Review of "The Moisture Mode-to-Gravity Wave Spectrum as a Framework to Define Tropical Weather Systems" by Muhamad Reyhan Respati et al.

Signed: Kuni Inoue

Summary and General Comments

This manuscript develops an object-based framework for defining tropical weather systems along a moisture-mode-to-gravity-wave spectrum. The authors combine phase-speed filtering of ISCCP OLR anomalies with Lagrangian object tracking, and then use ERA5 and GPM-IMERG data to examine the thermodynamic, dynamical, and rainfall characteristics of the resulting moisture-mode, mixed-system, and inertio-gravity-wave categories. The study argues that the observed ratio of DSE to moisture perturbations increases with phase speed, that slow and intermediate systems exhibit distinct equatorial and off-equatorial dynamical structures, and that these system types provide a useful way to relate large-scale tropical dynamics to heavy and extreme rainfall, with moisture modes especially important near moist margins and mixed systems important along the ITCZ.

I find the manuscript to be very valuable, clearly written, and scientifically interesting. In particular, it makes a worthwhile attempt to extend recent theoretical ideas on tropical motions into an observational, weather-system-oriented framework, and many of the presented results appear broadly convincing. However, I think the manuscript may need to address a few issues before publication, because the theoretical justification for using zonal phase speed alone as the basis for classification appears to be incomplete. The omission of the Burger-number contribution in the underlying scaling is potentially important for broad, off-equatorial, or subtropical disturbances, and it affects the interpretation of several central results. In addition, the composite analyses need clearer treatment of propagation direction, since combining eastward- and westward-propagating systems may obscure or bias the phase relationships on which some of the physical interpretation rests. These concerns do not undermine the potential value of the study, but they require some clarification, qualification, and possibly additional analysis before the conclusions can be fully supported. I therefore recommend **major revision**.

1. Major Comments

Major Comment 1.

The manuscript presents a compelling and well-written framework for classifying tropical convectively coupled systems by phase speed, but the theoretical basis for using phase speed alone as a proxy for N_{mode} needs further qualification, particularly for off-equatorial disturbances. As written, Eq. (2) and the subsequent classification in Section 3 implicitly assume that the Froude-number contribution controls N_{mode}. However, in the scaling of Adames (2022), the modal thermodynamic parameter is more generally controlled by the larger of the Froude-number and inverse-Burger-number contributions, i.e.,

$$N_{\mathrm{mode}} \equiv \max\left(\mathrm{Fr}_\tau^2, \mathrm{Bu}^{-1}\right)\frac{1}{\hat{\alpha}(1-\hat{\alpha})}, \qquad \mathrm{Fr}_\tau \equiv \frac{c_p}{c}, \qquad \mathrm{Bu} \equiv \left(\frac{L_d}{L_y}\right)^2, \qquad L_d = \frac{c}{f}.$$

Thus, the reduction to Eq. (2),

$$N_{\mathrm{mode}} \approx \left(\frac{c_p}{c}\right)^2 \frac{1}{\hat{\alpha}(1-\hat{\alpha})},$$

is appropriate only when $\mathrm{Fr}_\tau^2 \gtrsim \mathrm{Bu}^{-1}$. When the Burger number is sufficiently small, as can occur at higher latitude, for larger meridional scales, or for disturbances with substantial off-equatorial structure, N_{mode} is no longer determined primarily by the zonal phase speed, as demonstrated below with concrete examples. This matters because if Bu^{-1} is non-negligible, some slowly propagating off-equatorial disturbances may have thermodynamic scaling more consistent with mixed or moist-QG-like systems than with moisture modes.

(i) The theoretical reduction from the full scaling to Eq. (2) should be stated and justified explicitly. In Section 2, immediately before and after Eq. (2), the manuscript states that N_{mode} is a function of phase speed and that "systems with slow propagation will have a small value of N_{mode} and more dominant moisture perturbations, while fast-moving systems will have a large value of N_{mode}." This is a useful interpretation under the Froude-number-dominated limit, but it omits the Burger-number dependence that appears in the more general scaling. A short theoretical discussion may be beneficial to clarify the assumptions under which Eq. (2) is valid, and to state whether or not those assumptions are expected to hold for the equatorial, off-equatorial, and subtropical parts of the analysis domain.

