

Response to Anonymous 1

Retrieving raindrop size distribution (DSD) using disdrometers has become a research hotspot in the field of weather radar in recent years, as DSD is crucial for understanding cloud and precipitation microphysical processes. This study combines disdrometer measurements with X-Ka band vertically pointing radar to perform DSD retrieval. To address the ambiguity problem in X-Ka band retrieval, the Ka-W band is further introduced as a joint constraint to reduce the ambiguous region between the dual-frequency ratio (DFR) and the mass-weighted diameter (D_m), thereby improving DSD retrieval accuracy. Multi-frequency radar retrieval is an important research direction, and the DFR- D_m ambiguity is a key issue in DSD retrieval. Although this study has practical significance, the methodology section suffers from unclear descriptions and logical issues related to circular reasoning.

Major issues:

- 1. The radar used in this study is an X-Ka dual-frequency radar. The proposed method employs the Ka-W dual-band to constrain the retrieval process of the X-Ka dual-frequency radar in order to mitigate the ambiguity problem inherent in X-Ka dual-frequency observations. However, no measured data from W-band radar are provided in the paper. It remains unclear how this constraint can be implemented in practical applications.*

Response: We thank the reviewer for raising this critical point regarding the practical implementation of the W-band constraint.

Under the current X/Ka dual-frequency observational configuration, $DFR(Ka,W)$ cannot be directly obtained. Therefore, the Ka-W constraint is not operationally applied in the retrieval itself, but is used here as a theoretical framework to evaluate the potential of a future three-frequency radar system for resolving the DFR- D_m ambiguity.

The W-band reflectivities employed in this study were obtained entirely through T-matrix scattering simulations based on measured raindrop size distributions (Section 5.1). Therefore, the $DFR(Ka, W)$ constraint should currently be regarded as a theoretical assessment of its potential to mitigate the $DFR(X, Ka)-D_m$ ambiguity, rather than a fully operational retrieval constraint that has been validated with real three-frequency radar data.

In the proposed retrieval framework (Figure 1 and Section 2.1), the W-band constraint is introduced through the scattering simulation branch (blue boxes). The procedure is as follows:

1. Step 2 (Section 2.2): T-matrix scattering calculations are performed to simulate theoretical radar reflectivities at X-, Ka-, and W-band frequencies based on 2DVD-measured DSDs.

2. Step 3: The DFR(Ka, W)-Dm retrieval relationship is established via fitting using these simulated reflectivities (Figure 13b).

3. Step 6: When actual X/Ka dual-frequency radar observations yield $DFR(X, Ka) \leq 0$ (the ambiguous zone), the pre-established simulated DFR(Ka, W)-Dm relationship is invoked to resolve the ambiguity and select the correct Dm solution.

Under the current observational conditions where only X/Ka dual-frequency radar data are available, the W-band constraint cannot be directly measured and must rely on the theoretical relationships derived from disdrometer data. This is explicitly why we characterize this study as a proof-of-concept demonstration (Section 5.1 and Section 6).

For operational application, this approach would require a three-frequency radar system with X/Ka/W bands (Section 5.1). With such a system deployed, the W-band reflectivity would be directly measured, enabling real-time computation of DFR(Ka, W) to constrain the retrieval without dependence on scattering simulations. However, the actual performance may be affected by several practical factors:

- W-band radar calibration accuracy;
- Atmospheric attenuation at W-band (which is significantly stronger than at X- or Ka-band);
- Signal-to-noise ratio limitations in weak precipitation regions.

This study provides a proof-of-concept framework that motivates the development of future three-frequency radar systems. The simulated DFR(Ka, W) constraint demonstrates the theoretical potential to increase the proportion of data permitting a unique Dm solution from 35.38% to 87.5% (Section 3.5 and Table 6). The practical applicability of this Ka-W constraint for operational systems requires further validation with real W-band observations and more extensive datasets covering diverse precipitation regimes (Section 6). We will clarify this distinction more explicitly in the revised manuscript to ensure readers understand the current simulation-based nature of the W-band constraint and the requirements for its future operational implementation.

2. This study uses DSD data observed by disdrometers during three rainfall events to fit the attenuation coefficients for the X-band and Ka-band radars. These coefficients are then used to correct attenuation in the X-band and Ka-band radar observations from the same three rainfall events. Subsequently, the DFR-Dm relationship is fitted, and DSD retrieval is performed. This algorithm may suffer from logical issues of self-consistency bias.

Response: We thank the reviewer for raising this critical concern regarding potential self-consistency bias in the validation procedure.

We agree that the use of the same DSD dataset for both relationship development and preliminary evaluation may lead to optimistic estimates of retrieval performance,

regardless of whether the underlying relationships are physically based or empirically fitted. Therefore, an independent event in 2025 was introduced, which was not included in the relationship development. The independent validation results (Table 7) demonstrate comparable accuracy to the training-period evaluation, suggesting that the proposed relationships possess a degree of generalizability beyond the training data.

Despite the independent validation for the DSD retrieval, we recognize that the attenuation correction coefficients and the DFR-Dm fitting were both derived from the same 2022 events used for the initial evaluation. This represents a limitation of the current proof-of-concept study. As discussed in Section 5.4, future studies will employ event-independent validation using leave-one-event-out cross-validation with a larger and more diverse dataset to further quantify the robustness and generalization capability of the proposed method. We will also explicitly separate the datasets used for establishing attenuation relationships from those used for validating retrieval results to completely avoid any self-consistency issues.

Thank you for this valuable suggestion, which has prompted us to more carefully consider the validation strategy and its limitations in the revised manuscript.

Specific comments:

- 1. The flowchart shown in Figure 1, along with its corresponding steps and the formulas contained therein, requires additional explanatory clarification.*

Response: We acknowledge that Figure 1 and its corresponding workflow require more detailed explanatory clarification to ensure readers can fully understand the retrieval framework. In the revised manuscript, we have enhanced the explanatory content as follows:

(1) A new Section 2.1 (Retrieval Framework) is included to systematically elaborate on the three core components of the overall retrieval framework: observational inputs, scattering simulations and multi-constraint retrieval. The corresponding retrieval flowchart has also been redrawn. Blue boxes denote the results from scattering simulations, and green boxes represent those derived from radar observation data.

(2) A new Section 2.2 (Retrieval Procedure) presents the complete workflow from 2DVD observations to the final retrieval of Dm, LWC and Nw in eight steps. The input, processing and output of each step are clearly specified.

We have clarified that blue arrows represent the scattering simulation workflow (physics-based theoretical derivation using 2DVD data), while green arrows represent the operational retrieval workflow applied to real radar observation data. This distinction is critical because the W-band constraint ($DFR(Ka, W)$) is currently derived from the simulation branch (blue), whereas the actual retrieval for radar observations (green) relies on X/Ka-band measurements augmented by this

pre-established theoretical constraint.

These revisions ensure that Figure 1 serves as a self-contained visual guide to the retrieval framework, with clear traceability between the flowchart, the step-by-step procedure in Section 2.2.

Thank you for this constructive comment, which significantly improve the accessibility and reproducibility of our retrieval method.

2. Table 1 indicates that the X-Ka dual-frequency radar has the capability to observe polarimetric parameters. If more polarimetric observation parameters could be used to constrain the DSD retrieval, the retrieval accuracy would theoretically be higher than that achieved using only dual-frequency parameters.

Response: We thank the reviewer for this insightful suggestion regarding the potential use of polarimetric parameters to further improve DSD retrieval accuracy.

The reviewer is correct that the X/Ka dual-frequency radar system listed in Table 1 has the capability to observe polarimetric parameters, including differential reflectivity (Z_{DR}), differential propagation phase (Φ_{DP}), specific differential phase (K_{DP}), and co-polar correlation coefficient (ρ_{HV}). However, the radar was operated in a vertically pointing mode throughout this study (Section 3.1: "vertical pointing mode"). As explicitly noted in Section 5.4, this observation geometry largely eliminates polarimetric differences because the radar beam is aligned with the principal axis of symmetry of the raindrops (near-zenith incidence). Under vertically pointing conditions, the differential reflectivity (Z_{DR}) approaches 0 dB, and the specific differential phase (K_{DP}) becomes negligible, rendering these parameters ineffective as discriminators for DSD retrieval. Consequently, polarimetric constraints could not be meaningfully applied in this specific observational configuration.

We fully agree with the reviewer that incorporating polarimetric parameters would theoretically enhance retrieval accuracy beyond what is achievable with dual-frequency parameters alone. In scanning or slant-path observations, polarimetric variables provide complementary microphysical information:

Z_{DR} is sensitive to the median volume diameter (D_0) and drop shape, offering an independent constraint on D_m .

K_{DP} is proportional to the liquid water content and is immune to absolute calibration errors, providing a robust constraint for N_w retrieval.

ρ_{HV} can help identify non-meteorological echoes, mixed-phase regions, and DSD broadening.

These properties have been widely demonstrated in the literature for improving DSD retrievals and attenuation correction.

