Supplement

S1. Longuet-Higgins Radiation Stress Coupling:

The concept of radiation stress, introduced by Longuet-Higgins and Stewart (1962), quantifies the transfer of momentum by surface gravity waves. This momentum transfer is particularly important for affecting coastal water levels and nearshore currents.

Due to wave motion, the radiation stress terms which induce wave setup and drive nearshore currents, are added to the Navier–Stokes momentum equation of (Longuet-Higgins and Stewart, 1962, 1964; Roland et al., 2012).

$$R_s = \left(R_{sx}, R_{sy}\right) \quad \text{(S1)}$$

$$R_{sx} = -\frac{1}{\rho_0 H}\frac{\partial S_{xx}}{\partial x} - \frac{1}{\rho_0 H}\frac{\partial S_{xy}}{\partial y} \quad \text{(S2)}$$

$$R_{sy} = -\frac{1}{\rho_0 H}\frac{\partial S_{yy}}{\partial y} - \frac{1}{\rho_0 H}\frac{\partial S_{xy}}{\partial x} \quad \text{(S3)}$$

where:

ρ_0 is the water density, η is the surface elevation, h is the resting depth of the ocean, $H = h + \eta$ is the total water depth, S_{xx}, S_{xy}, S_{yy} are the components of the radiation stress tensor, the stress is uniform in the vertical dimension.

In the Longuet-Higgins Coupling, the ocean component (e.g. SCHISM) provides current velocity and water level to the wave component (e.g. WW3), and WW3 provides the radiation stress to SCHISM (Fig. S1).

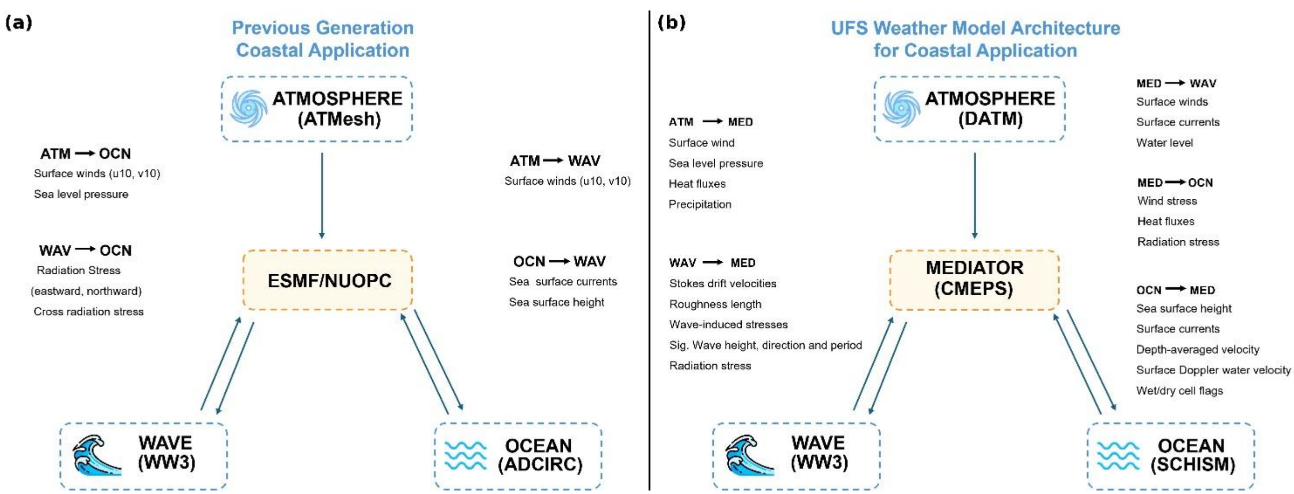

Figure S1. (a) Longuet-Higgins Ocean Wave coupling diagram in UFS-Coastal, (b) 3D vortex Ocean Wave coupling diagram in UFS-Coastal

S2. 3D Vortex Coupling Formulation

The vortex formulation explicitly represents the 3D transfer of wave momentum into sub-surface currents. The primary equation is also adding the vortex term to the modified Navier–Stokes momentum equation (Uchiyama et al., 2010; Bennis et al., 2011; Kumar et al., 2012).

Here we have the concept $\vec{U} = \overrightarrow{U_l} - \vec{U}_s$

Where $\vec{U}$ is the quasi-Eulerian velocity, (u, v, w) are its components in x, y, z directions; $\overrightarrow{U_l}$ is the mean Lagrangian velocity; and $\overrightarrow{U_s}$is the wave-introduced stoke drift, (u_s, v_s, w_s) are the U_s components in x, y, z directions.

$$(u_s, v_s) = \sigma\, k(cos\theta, sin\theta)E\,\frac{cosh\,(2kz+s2kh)}{sinh^2(kD)} \qquad \text{(s4)}$$

Where D is the water depth equal to the bathymetry adding the surface elevation $D = h + \varsigma$, $k(cos\theta, sin\theta)$ is the wave number vector, σ is the intrinsic radiation frequency, E is the spectral density of the surface wave.

w_s could be calculated from the continuity equation:

$$\frac{\partial u_s}{\partial x} + \frac{\partial v_s}{\partial y} + \frac{\partial w_s}{\partial z} = 0 \qquad \text{(s5)}$$

To enable the 3D vortex coupling, three major terms are added to the right side of the primitive ocean momentum equation: $-\vec{\nabla}J$, $-w_s\frac{\partial\vec{U}}{\partial z}$ and $\overrightarrow{F_{vortex}}$. Other terms such as mixing F_m, wave breaking, wave turbulence F_d, bottom frictions F_b are optional to use to improve the simulation results (Bennis et al., 2011).