(ii) The off-equatorial composites may lie near a regime in which the Burger-number term is not negligible. The concern may not be merely formal. First, consider an example in which the Froude-number scaling works well. A similar computation is given in Section 5b of Adames (2022). Suppose that the latitude is approximately 12°, for which $f \equiv 2\Omega\sin\theta \simeq 3.0\times10^{-5}\mathrm{s}^{-1}$. We then compute the meridional wavelength as $\lambda_y = 2\pi L_y$. The factor of 2π is needed because L_y represents the inverse meridional wavenumber, so converting this scale to a wavelength requires multiplication by 2π; see the definitions in Eqs. (30a) and (30b) of Adames (2022). If the meridional wavelength of the convective disturbance is $\lambda_y = 1000\mathrm{km}$ and $c = 50\mathrm{m/s}$, the Burger number becomes:

$$\mathrm{Bu} = \left(\frac{2\pi c}{\lambda_y f}\right)^2 \simeq 110 \tag{1.1}$$

Thus, $\mathrm{Bu}^{-1} \sim 0.01$. Therefore, over most of the range of c_p, we have $\mathrm{Fr}_\tau > \mathrm{Bu}^{-1}$, and the Foude-number scaling is adequate.

However, the off-equatorial moisture-mode composite in Fig. 6c appears to have a substantially larger meridional scale, of order 16–17°, i.e. roughly $\lambda_y \simeq 1800$ km, at about 15° latitude. With $f \simeq 3.77 \times 10^{-5}$ s^{-1}, this gives Bu $\simeq 21$ and Bu$^{-1} \simeq 0.047$. By comparison, the upper phase-speed threshold for the manuscript's moisture-mode category is $c_p = 9$ m s^{-1}, for which $(c_p/c)^2 \simeq 0.032$. In such a case, the Burger-number contribution can exceed the Froude-number contribution, giving

$$N_{\text{mode}} \simeq \text{Bu}^{-1} \frac{1}{\hat{\alpha}(1-\hat{\alpha})} \simeq 0.52 \quad \text{for } \hat{\alpha} = 0.9.$$

This value is not obviously in the same regime as the low-N_{mode} moisture-mode criterion implied by the manuscript's phase-speed threshold. Consequently, some off-equatorial systems classified as moisture modes by zonal phase speed may instead be closer to the manuscript's mixed-system category in terms of the full thermodynamic scaling, or may correspond more closely to moist-QG-like disturbances.

(iii) The issue has direct implications for the interpretation of off-equatorial and subtropical features in several figures. In Fig. 2c, for example, the feature labelled as a moisture mode near Mexico appears as a meridionally slanted, synoptic-scale disturbance; dynamically, such a structure could plausibly reflect Rossby-wave or moist-QG influence rather than a pure moisture mode defined by Froude-number scaling alone. Similarly, Fig. 3 shows occurrences extending toward the poleward edges of the plotted domain, and the manuscript notes in Section 4 that some mixed-system and IG-wave occurrences along the northern and southern boundaries of the domain, near 30°N/S, are "likely to be the imprints of extratropical frontal bands and other systems moving with similar propagating speeds: the moist QG systems." The same point reappears in the Summary and Discussion:

> There are also some mixed system occurrences along the subtropical edges (i.e., 30°N/S), and these are likely to be the imprints of the moist QG system, which may be considered to be a mixed system with a small Rossby number and strong influences from the extratropics (Adames, 2022).