As stated in Section 5.4, incorporating polarimetric constraints will be one of the priorities for future work. When the radar operates in scanning mode (non-zenith angles), the full polarimetric capability of the system will be leveraged to:

1. Provide additional independent constraints on D_m and N_w , reducing the reliance on the DFR-based retrieval alone.
2. Improve attenuation correction through the use of the $\Phi \sim DP$ constraint, which is particularly valuable at Ka-band where attenuation is significant.
3. Enhance the discrimination of hydrometeor types and melting layer identification.

Thank you for this valuable suggestion, which reinforces the importance of utilizing the full polarimetric capabilities of the instrument in future observational campaigns.

3. Figure 3 shows the radar reflectivity factors observed by the X-band and Ka-band radars below the melting layer. The paper does not specify how the position of the melting layer is determined. Furthermore, does the retrieved DSD correspond to the entire rainfall region below the melting layer or only to the near-surface layer? Since the algorithm is based on ground-based disdrometer observations, it is inferred that the retrieval results should represent near-surface conditions. However, the radar observations are presented as corresponding to the rainfall region below the melting layer, indicating an inconsistency.

Response: We thank the reviewer for raising these important points regarding the melting layer identification and the vertical representativeness of the retrieved DSDs.

The melting layer position was identified from the radar reflectivity observations based on the bright band signature. As noted in the caption of Figure 3, "The height of the melting layer bright band indicates that the zero-degree layer is approximately 5 km." The bright band, characterized by enhanced reflectivity near the 0°C isotherm due to melting hydrometeors, provides a clear marker for the melting layer top. Below this height, the radar observations are considered to be in the rain region. We acknowledge that the manuscript did not explicitly detail the specific algorithm used for bright band detection (e.g., peak reflectivity identification or temperature profile constraints), and we have added this clarification in the revised manuscript.

The retrieved DSDs correspond to the entire rainfall region below the melting layer, not just the near-surface layer. As shown in Figure 16, the retrieval framework produces time-height profiles of D_m and N_w throughout the rain column below the melting layer, capturing the vertical microphysical structure of precipitation (Section

4.3). The radar-based retrievals reveal distinct characteristics at different heights, such as the local enhancement of Dm and Nw near the melting layer due to melting hydrometeors.

The reviewer correctly identifies a potential inconsistency. The retrieval relationships (DFR-Dm, LWC- Z_{ka} , and attenuation correction coefficients) were established using ground-based 2DVD measurements (Sections 3.2 – 3.5), yet they are applied to radar observations at all heights below the melting layer. This procedure implicitly assumes that the microphysical relationships derived from surface DSDs are representative of the entire rain column below the melting layer.

We acknowledge that this is a simplifying assumption with the following limitations:

1. The DSD may evolve vertically due to processes such as evaporation, coalescence, and breakup, meaning surface-based relationships may not perfectly represent upper-level conditions.

2. The quantitative validation presented in Figures 8, 17 – 20 and Table 7 is indeed limited to near-surface comparisons (reflectivity at 120 m height is used as a proxy for ground-level values, Section 3.3), as the 2DVD only provides ground-truth measurements at the surface.

We have explicitly noted this limitation in Section 5.3 ("Representativeness of 2DVD Validation"), where we discuss that wind drift, fall time, and sampling volume differences between radar and 2DVD limit the accuracy of quantitative comparisons. Furthermore, in Section 6, we state that future work should "incorporate height-dependent DSD models for upper-level retrievals" to address the vertical variability of DSDs and reduce the reliance on surface-derived relationships at higher altitudes.

In summary, while the current framework retrieves DSDs throughout the rain column below the melting layer, the underlying relationships are surface-based, and the quantitative validation is restricted to near-surface levels. We have clarified this distinction more explicitly in the revised manuscript and strengthen the discussion of this limitation. Consequently, the retrievals above the near-surface layer should be interpreted qualitatively rather than as fully validated quantitative estimates.

4. In Figure 5, the attenuation coefficients are fitted based on disdrometer-observed drop size data and used for radar attenuation correction. Figure 8 then compares the corrected radar retrieval results with the disdrometer observation data from the same several rainfall events. This procedure may suffer from a self-consistency trap. Similar issues exist in the DSD retrieval section.

Response: We thank the reviewer for raising this critical concern regarding potential self-consistency bias in the validation procedure.

We acknowledge the potential self-consistency bias. The current study is therefore positioned as a proof-of-concept investigation. Independent-event (2025) validation

has been added. The independent validation results (Table 7) demonstrate comparable accuracy to the training-period evaluation, suggesting that the proposed relationships possess a degree of generalizability beyond the training data.

Despite the independent validation for the DSD retrieval, we recognize that the attenuation correction coefficients and the DFR-Dm fitting were both derived from the same 2022 events used for the initial evaluation. This represents a limitation of the current proof-of-concept study. As discussed in Section 5.4, future studies will employ event-independent validation using leave-one-event-out cross-validation with a larger and more diverse dataset to further quantify the robustness and generalization capability of the proposed method. We will also explicitly separate the datasets used for establishing attenuation relationships from those used for validating retrieval results to completely avoid any self-consistency issues.

Thank you for this valuable suggestion, which has prompted us to more carefully consider the validation strategy and its limitations in the revised manuscript.

5. Figure 6 presents DFR results for the Ka-band and W-band obtained through simulations, but no measured W-band data are provided in the paper. In practical applications, how can the W-band be used to constrain the retrieval process?

Response: We fully acknowledge that this study does not include measured W-band radar observations. The W-band reflectivities used in this study were obtained entirely through T-matrix scattering simulations based on measured raindrop size distributions (Section 5.1). Therefore, the $\text{DFR}(\text{Ka}, \text{W})$ constraint presented here should currently be regarded as a theoretical assessment of its potential to mitigate the DFR-Dm ambiguity, rather than a fully operational retrieval constraint validated with real three-frequency radar data.

In practical applications, the implementation of this approach would require a three-frequency radar system with X/Ka/W bands (Section 5.1). The physical basis for using W-band as a constraint lies in the fact that the $\text{DFR}(\text{Ka}, \text{W})$ -Dm relationship exhibits a significantly narrower ambiguous zone compared to $\text{DFR}(\text{X}, \text{Ka})$ -Dm (Section 3.5 and Figure 12). While the ambiguous zone for $\text{DFR}(\text{X}, \text{Ka})$ covers a wide Z_{Ka} range of approximately -20 to 45 dBZ (corresponding to Dm from 0.3 to 1.6 mm), the ambiguous zone for $\text{DFR}(\text{Ka}, \text{W})$ is confined to a much narrower Z_{Ka} range of approximately -20 to 20 dBZ (corresponding to Dm between 0.3 and 0.8 mm). This characteristic allows $\text{DFR}(\text{Ka}, \text{W})$ to effectively resolve the dual-solution ambiguity when $\text{DFR}(\text{X}, \text{Ka}) \leq 0$.

However, for operational implementation, several practical challenges must be addressed (Section 5.1):

1. W-band radar calibration accuracy: Calibration errors at W-band would propagate into $\text{DFR}(\text{Ka}, \text{W})$ estimates.
2. Atmospheric attenuation at W-band: W-band signals are subject to stronger atmospheric attenuation, which must be carefully corrected.

3. Signal-to-noise ratio limitations: The performance may be constrained by SNR limitations at W-band, particularly in weak precipitation regions.

This study provides a proof-of-concept demonstration that motivates the development of future three-frequency radar systems for operational DSD retrieval. The practical applicability of the Ka-W constraint requires further validation with real W-band observations and more extensive datasets covering diverse precipitation regimes (Section 6). Future work should also quantify the impact of radar calibration and attenuation correction uncertainties on the retrieval accuracy when real W-band data become available (Section 5.2).

6. Figure 12 shows the DFR-Dm relationship curves, indicating that DFR(Ka, W) exhibits a narrower ambiguous region. However, even if DFR(Ka, W) can effectively reduce the ambiguous region, actual observations only include X-band and Ka-band dual-frequency radar data. Under these conditions, how the W-band radar can be introduced to achieve retrieval constraints requires further explanation.

Response: We thank the reviewer for this important question regarding the practical implementation of the W-band constraint under current observational limitations.

It is important to explicitly acknowledge that this study does not include W-band radar observations. The W-band reflectivities used herein were obtained entirely through T-matrix scattering simulations based on measured raindrop size distributions (2DVD data) (Section 5.1). Therefore, the DFR(Ka, W) constraint should currently be regarded as a theoretical assessment of its potential to mitigate the DFR-Dm ambiguity, rather than a fully operational retrieval constraint validated with real three-frequency radar data.

In the proposed retrieval framework (Section 2.1 and Figure 1), the W-band constraint is introduced through the scattering simulation branch (blue boxes). Specifically:

1. The 2DVD-measured DSDs are used as ground-truth inputs (Step 1 in Section 2.2).
2. T-matrix scattering calculations are performed to simulate theoretical radar reflectivities at X-, Ka-, and W-band frequencies (Step 2).
3. The DFR(Ka, W)-Dm retrieval relationship is established via fitting (Step 3).
4. When actual X/Ka dual-frequency radar observations yield $\text{DFR}(X, \text{Ka}) \leq 0$ (the ambiguous zone), the simulated DFR(Ka, W)-Dm relationship is invoked to resolve the ambiguity and retrieve the correct Dm value (Step 6).

