aside from an orientational error (Eq. (4) in M24) and a typo in M24 using lower case ‘z’ instead of upper case ‘Z’ as a partial derivative subscript in his Eq. (11) [both of which have been noted and corrected by McWilliams (2026, PNAS; hereafter M26)], I can state for the record that I have determined independently that all key numbered equations in M24 (i.e., Eqs. (1) - (3), (5) – (22)) are correct. C26’s claim that M24 is in error because tensor calculus was not used to derive the transformed pressure gradient force per unit mass is mistaken. Moreover, tensor calculus is a red herring as M24’s demonstration is a direct demonstration of the lack of the horizontal gravity force in the governing fluid dynamical equations in geopotential coordinates. I am surprised by the still ongoing controversy on this matter and the discussion whether there is an important horizontal force of gravity that has been neglected in the many decades since the birth of Geophysical Fluid Dynamics.

I will close my comment by re-stating a few conclusions from two reviewers, which are public.

Reviewer J. Thuburn succinctly describes the matter at hand.

"For the present discussion, the important terms in (8) are the pressure gradient and geopotential gradient. In this approach we do indeed obtain a nonzero horizontal component to the geopotential gradient. However, the crucial point is that, to an excellent approximation, *it is compensated by an (almost) equal and opposite horizontal pressure gradient term, because the atmosphere and ocean are very close to hydrostatic balance (emphasis added by me)*. In this approach, if we neglect the horizontal component of gravity, then the pressure gradient would also lose its horizontal hydrostatic component. Thus, we effectively omit two terms or contributions whose sum is virtually zero and so make very small error overall."

This observation was, in fact, demonstrated analytically in M24's Eq. (21)! (See point 5. in Thuburn's review, quoted verbatim below.)

Reviewer Thuburn continues by providing a succinct mathematical analysis:

"5. As a result of the above errors, the expression (31) and its approximations (34a,b) for the gradient in geopotential coordinates are incorrect.

... and then using the above expression for $\hat{Z}$leads, after some cancellation to

∇f = $f_x \hat{x}$ + $f_y \hat{y}$ + |∇Z| f_Z $\hat{Z}$

This last expression agrees with equation (21) of M24. Thus, M24 is correct, and the last sentence of the abstract of the manuscript under review is incorrect.

There are clearly significant problems with the manuscript that mean it is not publishable."

Reviewer Baylor Fox Kemper's review concurs with J. Thuburn's review. One comment in particular rings clear and finds resonance with me since it conveys the trusted experience of Carpenters and Surveyors:

"One **can** define a "vertical", or really quasi-vertical, direction in such a way that the geopotential direction is not aligned with it. However, this is an unnecessarily complex and computationally costly choice:

1) The natural way to define the vertical is the direction where a plumb bob hangs (https://en.wikipedia.org/wiki/Plumb_bob). Carpenters have been using this trick to build strong homes for millennia. This combines all forces experienced by a body which is in static location relative to the surface of the earth, i.e., the vector sum of gravity and centrifugal forces.

2) If one insists on having "horizontal" components of gravity, then the hydrostatic balance becomes a partial differential equation rather than an ordinary differential equation, as now geopotential gradient has derivatives in both the "horizontal", latitude-like direction, as well as the "vertical". This engenders a spurious waste of computation."

Signed,

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