This is a useful caveat, but it may be too limited as currently phrased. The Burger-number estimate above suggests that the ambiguity is not confined to the 30°N/S edges of the displayed maps; it can already become relevant within the off-equatorial band used in the manuscript, especially between about 15° and 20° latitude and for broad meridional structures. The interpretation of Figs. 6b–c, 8b–c, 8e–f, 12b–c, and 13a–b as showing off-equatorial moisture modes should therefore be presented with more caution unless the authors can demonstrate that the sampled disturbances remain in a Froude-number-dominated regime.

(iv) The manuscript should either constrain the displayed domain and language or provide a quantitative justification for retaining the current classification. I do not think this concern necessarily requires the authors to redo the full analysis, since the results are scientifically interesting and the phase-speed-based decomposition is valuable. However, the paper would be considerably stronger if the authors added an explicit

Burger-number discussion and used it to justify the applicability and limitations of the classification, particularly off the equator. At minimum, I recommend that the authors:

i. state the full scaling involving $\max(\mathrm{Fr}_\tau^2, \mathrm{Bu}^{-1})$ and explain why Eq. (2) is adopted;

ii. assess whether or not typical meridional scales and latitudes of the tracked objects imply $\mathrm{Fr}_\tau^2 \gtrsim \mathrm{Bu}^{-1}$ in the equatorial and off-equatorial composites;

iii. revise the discussion of off-equatorial "moisture modes" to acknowledge that some slowly propagating systems, especially near 15°–20° and beyond, may be better interpreted as mixed or moist-QG-like disturbances when the Burger-number term is considered.

A conservative presentation choice would also be to truncate Figs. 2, 3, 9, and 13 to 20°S–20°N, since the main analyses are conducted within that latitude range and the phase-speed definition of N_{mode} becomes increasingly questionable poleward of it. The authors might also consider whether the off-equatorial composite region should be restricted to 10°–15° rather than 10°–20°, although I leave that choice to the authors; the essential requirement is that the theoretical limitation introduced by the Burger number be clearly articulated and reflected in the interpretation of the off-equatorial results.

Major Comment 2.

The composite diagnostics appear to mix eastward- and westward-propagating disturbances within each phase-speed class, and this potentially affects the interpretation of the zonal–vertical and horizontal structures in Figs. 4–8 in which structures with opposite zonal phase tilts and opposite zonal phase relationships can be averaged together after centering on the OLR minimum. This issue may matter: several of the manuscript's physical interpretations rely on the phase relationships among convection, moisture, geopotential, winds, and vertical velocity. If eastward and westward disturbances are combined, the resulting composite may partly cancel physically meaningful asymmetries or may display the structure of whichever propagation direction dominates the sample, rather than the structure of a representative individual disturbance.

For example, Fig. 4c shows little apparent zonal tilt in the IG waves, even though convectively coupled IG waves are commonly associated with pronounced vertical/zonal phase tilts (e.g., Inoue et al. 2020, and references therein). A plausible explanation is that eastward- and westward-propagating IG-wave structures have opposing tilts that are cancelled by the composite procedure. Conversely, the apparent tilt in Fig. 4a may reflect dominance of the eastward-propagating MJO over westward-propagating ER-like disturbances, rather than a generic moisture-mode structure.

The same concern affects the interpretation of Fig. 6 and the related discussion in Section 4.2. The manuscript states:

> This low-level dynamical structure very much resembles the theoretical schematic of the equatorial moisture mode in Fig. 5 of Adames (2022), although the strong zonal pressure gradient is not as apparent here in Fig. 6a.

The weak zonal pressure gradient in Fig. 6a may not necessarily indicate a mismatch between the observed equatorial moisture-mode structure and the theoretical schematic;

it may instead arise because eastward- and westward-propagating systems with opposite zonal phase relationships have been averaged together. This also bears on later statements about the location of convection and ascent relative to the vortex center, the poleward/equatorward flow, and moisture advection in the off-equatorial composites, including the interpretation of Figs. 6 and 8 in terms of moisture–vortex instability. Some of these horizontal structures may indeed be physically meaningful, but with the present compositing strategy it is difficult to know whether a displayed phase relationship is representative of the individual disturbances, the residual after cancellation between opposite-propagating disturbances, or the structure of the more numerous or stronger propagation-direction subset.