Under current conditions where only X/Ka dual-frequency radar data are available, the W-band constraint cannot be directly measured and must rely on the simulated relationships derived from disdrometer data. However, the practical implementation of this approach would require a three-frequency radar system with X/Ka/W bands (Section 5.1). With such a system deployed, the W-band reflectivity would be directly measured, allowing real-time computation of $DFR(Ka, W)$ to constrain the retrieval without dependence on scattering simulations.

As emphasized in Section 5.1 and Section 6, the actual performance of a real three-frequency system may be affected by W-band radar calibration accuracy, atmospheric attenuation at W-band, and signal-to-noise ratio limitations. This study serves as a proof-of-concept demonstration that motivates the development of future three-frequency radar systems. Once real W-band observations become available, the retrieval framework established in this study can be directly applied, with the simulated relationships serving as an initial theoretical foundation until sufficient observational data are accumulated to refine the empirical $DFR(Ka, W)$ -Dm fitting.

Thank you for this valuable comment, which has prompted us to clarify the distinction between the current simulation-based approach and future operational implementation with actual W-band measurements.

Response to Anonymous 2

The manuscript addresses an important problem in dual-frequency radar retrieval of raindrop size distributions, namely the ambiguity in the DFR-Dm relationship. The proposed use of an additional Ka/W-band constraint is an interesting idea and may have value for improving retrievals in weak-echo or small-drop regimes. However, in its current form, the manuscript requires substantial revision before the method and conclusions can be fully evaluated.

My primary concern is that the retrieval framework is not described clearly enough to determine how independent the information used in the retrieval actually is. In particular, the Ka/W-band constraint appears to be based on W-band reflectivities simulated from measured DSDs rather than independent W-band radar observations. This creates a potential circularity problem, especially because the same or closely related DSD information appears to be used to construct retrieval relationships and evaluate the retrieved parameters. The manuscript should more clearly distinguish observed quantities from simulated or fitted quantities and explain how the method would be applied to independent radar observations.

More broadly, the study is based on a small dataset from a single location and relies heavily on fitted relationships derived from that dataset. The authors should therefore be more cautious in presenting the method as generally applicable. Additional discussion is needed regarding calibration uncertainty, attenuation correction, FMCW radar-specific issues, beam mismatch, representativeness of the 2DVD validation, and the practical limitations of using W-band observations in rain. The discussion and conclusion should also be expanded to address these limitations and to identify the additional validation required before the method can be considered robust across different precipitation regimes, locations, and radar systems.

1) The abstract is too general and should quantify the claimed improvement. The authors state that the proposed method substantially reduces the DFR-Dm ambiguity and performs better than conventional approaches, especially for weak echoes, but the abstract does not report any specific performance metrics.

Response: We thank the reviewer for this valuable suggestion. The revised abstract has been completely rewritten to include specific quantitative metrics that substantiate the claimed improvements:

- a. Ambiguity reduction: We now explicitly state that the proportion of data permitting a unique Dm solution increased from 35.38% (using DFR(X, Ka) alone) to 87.5% after incorporating the DFR(Ka, W) constraint.
- b. Weak-echo performance: We specify that the proposed framework maintains

- robust accuracy for weak echoes (< 20 dBZ), where conventional methods fail.
- c. Proof-of-concept scope: We have re-framed the study as a "proof-of-concept" investigation to appropriately constrain the claims.

2) The authors should clarify that the method retrieves DSD parameters under an assumed normalized gamma distribution with fixed μ , rather than independently retrieving the full DSD shape. Since μ is fixed and LWC is obtained from a fitted ZKa-LWC relationship, the resulting DSD is strongly constrained by the assumed distribution form and fitted relationships. The manuscript should avoid implying that the full DSD is independently retrieved without these assumptions.

Response: This is an important clarification. We have made the following revisions to explicitly state the assumptions and constraints:

- a. Figure 1 caption (Section 2.1): The caption now explicitly reads: "Flowchart of X/Ka dual-frequency radar retrieval for normalized gamma raindrop size distribution." This makes the assumed DSD model immediately visible to the reader.
- b. Section 2.2, Step 8: The final step of the retrieval procedure now explicitly states that D_m -est and N_w -est are substituted into the normalized gamma distribution function with $\mu = 3$.
- c. New Section 5.1 ("Practical Applicability of Ka-W Constraints"): We added a paragraph discussing the limitations of the fixed- μ assumption and the trade-off between D_m and N_w retrieval accuracy for different μ values. We acknowledge that the full DSD is not independently retrieved, but rather reconstructed under the normalized gamma assumption.
- d. Title change: The title has been revised to "The Proof-of-Concept for Raindrop Size Distribution Retrieval..." to better reflect the constrained nature of the retrieval framework.

3) The use of an FMCW radar system should be discussed in more detail. The proposed retrieval depends on accurate X- and Ka-band reflectivity measurements, so the authors should explain how the two radar channels were calibrated and matched in range, timing, sensitivity, and beam volume.

This is especially important because the method relies on DFR, which can be strongly affected by small calibration offsets or system artifacts. Since the authors claim improved performance in weak-echo conditions, they should also discuss FMCW-specific issues such as signal-to-noise limits, leakage/clutter suppression, range sidelobes, and range-Doppler effects. These factors could affect the computed DFR and therefore the retrieved D_m and N_w .

Response: We have significantly expanded the discussion of the FMCW system and its implications for retrieval accuracy:

- a. Section 3.1 ("Instruments"): We added explicit text stating that the X/Ka dual-frequency radar, developed by the 38th Research Institute of China Electronics Technology Group Corporation (CETC-38), employs a fully solid-state frequency-modulated continuous wave (FMCW) system with **coaxial scanning and has undergone absolute calibration prior to field deployment.**
- b. New Section 5.2 ("Radar Calibration and DFR Uncertainty"): We added a dedicated subsection that explicitly discusses four critical uncertainty sources:
 - Calibration error propagation: How calibration errors at either frequency directly propagate into DFR estimates.
 - Beam mismatch: The sampling volume differences between the X-band (1°) and Ka-band (0.4°) antennas, and how temporal smoothing mitigates this.
 - Attenuation correction uncertainties: Particularly at Ka-band where both attenuation and Mie scattering are significant.
 - DFR error amplification: How small errors in individual reflectivity measurements can be amplified in DFR calculation, especially in weak echo regions with low SNR.

We emphasize that this study serves as a proof-of-concept investigation, and that for operational implementation, these uncertainties require quantitative assessment and the development of corresponding mitigation strategies.

4) The manuscript states that the Ka-band beamwidth is 0.4° and the X-band beamwidth is 1° , and that a 3-point moving average along the time dimension adjusts the Ka-band beamwidth to 1.2° . This statement needs clarification. Temporal smoothing does not directly change antenna beamwidth.

The authors should revise this section to describe the procedure more accurately. If the intent is to reduce scale mismatch between the two frequencies by temporal averaging, then the authors should say so and quantify the effective sampling distance associated with the averaging window. If the authors are claiming an effective beamwidth adjustment, they need to justify how this was calculated.

Response: The reviewer is absolutely correct. We have revised the statement in Section 3.1 and removed the misleading claim regarding "effective beamwidth adjustment." The revised text now accurately describes the procedure:

"To mitigate the scale mismatch between the two frequencies, Ka-band reflectivities are temporally smoothed using a 3-point running mean. This increases the effective averaging scale along the time dimension, rendering the smoothed Ka-band sampling volume approximately equivalent to that of the X-band."

This revision clarifies that the intent is to reduce scale mismatch by increasing the effective sampling volume through temporal averaging, not to alter the physical antenna beamwidth.

5) A major concern is that the Ka/W-band constraint does not appear to come from independent W-band radar observations. Instead, the W-band reflectivity appears to be simulated from the same measured DSD dataset used to build the retrieval relationships. This creates a risk of circular reasoning: the method appears to use DSD observations to create a simulated W-band constraint, then evaluates the retrieval against DSD-derived quantities.

The authors should clearly separate observed radar quantities from simulated quantities. They should also show that the method works on independent data that were not used to build the simulated Ka/W relationships, preferably through event-independent or site-independent validation.

Response: This is indeed the most critical concern, and we have addressed it directly and comprehensively throughout the revised manuscript:

- a. Abstract: We now explicitly state: "W-band reflectivities obtained through scattering simulations based on measured raindrop size distributions, combined with the DFR(Ka, W) constraint..."
- b. Section 2.1 (Retrieval Framework, Figure 1): We added a clear visual and textual distinction: blue boxes represent scattering simulation results, and green boxes represent radar observation data. The caption reads: "Blue arrows indicate the scattering simulation workflow, while green arrows represent the retrieval workflow for real radar observation data."
- c. Section 2.2 and Section 5.1: We explicitly acknowledge that this study does not include W-band radar observations, and that the DFR(Ka, W) constraint is currently a theoretical assessment of its potential to mitigate ambiguity.
- d. New Section 4.4 ("Independent Validation"): To address the circularity concern, we explicitly separated the dataset:
 - Development data: Three precipitation events in 2022 (May 1, May 11, June 11) used to construct the retrieval relationships.
 - Independent validation data: A separate precipitation event on June 14, 2025, which was not included in the relationship development.
 - The independent validation results (Dm-validation and Nw-validation) demonstrate comparable accuracy to the training-period evaluation (Table 7), supporting generalizability to unseen data.
- e. Section 6 (Conclusions): We explicitly state that the practical applicability of the Ka-W constraint for operational three-frequency radar systems requires further validation with real W-band observations.

