

Reply to CC5 by Chris Wolfe – 2026-06-09

09 June 2026

(1) General Comment

“The other reviewers and commenters have done a commendable job pointing out the flaws in the manuscript under consideration---primarily that, since "horizontal" is defined to be perpendicular to gravity, there can be no horizontal component to gravity. As pointed out by John Thuburn, some of the author's confusion may stem from the imprecise way that coordinate systems are discussed in numerical model documentation. While the modeling community could do better at specifying exactly what is meant by "spherical" and "geopotential" coordinates, the truth is that most members of the field do not find this issue to be confusing.”

Response: My major point is the **coordinate invariance of combined gravity and pressure gradient forces**, which leads to the emergence of bumpy-geoid gradient in horizontal equation of motion, no matter how the “horizontal” is defined, even in the geopotential coordinates. I admit some deficiency in my manuscript and plan to rewrite.

(2) Special Comment

“I'll just add yet another quick derivation of the momentum equations in orthogonal geopotential coordinates demonstrating that the horizontal components contain no contributions from gravity in the hopes that some may find this derivation compelling. Please see the attached PDF.”

Response: Your concept with geopotential coordinates and your derivation are incorrect.

The geopotential coordinates are nonorthogonal.

Following your definition, let Φ be the geopotential (contributions from the bumpy geoid included) and be represented as $\Phi(\xi, \eta, \zeta)$ in local Cartesian coordinates with unit vectors $(\hat{\xi}, \hat{\eta}, \hat{\zeta})$. Unit vectors of your nonorthogonal curvilinear coordinate system $(\hat{e}_1, \hat{e}_2, \hat{e}_3)$ are connected to unit vectors $(\hat{\xi}, \hat{\eta}, \hat{\zeta})$ by

$$\hat{e}_1 = \hat{\xi}, \quad \hat{e}_2 = \hat{\eta}, \quad \hat{e}_3 = \nabla\Phi/|\nabla\Phi| \tag{R1}$$

The gradient operator can be defined by

$$\nabla = \hat{\xi} \frac{\partial}{\partial \xi} + \hat{\eta} \frac{\partial}{\partial \eta} + \hat{\zeta} \frac{\partial}{\partial \zeta} \tag{R2}$$

In local Cartesian coordinates since their unit vectors $(\hat{\xi}, \hat{\eta}, \hat{\zeta})$ are CONSTANT. Similar expression is not valid for your nonorthogonal curvilinear coordinate system $(\hat{e}_1, \hat{e}_2, \hat{e}_3)$ since \hat{e}_3 varies with space. You cannot write the gradient operator as

$$\nabla_G \neq \hat{\mathbf{e}}_1 \frac{\partial}{\partial x} + \hat{\mathbf{e}}_2 \frac{\partial}{\partial y} + \hat{\mathbf{e}}_3 \frac{\partial}{\partial z}$$

Here, ∇_G is used for the geopotential coordinates (your “orthogonal” curvilinear coordinates) to distinguish from Cartesian coordinates. Obviously, your equation (3) is incorrect.

The gradient operator ∇_G is given by [see Eq.(34b) in the manuscript],

$$\nabla_G = (\partial_x + \partial_x N \partial_z) \hat{\mathbf{e}}_1 + (\partial_y + \partial_y N \partial_z) \hat{\mathbf{e}}_2 + (\partial_z) \hat{\mathbf{e}}_3, \quad Z = \Phi/g \quad (\text{R3})$$

where N is the geoid,

$$\partial_x \Phi = -g \partial_x N, \quad \partial_y \Phi = -g \partial_y N \quad (\text{R4})$$

Here, the negative sign is due to the definition of $\mathbf{g} = \nabla \Phi$ in your comments.

Gradients of Φ and p in the geopotential coordinates are given by

$$\nabla_G \Phi = (\partial_x \Phi + \partial_x N \partial_z \Phi) \hat{\mathbf{e}}_1 + (\partial_y \Phi + \partial_y N \partial_z \Phi) \hat{\mathbf{e}}_2 + (\partial_z \Phi) \hat{\mathbf{e}}_3 = g \hat{\mathbf{e}}_3 \quad (\text{R5})$$

$$\nabla_G p = (\partial_x p + \partial_x N \partial_z p) \hat{\mathbf{e}}_1 + (\partial_y p + \partial_y N \partial_z p) \hat{\mathbf{e}}_2 + (\partial_z p) \hat{\mathbf{e}}_3 \quad (\text{R6})$$

The combined gravity and pressure gradient forces are obtained from (R5) and (R6),

$$-\frac{1}{\rho} \nabla_G p - \nabla_G \Phi = -[(\partial_x p + \partial_x N \partial_z p)/\rho] \hat{\mathbf{e}}_1 - [(\partial_y p + \partial_y N \partial_z p)/\rho] \hat{\mathbf{e}}_2 - (\partial_z p/\rho + g) \hat{\mathbf{e}}_3 \quad (\text{R7})$$

With the hydrostatic balance,

$$\partial_z p/\rho + g = 0$$

Eq.(R7) is reduced to

$$-\frac{1}{\rho} \nabla_G p - \nabla_G \Phi = -[(\partial_x p)/\rho - g \partial_x N] \hat{\mathbf{e}}_1 - [(\partial_y p)/\rho - g \partial_y N] \hat{\mathbf{e}}_2 \quad (\text{R8})$$

which shows the emergence of bumpy-geoid gradient in horizontal equation of motion in the geopotential coordinates.