Suggested clarification or additional analysis. I recommend that the authors either revise the analysis or more clearly state this limitation when interpreting composite structures. A cleaner approach would be to separate eastward- and westward-propagating signals before compositing. Alternatively, for one propagation direction the longitude axis could be reflected before averaging, so that positive relative longitude consistently denotes the downstream direction of propagation. If the authors choose not to redo the composites, the manuscript should at minimum add a clear caveat in Sections 3, 4.1, 4.2, and/or 6 explaining that the composites are centered on convective objects but are not oriented by propagation direction, so some zonal–vertical tilts, zonal pressure gradients, and horizontal phase relationships may be weakened, cancelled, or biased by the relative amplitude and sample size of eastward- versus westward-propagating disturbances.

2. Minor Comments

Minor Comment 1.
Line 78: Please add Sobel and Maloney (2013) as a citation for the MJO moisture-mode theory.

Minor Comment 2.
It might be better to make the definition and interpretation of N_{mode} more precise. Equation (1) defines

$$N_{\mathrm{mode}} \equiv \frac{s'}{Lq'} \, ,$$

and the text then interprets this as the ratio of DSE to latent energy perturbations. However, in the scaling arguments of Adames et al. (2019) and Adames (2022), the relevant numerator is more accurately the enthalpy perturbation scale, i.e. C_pT', with the approximation $C_pT' \simeq s'$ requiring an implicit assumption that the perturbation in DSE is dominated by the enthalpy rather than by the geopotential (as proven below). Thus, a more precise statement is that N_{mode} represents the ratio of the enthalpy perturbation scale to the latent-energy perturbation scale.

I emphasize this point not because I try to be strict, but because the analyses in this manuscript are very valuable, and I hope that future studies will adopt similar approaches.

In that context, an accurate definition of this quantity would be especially beneficial for future studies.

(i) Clarify the definition in Eq. (1). As written, it is more accurate to write Eq. (1) as

$$N_{\mathrm{mode}} \equiv \frac{C_p T'}{Lq'} \simeq \frac{s'}{Lq'}.$$

This is proven below.

(ii) Show how the simplified diagnostic follows from the published scaling theory. The relation between the intuitive diagnostic $C_pT'/(Lq')$ (or $s'/(Lq')$) and the expression for N_{mode} in Adames et al. (2019) is not completely transparent to readers who have not worked through the algebra in that paper. I therefore recommend adding a short appendix derivation showing how the diagnostic used here follows from the theoretical scaling. For example, starting from hydrostatic balance,

$$\frac{\partial \phi}{\partial p} = -\frac{R_d T}{p},$$

and assuming separable vertical structures,

$$T = \tilde{T}(x, y, t)a(p), \qquad \phi = \tilde{\phi}(x, y, t)\Lambda(p),$$

one obtains

$$C_p \tilde{T} a = -\frac{C_p}{R_d}\tilde{\phi}\, p \frac{\partial \Lambda}{\partial p}.$$

With $a = p\,\partial\Lambda/\partial p$, this gives

$$C_p T = -\frac{C_p}{R_d} a\tilde{\phi}.$$

Using the first-baroclinic gravity-wave speed written in terms of the vertical structure function,

$$c = \left(\frac{R_d \overline{M_s}}{C_p \langle a \rangle}\right)^{1/2},$$

where $\langle\cdot\rangle$ denotes vertical integration and $\overline{M_s}$ is the dry stratification, vertical integration and rearrangement give the enthalpy scale in terms of the geopotential scale,

$$C_p T \sim \frac{\overline{M_s}}{c^2}\tilde{\phi}.$$

Combining this with the scale estimates in Adames et al. (2019),

$$\tilde{\phi} \sim U c_p, \qquad \tilde{q} \sim \frac{N_c \overline{M_q} U}{L c_p},$$

then yields

$$N_{\mathrm{mode}} \equiv \frac{C_p T'}{Lq'} \sim \left(\frac{c_p}{c}\right)^2 \frac{1}{N_c(1 - \tilde{M})},$$

which is the same quantity given by the theoretical scaling.