We agree that the independent-event validation does not completely eliminate

the conceptual dependence introduced by the simulated W-band constraint, because the DFR(Ka,W) relationship is still derived from DSD-based scattering simulations. Therefore, the present study should be regarded as a proof-of-concept investigation demonstrating the potential value of an additional high-frequency constraint, rather than a fully independent validation of an operational triple-frequency retrieval framework.

6) *Although a retrieval flow chart is provided, the manuscript does not include a clear narrative description of the algorithm. As a result, the operational retrieval procedure remains difficult to follow. The authors should walk the reader through the method step by step, identifying the required inputs, which quantities are directly observed, which are simulated or fitted, and the order in which Dm, LWC, and Nw are retrieved.*

The manuscript should also clarify the mathematical form of the retrieval. Is the method simply direct evaluation of fitted relationships, lookup-table interpolation, numerical inversion, optimization, or another approach? This clarification is important for reproducibility and for understanding how the algorithm would be applied to independent radar observations.

Response: We have substantially revised Section 2.2 ("Retrieval Procedure") to provide a comprehensive, step-by-step algorithmic description. The retrieval now consists of eight clearly numbered steps:

Step 1: 2DVD observations → ground-truth DSDs, Dm, LWC, Nw.

Step 2: T-matrix scattering simulations → theoretical Z_X , Z_{Ka} , Z_W , A_X , A_{Ka} , $k(X, Ka)$.

Step 3: Fitting of retrieval relationships (DFR-Dm, DFR-Z, $k(X, Ka)$ -A, LWC- Z_{Ka}).

Step 4: Attenuation correction of observed reflectivities using differential specific attenuation, constrained by theoretical DFR(X, Ka)- Z_X .

Step 5: Calculation of DFR*(X, Ka) and DFR*(Ka, W) from corrected reflectivities.

Step 6: Dm retrieval using combined DFR*(X, Ka) and DFR*(Ka, W).

Step 7: LWC retrieval from Z_{Ka-cor} , then Nw computation from Dm and LWC.

Step 8: DSD reconstruction via normalized gamma function ($\mu = 3$).

We also clarify that the method uses direct evaluation of fitted polynomial relationships (Eq. 12, 17) rather than numerical inversion or optimization, which improves reproducibility and operational clarity.

7) *The comparison between radar retrievals at approximately 120 m and surface 2DVD measurements requires more discussion. The authors should address the representativeness error caused by fall time, wind drift, vertical air motion, radar sampling volume, and the much smaller sampling area of the disdrometer. These effects may contribute to differences between retrieved and observed DSD parameters and should be considered in the validation.*

Response: We have added a new subsection, Section 5.3 ("Representativeness of 2DVD Validation"), that comprehensively discusses these issues. We explicitly address:

- a. Wind drift: Horizontal displacement between the radar sampling volume and the disdrometer location during raindrop fall, particularly under strong wind conditions.
- b. Temporal lag: The time required for raindrops to fall from the radar sampling height (120 m) to the ground, which may result in sampling different parcels in rapidly evolving precipitation systems.
- c. Sampling volume mismatch: The fundamental difference in sampling volumes (radar: ~30 m resolution; 2DVD: ~10 cm), which can lead to statistical representativeness errors in inhomogeneous precipitation fields.

We acknowledge that these factors limit the absolute accuracy of quantitative comparisons and clarify that the 2DVD measured DSDs are used to ensure that the radar retrieved results are reasonable and relatively accurate, rather than to claim perfect agreement.

8) A large fraction of the dataset falls in the $DFR(X, K_a) \leq 0$ ambiguous regime. This may be physically reasonable for light rain or small-drop populations, but the authors should show the distribution of DFR, D_m , rain rate, and reflectivity and explain why this regime dominates the dataset. They should also demonstrate that the large negative-DFR fraction is not caused by relative calibration bias, attenuation-correction error, beam mismatch, or limited sampling of precipitation regimes.

Response: We have analyzed the DFR distribution and its physical basis in Sections 3.5 and 4.1, with supporting evidence from Figure 4 and Table 6:

Physical explanation: The high proportion (64.62%) of $DFR(X, K_a) \leq 0$ is physically attributable to the predominance of small-to-moderate raindrops in the subtropical precipitation regime of Yangjiang, Guangdong. As shown in Figure 4 and Table 6, the dataset includes extensive stratiform and light-rain periods (e.g., May 1, 2022; the weakening stratiform phase of June 11, 2022), characterized by narrow DSDs with small mass-weighted mean diameters.

Exclusion of instrumental artifacts: Since the DFR values used to define the ambiguous zone are theoretically derived from T-matrix scattering simulations based on measured DSDs, negative DFR values cannot be caused by relative calibration bias, attenuation correction errors, beam mismatch, or limited sampling of precipitation types. The ambiguity is an intrinsic physical property of the $DFR(X, K_a)$ - D_m relationship for small drops.

Impact assessment: In Section 4.1 and Figure 14, we show that rain rates do not exceed 0.6 mm h^{-1} when $D_m < 0.5 \text{ mm}$, confirming that the residual ambiguity in the $DFR(K_a, W)$ regime corresponds to negligible precipitation intensities.

9) *The choice of fixed $\mu = 3$ needs stronger justification. The sensitivity test shows that μ affects both Dm and Nw retrievals. However, the results appear to show that $\mu = 0$ gives the smallest Dm retrieval error, while larger μ values improve Nw error. The authors choose $\mu = 3$, citing consistency with prior work. This may be reasonable, but the justification should be expanded.*

The authors should clarify the objective function used to select μ . Is the priority minimizing Dm error, Nw error, DSD shape error, or some combined metric? Since the proposed method is intended to retrieve DSDs, not only Dm, the authors should consider evaluating full DSD reconstruction error as a function of μ . The authors should also discuss whether μ is expected to vary by precipitation regime and whether fixing $\mu = 3$ limits performance in convective or light-rain cases.

Response: Thanks for your suggestion, which has made our work more rigorous. We have added an cost function. Although a full DSD reconstruction error metric would be desirable, the primary purpose of the retrieval framework is to estimate Dm and Nw, which are the governing parameters of the normalized gamma distribution. Therefore, Dm and Nw retrieval errors were selected as the optimization criteria in this study.

To determine the optimal μ that minimizes the retrieval errors of both Dm and Nw against the ground-truth values, the Euclidean distance from each candidate μ to ideal point —serving as the cost function:

$$J = \sqrt{[RMSE(Dm) - \min(RMSE_Dm)]^2 + [RMSE(Nw) - \min(RMSE_Nw)]^2}$$

Here, $\min(RMSE_Dm) = 0.0006$ and $\min(RMSE_Nw) = 0.0412$ (Table 5), defining the coordinates of the ideal point in the objective space. The Euclidean distance from each candidate μ is presented in Fig. 11. The results indicate that $\mu=3$ corresponds to the shortest distance (0.061), closely followed by $\mu=6$ (0.063), while $\mu=-3$ exhibits the largest deviation. Therefore, $\mu=3$ represents the optimal value that minimizes the retrieval errors of Dm and Nw, consistent with the value adopted by Meneghini et al. (2022).

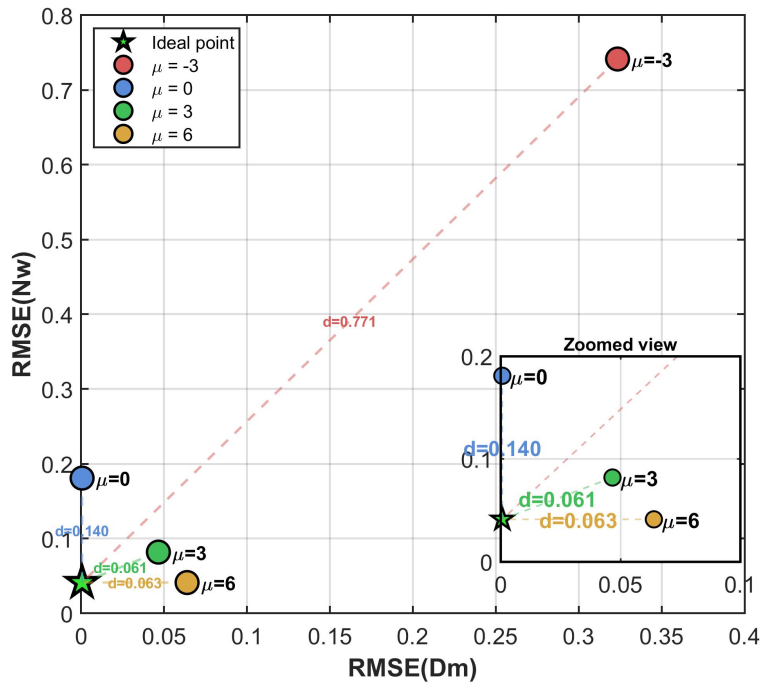


Figure 11. The Euclidean distance between each candidate μ and ideal point.

