Review of Mohr–Coulomb yield curves for viscous-plastic sea ice models: flow rules and failure angles by Damien Ringeisen, L. Bruno Tremblay, Jean-François Lemieux, and Martin Losch

The paper discusses various versions for a constitutive model for sea ice sheets. This model is based on a Mohr-Coulomb relationship for the admissible stresses and several models to define the flow directions in case of yielding. The model is implemented as Reiner-Rivlin equation in a depth integrated momentum equation for the ice sheet. The shear deformation field from numerical experiments on a strip of sea ice (i.e. uniaxial loading) is analysed graphically to determine the resulting angles of the shear bands. These angles are in the range of observations depending on the used formulation for the flow directions. The article is logically structured and clearly presented. However, there are a few issues that remain unclear.

1. The Mohr–Coulomb criterion is originally defined in three dimensions as

$$\frac{\sigma_1 - \sigma_3}{2} = -\frac{\sigma_1 + \sigma_3}{2} \sin\varphi + c\cos\varphi\,, \tag{1}$$

with the principal stresses $\sigma_1 \geq \sigma_2 \geq \sigma_3$. The authors use a two-dimensional version that can only be applied without restriction to an axisymmetric condition ($\sigma_1 \geq \sigma_2 = \sigma_3$ or $\sigma_1 = \sigma_2 \geq \sigma_3$) or to a plane strain condition. Plane strain would mean that the vertical strain in the ice sheet is zero, which does not seem realistic. Such thin structures are usually modelled under plane stress, i.e., the vertical stress is zero (gravity neglected). For a shear stress free horizontal surfaces, the two-dimensional formulation presented in this paper can only be used if the major and minor principal stresses have opposite signs or if one of them is zero – such as the vertical stress. Small surface shear stresses do not change this situation a lot. The authors should clearly state if the use a pane strain or a plain stress boundary condition to reduce 3D to their 2D problem.

2. Eq. (1) does not include failure for isotropic compression. However, there is a compressive failure possible under plane stress, which is equal to the uniaxial compressive strength ($P = -\sigma_3$ for $\sigma_3 < 0$ and $\sigma_1 = 0$, i.e. $\sigma_1 > \sigma_3$, in (1)):

$$P = \frac{2c\cos\varphi}{1 - \sin\varphi}\,. \tag{2}$$

Including such a compression failure cap in the model suggests that the authors assume a plane stress condition. If this is the case, their two-dimensional formulation of the Mohr-Coulomb criterion cannot be applied to arbitrary stress states.

3. Rewriting (1) in terms of the invariants used by the authors yields

$$\sigma_{\mathrm{II}} = -\sigma_{\mathrm{I}} \sin\varphi + c\cos\varphi = -\mu\sigma_{\mathrm{I}} + c\sqrt{1-\mu^2}\,. \tag{3}$$

The variable c used the paper therefore does not correspond to cohesion in the usual sense, which defines the shear strength of the material without normal stress.

4. What is the variable $\dot{\boldsymbol{\epsilon}}$? I assume that that is the symmetric part of the velocity gradient, i.e. the stretching tensor, and not the time derivative of a strain. If that is true, it would be better to call $\dot{\epsilon}$ stretching and not strain rate.

5. For the presented Mohr-coulomb rheology with pure shear flow rule (MC-PS) the stress invariants

$$\sigma_{\mathrm{I}} = \zeta \dot{\epsilon}_{\mathrm{I}} - p \tag{4}$$
$$\sigma_{\mathrm{II}} = \eta \dot{\epsilon}_{\mathrm{II}} \tag{5}$$

yield (for $\eta < \eta_{\mathrm{max}}$ and $\zeta < \zeta_{\mathrm{max}}$, i.e. stretching above a certain threshold, version Ip et al. (1991))

$$\sigma_{\mathrm{I}} = \frac{P(1+k_t)}{2} \operatorname{sign} \dot{\epsilon}_{\mathrm{I}} - \frac{P(1+k_t)}{2} \tag{6}$$
$$\sigma_{\mathrm{II}} = \mu \frac{P(1+k_t)}{2} (1 - \operatorname{sign} \dot{\epsilon}_{\mathrm{I}})\,. \tag{7}$$

The stress is therefore independent of the magnitude of the stretching, which is consistent with ideal plastic behaviour. However, this also means that only stress states at the two limbs (T,0) and ($-P$, $\mu(P+T)$) in Fig. 1 can occur. How can failure states on the line between these limbs can be reached? I did not manage to find any stretching tensor such that stress states on the inclined line between these two limbs occur.

6. If the regularisation in the paper with η_{max} takes place (stretching below a certain threshold), the stress state is either at (T,0) if $\dot{\epsilon}_{\mathrm{I}} > 0$ (tension), or in the vertical line between ($-P$, $\mu(P+T)$) and ($-P$, 0) if $\dot{\epsilon}_{\mathrm{I}} < 0$ (compression). Such states on this vertical line does not have $\dot{\epsilon}_{\mathrm{II}} = 0$, as indicated by a horizontal vectors in Fig. 1, except at ($-P$, 0).

7. Please include a table listing all the parameters used in the numerical computations, so that readers do not have to look them up in various places throughout the article. Furthermore, I was unable to find some values, such as the value used for η_{max}.

8. The equations for the orientation of LKFs as a function of the angle of friction and the angle of dilatation were developed for granular materials. It is questionable to what extent these relationships can be applied to a solid material such as ice, in which shear zones develop through a fracture process (beginning with an initial crack, followed by wing cracks and comb cracks, which form ice granules within a shear band).

   The additional information provided by the graphical representation of these relationships in Figures 5 to 7 is limited. Furthermore, it is not immediately clear why θ_A in Figures 6 and 7 depends on μ (i.e. on the friction angle). The authors should consider omitting these angles, as the key point is that the rheological model predicts angles within the range of the observations.

If the authors decide to include the angles $\theta_{R,C,A}$ in Figures 5 to 7, they must explain how δ is computed for Figures 6 and 7, as δ depends on σ_{I}, which is not explicitly given in these figures.

9. Apart from the problems mentioned in the above item, the situation for the 2D problem under consideration is not quite so clear. If the authors are considering plane strain, then $\dot{\epsilon}_{\mathrm{I}}$ corresponds to the volumetric stretching, and, for example, $\delta = 0$ corresponds to isochoric behaviour. If, on the other hand, they apply plane stress (which seems more appropriate for an ice sheet that is relatively thin compared to its horizontal dimensions), $\dot{\epsilon}_{\mathrm{I}}$ is not equal to the volumetric stretching $\dot{\epsilon}_v = \dot{\epsilon}_1 + \dot{\epsilon}_2 + \dot{\epsilon}_3$, as there is an additional (unknown) stretching in the vertical direction. Such the computed dilatation angle does not describe the volumetric behaviour, and the use of θ_A is rather problematic.

10. Fig. 6: use same labels for axis

11. typos:
p 4 l 87: missing d at defined
Eq. (18), delete comma before full stop
p 10 l 239. Capital W after full stop