Reply to RC6 by Chris W. Hughes

26 May 2026

General Response: The reviewer's analysis is incorrect.

Special Comments:

(1) "*The author argues that a consistent hydrostatic balance can be found using his two equations:*

$$\nabla_G p = (\partial_x p + \partial_x N \partial_Z p)\hat{\mathbf{x}} + \left(\partial_y p + \partial_y N \partial_Z p\right)\hat{\mathbf{y}} + (\partial_Z p)\hat{\mathbf{Z}} \qquad (35)$$

$$\nabla_G \Phi = (\partial_x \Phi + \partial_x N \partial_Z \Phi)\hat{\mathbf{x}} + \left(\partial_y \Phi + \partial_y N \partial_Z \Phi\right)\hat{\mathbf{y}} + (\partial_Z \Phi)\hat{\mathbf{Z}} \qquad \text{(R3)}$$

I agree that the two equations are consistent with the equation

$$-\frac{1}{\rho}\nabla_G p + \nabla_G \Phi = 0$$

Response: Thanks.

(2) "*The problem is that each of them is wrong in a way which balances. ...*"

Response: Thank you very much for your critics. I admit I didn't explain clearly in AC6. Equations (35) and (R3) are correct. Here, I will explain it again.

The geopotential Φ and geoidal undulation *N* are defined in the preprint by

$$\Phi = -\mathrm{g}_0 Z \qquad (15)$$

$$Z = \zeta + N \qquad (16)$$

Substitution of (15) and (16) into (R3) gives

$$\nabla_G \Phi = -\mathrm{g}_0 \hat{\mathbf{Z}}$$

There are no Cartesian coordinates to be involved.

(3) "*We keep coming back to the same issue. Since gravity is the gradient of a potential, its component tangent to a constant potential surface is zero by construction. It is futile to keep searching for a way to claim otherwise*"

Response: This comment is incorrect.

The bumpy-geoid gradient $\left(\partial_x N, \partial_y N\right)$ emerges in the horizontal pressure gradient force along the geopotential surface. See equation (35).