10) The manuscript introduces a raindrop axis-ratio relationship after discussing polarization-dependent scattering from oblate raindrops. However, the radar observations used here are vertically pointing, for which the authors state that polarization-dependent differences are effectively eliminated and $ZH \approx ZV$. The authors should clarify the role of the assumed axis-ratio model in the vertically pointing T-matrix simulations. If the axis ratio is used only to compute absolute backscatter cross sections and frequency-dependent DFR behavior under vertical incidence, this should be stated explicitly. The authors should also quantify the sensitivity of the simulated DFR–Dm relationships to the assumed axis-ratio relation, especially at Ka and W band where non-Rayleigh scattering effects may be important.

Response: The reviewer is correct. The axis-ratio model is still required in the T-matrix calculations because the scattering and extinction properties depend on particle shape. However, under vertically pointing observations, polarization-dependent differences are negligible and do not directly affect the retrieval framework.

11) The manuscript should discuss the practical difficulty of using W-band radar measurements in rain. While the simulated Ka/W-band DFR relationship may help reduce the DFR(X,Ka)-Dm ambiguity, real W-band observations can

be strongly attenuated by rain, cloud liquid water, atmospheric absorption, and wet radome effects. In moderate or heavy precipitation, the W-band signal may become too weak or unreliable to provide a useful constraint.

This point is especially important because the manuscript claims that the proposed method is useful for weak-echo situations. Weak echoes are also where W-band detectability, calibration uncertainty, and signal-to-noise limitations may become especially important, while moderate to heavy rain introduces severe attenuation concerns. Since the manuscript uses simulated W-band reflectivity rather than actual W-band radar observations, the authors should clarify whether the method is intended for real triple-frequency radar measurements or only as a simulation-based constraint. If it is intended for real W-band use, they should show under what rain-rate, range, reflectivity, and attenuation conditions W-band observations would remain useful.

Response: We have significantly expanded this discussion in the new Section 5.1 ("Practical Applicability of Ka-W Constraints"). We explicitly state:

"It is important to explicitly acknowledge that this study does not include W-band radar observations. The W-band reflectivities used herein were obtained entirely through T-matrix scattering simulations based on measured DSDs. Therefore, the DFR(Ka, W) constraint should currently be regarded as a theoretical assessment of its potential to mitigate DFR-Dm ambiguity, rather than a fully operational retrieval constraint that has been validated with real three-frequency radar data. The practical implementation of this approach would require a three-frequency radar system with X/Ka/W bands, and the actual performance may be affected by W-band radar calibration accuracy, atmospheric attenuation at W-band, and signal-to-noise ratio limitations. This study provides a proof-of-concept demonstration that motivates the development of future three-frequency radar systems for operational DSD retrieval."

This statement makes clear the current theoretical nature of the constraint and the operational requirements for future implementation.

12) Figure captions should be much more descriptive and should pay careful attention to which values are simulated vs. measured.

Response: We have revised all figure captions to explicitly distinguish between simulated and measured quantities. For example:

Figure 1: Blue arrows indicate scattering simulation workflow; green arrows represent retrieval workflow for real radar observation data.

Figure 6: Scatterplots of simulated X-band reflectivity and DFR(X, Ka) from T-matrix calculations.

Figure 8: Comparison between observed/corrected radar reflectivities and reflectivity derived from measured DSDs.

Figure 11: Scattering simulation scatterplots showing theoretical DFR-Dm relationships.

Figure 15. Scatterplots of the Ka-band reflectivity (Z_{Ka}) and liquid water content (LWC)

through scattering simulation.

13) *The discussion and conclusion are too limited given the scope of the claims. The study is based on only three development cases and one validation case from a single observation site. This is a small and geographically limited dataset, and the manuscript does not adequately discuss whether the retrieval relationships would generalize to other locations, precipitation regimes, seasons, climates, or radar systems.*

This limitation is especially important because the proposed method relies heavily on curve fitting and regression relationships derived from the available DSD dataset, including the DFR-Dm relationships, the simulated Ka/W constraint, and the ZKa-LWC relationship. With such a small and localized dataset, these fitted relationships may reflect site-specific DSD characteristics rather than a generally applicable retrieval method.

The authors should add a more complete limitations section. In particular, they should discuss the limited sample size, the use of one site, the dependence on local DSD characteristics, the lack of independent W-band observations, and the uncertainty of applying near-surface DSD-derived relationships to elevated radar volumes. The future-work discussion should also be expanded. The introduction states that limitations and future research directions will be discussed, but the conclusion only briefly mentions height-dependent DSD models and does not provide a broader path for validating or extending the method.

Response: We fully acknowledge that the original discussion of limitations was insufficient. We have completely restructured the discussion and conclusion to address these concerns:

- a. New Section 5.4 ("Generalization and Sample Limitations"): We explicitly state:
 - The framework was developed using data from a single observational site (Yangjiang, Guangdong) and a limited number of precipitation events (four cases total: three for development, one for independent validation).
 - The fitted relationships (DFR-Dm, DFR(Ka,W)-Dm, LWC-Z~Ka~) may reflect site-specific DSD characteristics (e.g., maritime-influenced warm-rain processes in South China) rather than universally applicable physical laws.
 - The generalization capability to other climatic regions, seasons, and precipitation regimes remains to be demonstrated.
- b. Vertical representativeness (Section 5.3): We acknowledge that applying near-surface 2DVD-derived relationships to elevated radar volumes assumes vertical homogeneity that may not hold in practice.
- c. Expanded Conclusions (Section 6): We have significantly expanded the future-work roadmap to include:
 - Real W-band validation with actual three-frequency radar observations.
 - Leave-one-event-out cross-validation with a larger sample size.

- Multi-site and multi-season validation across diverse geographical locations.
- Height-dependent DSD models for upper-level retrievals.
- Polarimetric constraints using the full polarimetric capability of the radar system.
- Operational uncertainty quantification including calibration error propagation and attenuation correction sensitivity.

Response to Chen Yichen

This manuscript addresses a key challenge in dual-frequency radar retrievals of raindrop size distributions (DSD) — the dual-solution ambiguity in the DFR-Dm relationship. The authors propose a novel solution by incorporating Ka/W-band DFR as an additional constraint, combined with T-matrix scattering simulations. The research is well-structured, the experimental design is sound, and the validation using ground-based disdrometer data is rigorous. The proposed method demonstrates clear improvements, particularly for weak echo regions. Overall, The manuscript is well suited for Atmospheric Measurement Techniques and will be valuable to the community. Accordingly, I recommend this manuscript for publication; however, you may consider the minor comments as follows:

- 1. The manuscript's central assumption is the availability of W-band (93.75 GHz) data. However, W-band radar suffers from extreme attenuation in rainfall, potentially losing signal within hundreds of meters during moderate to heavy rain. It appears from Section 3.6 that Z_W was simulated using T-matrix calculations based on measured DSDs, rather than being directly observed. If true, this manuscript presents a proof-of-concept simulation study rather than a retrieval algorithm ready for operational radar data. The authors must clarify this point explicitly in the abstract, introduction, and conclusions. A more accurate framing would be "simulation-based retrieval framework" rather than simply "retrieval method."*

Response: We sincerely thank the reviewer for this critical and constructive comment. We fully agree with the reviewer's assessment that the W-band reflectivities used in this study were obtained through T-matrix scattering simulations based on measured DSDs, rather than from actual W-band radar observations. Consequently, this study is indeed a proof-of-concept investigation rather than an operational retrieval algorithm ready for direct application to real three-frequency radar data.

To address this concern, we have made comprehensive revisions throughout the revised manuscript to explicitly clarify the proof-of-concept nature of this work and the simulation-based origin of the W-band data. The specific modifications are as follows:

The original title was "An Improved Method for Raindrop Size Distribution Retrieval Using Combined Dual-frequency Radar DFR." In the revised manuscript, the title has been changed to "The Proof-of-Concept for Raindrop Size Distribution Retrieval Using Combined Dual-frequency Radar DFR." The explicit inclusion of "Proof-of-Concept" accurately reflects the nature of this study from the outset.

In the revised abstract, we have explicitly stated that "W-band reflectivities obtained through scattering simulations based on measured raindrop size distributions, combined with the DFR(Ka, W) constraint, effectively mitigate the DFR(X, Ka)-Dm dual-solution ambiguity." Furthermore, the abstract conclusion now clearly states: "This proof-of-concept study provides a reference framework for future three-frequency radar DSD retrieval." These revisions ensure that readers immediately understand the simulation-based and conceptual nature of the work.

A new subsection has been added to the Discussion and Limitations section to explicitly acknowledge this limitation. The text states: "It is important to explicitly acknowledge that this study does not include W-band radar observations. The W-band reflectivities used herein were obtained entirely through T-matrix scattering simulations based on measured DSDs. Therefore, the DFR(Ka, W) constraint should currently be regarded as a theoretical assessment of its potential to mitigate DFR-Dm ambiguity, rather than a fully operational retrieval constraint that has been validated with real three-frequency radar data. The practical implementation of this approach would require a three-frequency radar system with X/Ka/W bands, and the actual performance may be affected by W-band radar calibration accuracy, atmospheric attenuation at W-band, and signal-to-noise ratio limitations. This study provides a proof-of-concept demonstration that motivates the development of future three-frequency radar systems for operational DSD retrieval."