F_{vortex} is the vortex-force term, $-\vec{\nabla}J$ is the wave-induced mean pressure (Bernoulli head) (J is the wave-induced mean pressure), $-w_s\frac{\partial\vec{U}}{\partial z}$ is the vertical Stokes-drift advection term.

$$\overrightarrow{F_{vortex}} = (f + \vec{\nabla} \times \vec{U}) \times \overrightarrow{U_s} \qquad \text{(S6)}$$

Where f is the Coriolis force coefficient.

The three terms separated in x, y directions become as follows:

$$-\frac{\partial J}{\partial x} - w_s\frac{\partial u}{\partial z} + \left[f + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\right]u_s \qquad \text{(s7)}$$

$$-\frac{\partial J}{\partial y} - w_s\frac{\partial v}{\partial z} + \left[f + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\right]v_s \qquad \text{(s8)}$$

These terms are computed from wave model outputs (e.g., WW3) and integrated within hydrodynamic models (e.g., SCHISM), allowing for accurate representation of wave-induced currents, mixing, and turbulence. Uchiyama et al. (2010) provide a rigorous derivation and explanation of the vortex-force formalism. The detailed variable exchanges can be found in Fig. S1b and Table S1 &S2, the variable names are from the SCHISM and WW3 defintions.

Table S1. Variables exported from WW3 to SCHISM in the 3D vortex coupling configuration.

Wave Variable	Exported name	Purpose
Ustx	eastward_stokes_drift_current (stokes drift)	Calculate 2D stokes terms
Usty	northward_stokes_drift_current (stokes drift)	Calculate 2D stokes terms
Uorbx	eastward_wave_bottom_current (near bottom orbital velocity)	For wave bottom boundary layer calculations
Uorby	northward_wave_bottom_current (near bottom orbital velocity)	For wave bottom boundary layer calculations
Awb	exported_name (near bottom excursion wave amplitude)	For wave bottom boundary layer calculations
k	exported_name (spectrally averaged wavenumber vector)	Calculate wave induced terms in ocean model
θ	exported_name (spectrally averaged wave direction)	Calculate wave induced terms in ocean model
τwx	surface_eastward_wave_induced_stress	Calculate surface drag coefficients & boundary conditions
τwy	surface_northward_wave_induced_stress	Calculate surface drag coefficients & boundary conditions
τbwx	eastward_wave_to_bbl_stress (utbb)	Bottom boundary layer calculations & boundary conditions
τbwy	northward_wave_to_bbl_stress (vtbb)	Bottom boundary layer calculations & boundary conditions
Hs	Significant wave height (WW3__OHS)	Calculation on BL
TM1	Mean wave period (WW3_T0M1)	Wave BBL
BHD	wave-induced Bernoulli head pressure WW3__BHD (Ali Abdolali checked unit: m2/s2) J term According to Smith JPO 2006	Momentum
TWO	wave-ocean momentum flux WW3_TWOX &WW3_TWOY	Momentum

TBB	Momentum flux due to bottom friction WW3_TBBX & WW3_TBBY	Momentum
Rollers	exported_name (dissipation term)	Calculate wave roller terms (Bennis et al. 2011). This is one way to calculate the roller terms.

Table S2. Variables exported from SCHISM to WW3 in the 3D vortex coupling configuration.

Ocean Variable	Exported name	Purpose
ζ	sea_surface_height_above_sea_level (surface height above geoid)	Calculate WW3 terms
usx	surface_eastward_sea_water_velocity (surface water velocity)	Calculate WW3 terms
usy	surface_northward_sea_water_velocity (surface water velocity)	Calculate WW3 terms
dav	Depth-averaged velocity (x,y)	2D radiation stress formulation
υdx	exported_name (surface doppler water velocity)	Calculate WW3 terms
υdy	exported_name (surface doppler water velocity)	Calculate WW3 terms
idry_e(mask)	exported_name=idry (wet (=0)/dry (=1) flags @ cell)	Skip dry cells (mask array, exported ocean_mask field)

Reference:

Bennis, A.-C., Ardhuin, F., and Dumas, F.: On the coupling of wave and three-dimensional circulation models: Choice of theoretical framework, practical implementation and adiabatic tests, Ocean Modelling, 40, 260-272, 10.1016/j.ocemod.2011.09.003, 2011.

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Uchiyama, Y., McWilliams, J. C., and Shchepetkin, A. F.: Wave–current interaction in an oceanic circulation model with a vortex-force formalism: Application to the surf zone, Ocean Modelling, 34, 16-35, 10.1016/j.ocemod.2010.04.002, 2010.