Reply to 2nd Review by Chris W. Hughes of "Revealing horizontal gravity force in geopotential coordinates via metric tensors"

12 May 2026

Thank you very much for your quick response.

General Comments

"*The author continues to insist that there are horizontal components of gravity, and simply contradicts without explanation (just providing more, unilluminating maths) the descriptions of errors in his analysis. My recommendation remains to reject the paper for publication*."

Response: I disagree because your simple analysis is incorrect.

More detailed analysis

Response: Your analysis is incorrect.

You present a simple example

$$\frac{D\mathbf{u}}{Dt} = -2\boldsymbol{\Omega} \times \mathbf{u} - \frac{1}{\rho}\nabla p + \nabla\Phi, \qquad \mathbf{u} = 0, \qquad \frac{D\mathbf{u}}{Dt} = 0, \;\; \mathbf{g} = \nabla\Phi \tag{R1}$$

and make the statement "This describes a situation in which pressure and density are each constant along every geopotential surface: hydrostatic equilibrium. Such a physical balance holds *independent of the coordinate system used to describe it*," which I like.

This equilibrium is represented by

$$-\frac{1}{\rho}\nabla p + \nabla\Phi = 0, \tag{R2}$$

Gradient along the geopotential surface for any scalar Q is given by Eq.(34b) in the preprint,

$$\nabla_G Q = (\partial_x Q + \partial_x N \partial_Z Q)\hat{\mathbf{x}} + (\partial_y Q + \partial_y N \partial_Z Q)\hat{\mathbf{y}} + (\partial_Z Q)\hat{\mathbf{Z}} \tag{34b}$$

which reads

$$\nabla_G p = (\partial_x p + \partial_x N \partial_Z p)\hat{\mathbf{x}} + (\partial_y p + \partial_y N \partial_Z p)\hat{\mathbf{y}} + (\partial_Z p)\hat{\mathbf{Z}} \tag{35}$$

$$\nabla_G \Phi = (\partial_x \Phi + \partial_x N \partial_Z \Phi)\hat{\mathbf{x}} + (\partial_y \Phi + \partial_y N \partial_Z \Phi)\hat{\mathbf{y}} + (\partial_Z \Phi)\hat{\mathbf{Z}} \tag{R3}$$

Geopotential gradients along each geopotential surface are given by

$$\partial_x \Phi = 0,\ \partial_y \Phi = 0,\ \partial_Z \Phi = -\mathrm{g} \tag{R4}$$

Constant pressure and density along every geopotential surface lead to

$$\partial_x p = \partial_y p = 0,\ \ \partial_Z p = -\rho \mathrm{g} \tag{R5}$$

Substitution of (R4) into (R3) reads

$$\nabla_G \Phi = \ -\mathrm{g}\partial_x N\hat{\mathbf{x}} - \mathrm{g}\partial_y N\hat{\mathbf{y}} - \mathrm{g}\hat{\mathbf{Z}} \tag{R6}$$

Substitution of (R5) into (35) reads

$$\nabla_G p = \ -\rho\mathrm{g}\partial_x N\hat{\mathbf{x}} - \rho\mathrm{g}\partial_y N\hat{\mathbf{y}} - \rho\mathrm{g}\hat{\mathbf{Z}} \tag{R7}$$

Eq.(R6) and Eq.(R7) give

$$-\frac{1}{\rho}\nabla_G p + \nabla_G \Phi = 0$$

which is the same equilibrium as shown by Eq.(R2). There is no spurious error as you mentioned. Thus, Eq.(35) is correct.