The conclusions have been thoroughly revised to emphasize the proof-of-concept nature. The opening now states: "This study presents a proof-of-concept DSD retrieval framework that explores the potential of combining X/Ka-band radar observations with simulated DFR(Ka, W) constraints to mitigate the DFR(X, Ka)-Dm ambiguity problem." A new paragraph explicitly emphasizes: "It is important to emphasize the W-band reflectivities were obtained through scattering simulations rather than actual radar observations, and the evaluation is based on a limited sample from a single observational site. The practical applicability of the Ka-W constraint for operational three-frequency radar systems requires further validation with real W-band observations and more extensive datasets covering diverse precipitation regimes."

In the retrieval framework description, we have added explicit clarification that "Blue boxes denote the results from scattering simulations, and green boxes represent those derived from radar observation data." This visual distinction in the flowchart (Figure 1) helps readers immediately identify which components are simulation-based and which are observation-based.

2. *When $DFR(X,Ka) \leq 0$, the algorithm switches to $DFR(Ka,W)$. However, Table 6 shows that 12.5% of data points still fall within the ambiguous zone of $DFR(Ka,W)$ (i.e., $DFR(Ka,W) \leq 0$). The authors do not specify how these remaining ambiguous cases are handled. Are they discarded? Assigned a default Dm value? Or constrained by another relationship*

(e.g., $Z_{\text{Ka-LWC}}$)? The authors should provide a complete decision tree or flow chart (as an enhancement to Figure 1) that explicitly states the retrieval strategy for all possible combinations of $DFR(X, \text{Ka})$ and $DFR(\text{Ka}, W)$. Additionally, the retrieval performance for these "doubly ambiguous" cases should be highlighted in Figure 17.

Response: We sincerely thank the reviewer for raising this important point regarding the handling of the remaining 12.5% of data points that fall within the ambiguous zone of $DFR(\text{Ka}, W)$.

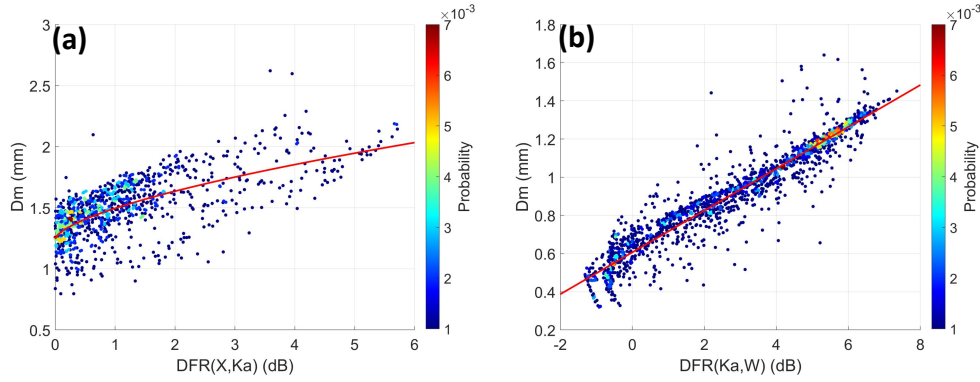


Figure 13. DFR-Dm fitting curve [a: $DFR(X, \text{Ka}) > 0$, b: $DFR(\text{Ka}, W)$ when $DFR(X, \text{Ka}) \leq 0$]

In the revised manuscript (Section 4.1), we present the combined retrieval formula (Eq. 19) as follows:

$$D_m = \begin{cases} 0.251DFR(X, \text{Ka})^{0.6391} + 1.2445 & DFR(X, \text{Ka}) > 0 \\ 0.11DFR(\text{Ka}, W) + 0.6064 & DFR(X, \text{Ka}) \leq 0 \end{cases}$$

This formulation implicitly applies the $DFR(\text{Ka}, W)$ - D_m relationship whenever $DFR(X, \text{Ka}) \leq 0$, regardless of whether $DFR(\text{Ka}, W)$ is positive or negative. We note that Section 4.1 of the revised manuscript includes the following statement: **"Specifically, the $DFR(\text{Ka}, W)$ - D_m accurately retrieves larger D_m values when $DFR(\text{Ka}, W)$ is less than 0, with retrieval errors remaining only for raindrops smaller than 0.5 mm (Fig. 13b). Furthermore, as indicated in Fig. 14, rain rates do not exceed 0.6 mm h^{-1} when D_m is below 0.5 mm. Therefore, the retrieval error associated with $DFR(\text{Ka}, W)$ in this range is almost negligible."**

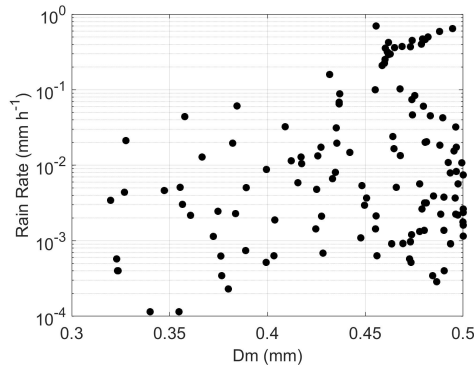


Figure 14. Scatterplots of rain rate versus D_m when D_m is below 0.5 mm based on measured DSDs

3. *In Section 3.1 (Instruments), the authors provide detailed specifications for the X/Ka-band radar and the 2DVD disdrometer. However, there is no mention of any W-band radar or the source of W-band reflectivity data. This is a critical omission for understanding the experimental setup. Please add a clear statement at the end of Section 3.1 or at the beginning of Section 3.3: "W-band reflectivities used in this study are simulated using the T-matrix method based on measured DSDs."*

Response: We sincerely thank the reviewer for this important observation. Since Section 3.1 provides a detailed description of the X/Ka dual-frequency radar and the 2DVD disdrometer without introducing retrieval methods, we have not added a statement regarding the source of W-band reflectivity within this section. Nevertheless, we have emphasized this point in the abstract, conclusions and Section 5.1.

Abstract: "..., This study demonstrates that W-band reflectivities obtained through scattering simulations based on measured raindrop size distributions, combined with the $DFR(Ka, W)$ constraint, effectively mitigate the $DFR(X, Ka)$ - D_m dual-solution ambiguity. ...".

Conclusions: "..., It is important to emphasize the W-band reflectivities were obtained through scattering simulations rather than actual radar observations, ...".

5.1 Practical Applicability of Ka-W Constraints

It is important to explicitly acknowledge that this study does not include W-band radar observations. The W-band reflectivities used herein were obtained entirely through T-matrix scattering simulations based on measured DSDs. Therefore, the $DFR(Ka, W)$ constraint should currently be regarded as a theoretical assessment of its potential to mitigate DFR - D_m ambiguity, rather than a fully operational retrieval constraint that has been validated with real three-frequency radar data. The practical implementation of this approach would require a three-frequency radar system with

X/Ka/W bands, and the actual performance may be affected by W-band radar calibration accuracy, atmospheric attenuation at W-band, and signal-to-noise ratio limitations. This study provides a proof-of-concept demonstration that motivates the development of future three-frequency radar systems for operational DSD retrieval.

4. Figure 6 shows dense scatterplots for Z_X -DFR(X,Ka) and Z_{Ka} -DFR(Ka,W). The 6th-order polynomial fitting curves are barely visible despite being provided in equation form. Please increase the line width or color contrast of the fitting curves. Also, add a note in the caption stating "The black curve represents the 6th-order polynomial fit."

Response: Thank you for your suggestion. Corresponding revisions have been made.

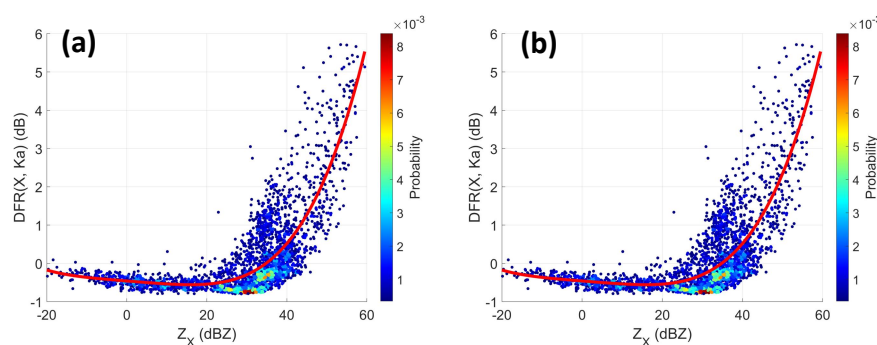


Figure 6. Scatterplots of the X-band reflectivity (Z_X) and X/Ka dual-frequency ratio [DFR(X, Ka)]; the Ka-band reflectivity (Z_{Ka}), and Ka/W dual-frequency ratio [DFR(Ka, W)]. The red curve represents the polynomial fit.