Note that although Adames (2022) begins with a DSE equation, that study scales the DSE perturbation by the enthalpy perturbation in its Eq. (30c), where $s' = C_p T \hat{s}$. Thus, although this point is not stated explicitly, the formulation effectively assumes that $s' \simeq C_p T'$.

Presenting this kind of derivation in Appendix would make clear why the ratio used in this manuscript is not merely an ad hoc empirical diagnostic but a simplified form of the theoretical N_{mode}.

Minor Comment 3.

The manuscript appears to overgeneralize the role of the background moisture gradient in moisture-mode dynamics, especially by treating the meridional moisture-gradient mechanism associated with moisture–vortex instability as if it were a necessary condition for moisture modes more generally.

In Section 4, where the manuscript interprets Fig. 3, the authors write:

> Figure 3 shows that moisture modes mainly occur along the edges of the deep tropics, around 15°N/S. These regions coincide with the climatological location of the moist margin (see e.g., Mapes et al., 2018; Robinson et al., 2024), where the background horizontal moisture gradient is large. This is thought to be a necessary condition for moisture modes to occur (Mayta and Adames Corraliza, 2024).

I find this phrasing too broad. A background moisture gradient is central to the moisture–vortex instability mechanism, but it is not a necessary condition for all moisture-mode theories. Moisture–vortex instability is better understood as one member of the broader class of moisture-mode dynamics, not as synonymous with moisture-mode dynamics as a whole. For example, moisture-mode theories of the MJO can involve different maintenance mechanisms and can be formulated in settings where the meridional moisture distribution is approximately parabolic about the equator, rather than requiring a large local meridional moisture gradient (e.g., Adames and Kim 2016; Wang and Sobel 2022).

A similar concern arises where the manuscript states later in Section 4.2:

> While the MVI mechanism described so far is useful in elucidating the dynamical structure of the off-equatorial moisture mode, it cannot, however, explain why the convectively active region of the equatorial moisture mode occurs where the background meridional moisture gradient is virtually zero (Figs. 8a,d).

Minor Comment 4.

In lines 211–214, the manuscript states:

> we do not see any significant differences in DSE anomaly magnitudes among the three systems. Unlike the Lq perturbation, the DSE anomaly varies only weakly with the system's propagating speed. Hence, although the ratio of DSE to latent energy anomalies (i.e., N_{mode}) increases with phase speed as hypothesised in Eq. 2, it is mainly due to the change in Lq' alone, rather than the changes in both s and Lq'.

This result appears to be consistent with the theory of Adames (2022). In Eq. (45), the enthalpy perturbation scale is set by the static stability, scale height, and dry-gravity-wave phase speed, all of which are independent of the relevant time scale. Thus, the enthalpy perturbation scale is also independent of that time scale. By contrast, the moisture perturbation is scaled by the Chikira scale and the wave time scale, as shown in Eq. (58) of Adames (2022). Therefore, as the time scale becomes longer, the moisture perturbation magnitude increases, consistent with the manuscript's finding that the increase in N_{mode} is primarily attributable to changes in Lq' rather than to changes in s'.

Minor Comment 5.
The manuscript states in lines 268–270:

> It clearly shows that the faster moving systems are more horizontally unbalanced than the slower moving ones, even for those in the equatorial region.

This result is also consistent with the theory of Adames (2022). In that theory, the degree of nonlinear balance is controlled by ϵ in Eq. (47). When ϵ is small, the deviation from WTG becomes less divergent, which corresponds to a state closer to nonlinear balance. Moreover, ϵ is determined by the Froude number in a manner analogous to N_{mode}. Thus, a reduction in N_{mode} is expected to correspond to improved nonlinear balance.

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