5. Several sentences could be clarified for better readability. For example, on Page 12, the sentence "...the corrected reflectivities were validated against those derived from ground-based raindrop size distributions measurements" — the word "those" is ambiguous. It should read "...validated against the theoretical reflectivity values derived from ground-based DSD measurements." Similarly, on Page 17, "This characteristic underpins the interpretation..." would be more accurate as "This characteristic guides the choice of retrieval algorithms..."

Response: Thanks for your valuable comment. Corresponding revisions have been made.

"The corrected reflectivities were validated against the theoretical reflectivity values derived from ground-based raindrop size distributions measurements."

In response to comments from other reviewers, the section concerning axis ratio has been removed, including the sentence "This characteristic underpins the interpretation...".

6. *In Table 1, the unit for Antenna Gain is listed as "dB". While this is common, the formal unit should be "dBi" (decibels relative to isotropic). Please consider this correction.*

Response: Thank you for the correction, it has been revised in the revised manuscript.

Response to Yu Tiantian

This manuscript proposes an improved method for raindrop size distribution retrieval by combining dual-frequency radar DFR information, with the aim of reducing the well-known dual-solution ambiguity in the DFR– D_m relationship. The topic is relevant to radar meteorology and precipitation microphysics, and the manuscript addresses an important limitation of conventional DFR-based DSD retrieval methods, especially under weak-echo and light-precipitation conditions.

The proposed approach appears promising, and the reported improvement in the proportion of uniquely retrievable D_m values is encouraging. However, several key methodological issues need to be clarified before the manuscript can be considered for publication. In particular, the source and practical role of the W-band reflectivity require much clearer explanation, and the novelty of the proposed method should be more explicitly distinguished from previous studies.

1. The source and practical role of the W-band reflectivity must be clarified

The most important issue in the manuscript concerns the use of W-band reflectivity and $DFR(Ka, W)$. The abstract states that W-band reflectivity is synthetically incorporated through scattering simulations based on measured DSDs. However, the instrument section only describes an X/Ka dual-frequency vertically pointing radar and a 2DVD disdrometer. No actual W-band radar measurement appears to be available.

Response : We sincerely thank the reviewer for raising this critical point regarding the source and practical role of W-band reflectivity. In the revised manuscript, we have made substantial modifications to explicitly address this concern, as detailed below:

(1) Title changed to reflect the proof-of-concept nature

The title has been revised to "The Proof-of-Concept for Raindrop Size Distribution Retrieval Using Combined Dual-frequency Radar DFR" (Revised manuscript, page 1). This explicitly signals to readers that the study explores a theoretical framework rather than an operational retrieval using actual three-frequency radar measurements.

(2) Abstract clarified the source of W-band reflectivity

The abstract now explicitly states: "This study demonstrates that W-band reflectivities obtained through scattering simulations based on measured raindrop size distributions, combined with the $DFR(Ka, W)$ constraint,

effectively mitigate the DFR(X, Ka)-Dm dual-solution ambiguity" (Revised manuscript, page 1). This directly clarifies that W-band reflectivities are synthetically derived, not observed.

(3) Retrieval framework diagram explicitly distinguishes simulation from observation

In Section 2.1 (Retrieval Framework), the caption of Figure 1 has been revised to explicitly indicate: "Blue arrows indicate the scattering simulation workflow, while green arrows represent the retrieval workflow for real radar observation data". This visual distinction ensures readers can immediately identify which components (including W-band reflectivity and DFR(Ka, W)) originate from T-matrix scattering simulations versus actual X/Ka-band radar observations.

(4) New dedicated discussion section (Section 5.1) addressing practical applicability

We have added a new subsection, "5.1 Practical Applicability of Ka-W Constraints", specifically to address this reviewer concern. This section explicitly states: "It is important to explicitly acknowledge that this study does not include W-band radar observations. The W-band reflectivities used herein were obtained entirely through T-matrix scattering simulations based on measured DSDs. Therefore, the DFR(Ka, W) constraint should currently be regarded as a theoretical assessment of its potential to mitigate DFR-Dm ambiguity, rather than a fully operational retrieval constraint that has been validated with real three-frequency radar data."

This section further discusses that practical implementation would require an actual X/Ka/W three-frequency radar system, and that real-world performance would be affected by W-band calibration accuracy, atmospheric attenuation, and signal-to-noise ratio limitations.

(5) Conclusions reinforced with explicit limitations

In Section 6 (Conclusions), we have added: "It is important to emphasize the W-band reflectivities were obtained through scattering simulations rather than actual radar observations, and the evaluation is based on a limited sample from a single observational site. The practical applicability of the Ka-W constraint for operational three-frequency radar systems requires further validation with real W-band observations and more extensive datasets covering diverse precipitation regimes".

These comprehensive revisions ensure that the manuscript now clearly distinguishes between the simulated W-band reflectivity (used as a theoretical constraint to resolve DFR ambiguity) and the actual X/Ka-band radar

observations, leaving no ambiguity about the data sources or the current limitations regarding operational application.

2. The novelty of the proposed method should be more clearly distinguished from previous studies

The manuscript reviews several previous approaches for addressing the DFR–Dm ambiguity, including generalized DFR methods and differential dual-frequency radar equation methods. However, the distinction between the present method and previous work is not yet sufficiently clear.

Response : Thanks for your valuable suggestion. In the revised manuscript, we have strengthened the distinction from previous studies through the following modifications:

(1) Clarified the fundamental limitations of existing methods in the Introduction

The revised Introduction (Section 1) retains the review of previous approaches but more clearly positions them as the motivation for this study:

Liao and Meneghini (2019) proposed DFR* to avoid ambiguity by redefining the dual-frequency ratio (introducing a tunable coefficient γ), but this sacrifices the key advantage that DFR depends solely on Dm and introduces an undesirable dependence on Nw.

Iguchi and Meneghini (2022) used a differential form of the dual-frequency radar equation, which primarily mitigates the dual-solution under strong attenuation (intense precipitation) conditions, leaving weak precipitation unresolved.

GPM operational algorithm simply circumvents the problem by using the R–Dm relationship instead of directly addressing the DFR–Dm ambiguity.

By clearly stating these limitations, the revised text establishes that existing methods either alter the physical definition of DFR or are limited to specific precipitation regimes, thereby highlighting the necessity of a new approach.

(2) Explicitly articulated the core innovation mechanism in Section 2.1

The newly revised Section 2.1 (Retrieval Framework) explicitly states the physical principle distinguishing our method:

"The core concept of this method is rooted in the physical principle that the DFR at distinct frequency pairs, specifically X/Ka and Ka/W bands, exhibits distinct responses to variations in Dm."

"To resolve this ambiguity, $\text{DFR}(K_a, W)$ is employed, which operates at higher frequencies and exhibits a significantly narrower ambiguity zone than $\text{DFR}(X, K_a)\text{-Dm}$."

This clearly differentiates our strategy from DFR: we do not modify the definition of $\text{DFR}(X, K_a)$; instead, we preserve its physical meaning and resolve its ambiguity by introducing an additional, higher-frequency constraint. Unlike the differential equation approach, our method leverages the scattering regime differences between frequency pairs to constrain Dm , particularly effective for weak echoes where $\text{DFR}(X, K_a) \leq 0$.

(3) Added the step-by-step retrieval procedure in Section 2.2 to demonstrate the unique architecture

The original manuscript only provided a brief overall strategy. The revised manuscript adds Section 2.2 (Retrieval Procedure) with an 8-step detailed workflow. This explicitly demonstrates the novel "dual-frequency observation + simulated third-frequency constraint" architecture:

Step 2: T-matrix simulations generate W-band reflectivity from measured DSDs.

Step 6: $\text{DFR}(K_a, W)$ is specifically used to resolve ambiguity when $\text{DFR}(X, K_a) \leq 0$.

This procedural detail makes it unambiguous that our method is not a simple extension of conventional dual-frequency techniques but a new framework that synergizes observed X/K_a data with theoretically simulated W-band constraints.

(4) Added Section 5.1 to explicitly define the proof-of-concept nature and theoretical innovation

The new Section 5.1 (Practical Applicability of Ka-W Constraints) states:

"The $\text{DFR}(K_a, W)$ constraint should currently be regarded as a theoretical assessment of its potential to mitigate $\text{DFR}\text{-Dm}$ ambiguity, rather than a fully operational retrieval constraint that has been validated with real three-frequency radar data."

(5) Strengthened the abstract and conclusions to highlight the unique contribution

The revised Abstract now leads with:

"W-band reflectivities obtained through scattering simulations... combined with the DFR(Ka, W) constraint, effectively mitigate the DFR(X, Ka)-Dm dual-solution ambiguity."

The Conclusions (Section 6) reiterate that this proof-of-concept framework "explores the potential of combining X/Ka-band radar observations with simulated DFR(Ka, W) constraints"—ensuring readers understand that the novelty lies in the simulation-based multi-frequency constraint strategy rather than in conventional dual-frequency retrieval.

3. The English language and grammar require careful revision

The manuscript contains a number of grammatical and stylistic issues that should be corrected before publication. For example, the section title "Compression the Ambiguous zone of DFR-Dm" is grammatically incorrect and could be revised to:

"Reduction of the Ambiguous Zone in the DFR–Dm Relationship."

Similarly, the conclusion begins with "A improved raindrop size distribution retrieval method," which should be corrected to:

"An improved raindrop size distribution retrieval method."

Response : We sincerely thank the reviewer for the careful reading and constructive suggestions regarding the English language and grammar. We have thoroughly revised the manuscript accordingly.

(1) Section title corrected. Following the reviewer's exact suggestion, the section title previously written as "Compression the Ambiguous zone of DFR-Dm" (Original manuscript, Section 3.6) has been revised to "Reduction of the Ambiguous Zone in the DFR–Dm Relationship" (Revised manuscript, Section 3.5). This revision corrects the grammatical error and uses the proper preposition and capitalization.

(2) Conclusion opening corrected. The sentence "A improved raindrop size distribution retrieval method was developed..." (Original manuscript, Section 5) contained an article error. In the revised manuscript, the Conclusions section (Section 6) has been rewritten to begin with: "This study presents a proof-of-concept DSD retrieval framework...", thereby eliminating the grammatical error entirely while also aligning the tone with the study's scope.

Beyond these specific corrections, the entire manuscript has been carefully proofread and polished to improve English grammar, style, and readability.

Other minor grammatical and stylistic issues throughout the text have been identified and corrected to meet the publication standards.

4. The figures should be improved for clarity and interpretation

Several figures are central to the manuscript but could be made clearer.

For example, Fig. 1 should explicitly distinguish between observed variables, simulated variables, intermediate retrieval products, and final outputs. This is especially important because the role of the W-band reflectivity is currently ambiguous.

Fig. 11 is one of the most important figures in the manuscript because it demonstrates the reduction of the ambiguous DFR–Dm region. The ambiguous zones, thresholds, and physical interpretation should be labeled more clearly.

Fig. 17 contains many panels and is somewhat difficult to read. The authors may consider separating the Dm and Nw results into different figures or improving the layout.

For Figs. 18 and 19, the number of samples in each reflectivity interval should be reported, either in the figure panels or in the caption.

Response : We sincerely thank the reviewer for the detailed and constructive suggestions on improving figure clarity. We have carefully revised the figures, captions, and associated text to address each concern.

Regarding Figure 1 (Retrieval Framework):

In the revised manuscript (Section 2.1), we have significantly improved the retrieval framework schematic to explicitly distinguish between different variable types. The figure caption now clearly states: "Blue arrows indicate the scattering simulation workflow, while green arrows represent the retrieval workflow for real radar observation data" (Revised manuscript, Figure 1). Additionally, the text explicitly explains: "Blue boxes denote the results from scattering simulations, and green boxes represent those derived from radar observation data." This color-coded and arrow-based distinction now explicitly separates: (1) observed variables (X/Ka-band radar reflectivities, 2DVD-measured DSDs); (2) simulated variables (W-band reflectivity, theoretical DFR–Dm relationships, attenuation coefficients); (3) intermediate retrieval products (attenuation-corrected reflectivities, DFR values); and (4) final outputs (Dm, Nw, LWC, retrieved DSD). This directly resolves the previous ambiguity regarding the source and role of W-band reflectivity.

Regarding Figure 12 (previously Figure 11, Reduction of the Ambiguous Zone):

Following the reorganization of the manuscript (with the addition of the μ cost-function analysis as Figure 11), this figure is now numbered as Figure 12. The caption has been enhanced to clearly label the ambiguous zones: "The black dashed area indicates the ambiguous zone when $DFR \leq 0$ " (Revised manuscript, Section 3.5). In panel (a), blue dots represent $DFR(X, K_a)$ and red dots represent $DFR(K_a, W)$, providing immediate visual distinction between the two frequency pairs. In the main text, we have added explicit quantitative thresholds and physical interpretation: "the ambiguous zone for $DFR(K_a, W)$ is confined to a Z_{K_a} range of approximately -20 to 20 dBZ, corresponding to D_m values between 0.3 and 0.8 mm... In comparison, the ambiguous zone for $DFR(X, K_a)$ covers a much wider Z_{K_a} range of about -20 to 45 dBZ, corresponding to D_m values from 0.3 to 1.6 mm" (Revised manuscript, Section 3.5). These quantitative boundaries and physical explanations are now explicitly stated to guide the reader's interpretation of the figure.

Regarding Figure 18 (previously Figure 17, Retrieval Comparison):

We acknowledge the reviewer's concern that the 8-panel layout is dense. In the revised manuscript, we have reorganized the figure with a clearer caption structure: "Retrieved mass-weighted mean diameter (D_m) and normalized intercept parameter [$\log_{10}(N_w)$]. (a, e: the proposed method; b, f: the first conventional method; c, g: the second conventional method; d, h: the retrieved D_m and N_w of June 14, 2025 by the proposed method)" (Revised manuscript, Figure 18). The panels are arranged in a logical 2×4 layout with D_m results in the top row (a–d) and N_w results in the bottom row (e–h), facilitating systematic comparison. If the editor and reviewer prefer, we are prepared to separate this into two distinct figures for final publication: one dedicated to D_m retrieval comparison (panels a–d) and one to N_w retrieval comparison (panels e–h).

Regarding Figures 19 and 20 (previously Figures 18 and 19, DSD and Error Statistics):

We agree that reporting sample sizes is essential for statistical transparency and reproducibility. We have added the number of samples for each reflectivity interval in the revised figure captions.

Figure 19. The mean raindrop size distributions retrieved by conventional methods (DSD-est1, DSD-est2) and the proposed method (DSD-est) against the true values (DSD). a: $Z < 20$ dBZ, 655 samples; b: $20 \text{ dBZ} \leq Z < 30 \text{ dBZ}$, 466 samples; c: $30 \text{ dBZ} \leq Z < 40 \text{ dBZ}$, 1281 samples; d: $40 \text{ dBZ} \leq Z < 50 \text{ dBZ}$, 394 samples.

5. Terminology and notation should be standardized

The manuscript should use consistent terminology and notation throughout. For example: D_m , N_w , and LWC should follow a uniform notation style.

Clear and consistent notation will make the retrieval procedure easier to follow.

Response : Thanks for your important suggestion. In the revised manuscript, we have systematically unified all symbols and terminology throughout the text, equations, and figures.

Response to Luo Li

This manuscript addresses the DFR-Dm dual-solution ambiguity in dual-frequency radar DSD retrievals by incorporating Ka/W-band DFR as an additional constraint. The method is well-designed and shows clear improvements, particularly for weak echoes. The following three key issues should be addressed prior to publication.

1. Clarify that W-band data are simulated, not observed

The manuscript's central innovation relies on Ka/W-band DFR, but it appears from Section 3.6 that W-band reflectivities are simulated using T-matrix calculations based on measured DSDs, rather than directly observed. Given that W-band radar suffers from extreme attenuation in rainfall, this distinction is critical. The authors must explicitly state in the abstract, introduction, and Section 3.1 that this is a simulation-based retrieval framework, and add a clear statement: "W-band reflectivities used in this study are simulated using the T-matrix method based on measured DSDs."

Response: We sincerely thank the reviewer for this critical and constructive comment. We fully agree with the reviewer's assessment that the W-band reflectivities used in this study were obtained through T-matrix scattering simulations based on measured DSDs, rather than from actual W-band radar observations. Consequently, we have emphasized this point in the abstract, conclusions and Section 5.1.

Abstract: "..., This study demonstrates that W-band reflectivities obtained through scattering simulations based on measured raindrop size distributions, combined with the DFR(Ka, W) constraint, effectively mitigate the DFR(X, Ka)-Dm dual-solution ambiguity. ...".

Conclusions: "..., It is important to emphasize the W-band reflectivities were obtained through scattering simulations rather than actual radar observations,..."

5.1 Practical Applicability of Ka-W Constraints

It is important to explicitly acknowledge that this study does not include W-band radar observations. The W-band reflectivities used herein were obtained entirely through T-matrix scattering simulations based on measured DSDs. Therefore, the DFR(Ka, W) constraint should currently be regarded as a theoretical assessment of its potential to mitigate DFR-Dm ambiguity, rather than a fully operational retrieval constraint that has been validated with real three-frequency radar data. The practical implementation of this approach would require a three-frequency radar system with X/Ka/W bands, and the actual performance may be affected by W-band radar calibration accuracy, atmospheric attenuation at W-band, and signal-to-noise ratio

limitations. This study provides a proof-of-concept demonstration that motivates the development of future three-frequency radar systems for operational DSD retrieval.

2. Specify handling of remaining ambiguous cases ($DFR(Ka, W) \leq 0$)

Table 6 shows that 12.5% of data points still fall within the ambiguous zone of $DFR(Ka, W)$ (i.e., $DFR(Ka, W) \leq 0$). The authors do not specify how these "doubly ambiguous" cases are handled (discarded? default value? constrained by Z_{Ka} -LWC?). Please provide a complete decision tree or flow chart (as an enhancement to Figure 1) and evaluate the retrieval performance for these cases.

Response: We sincerely thank the reviewer for raising this important point regarding the handling of the remaining 12.5% of data points that fall within the ambiguous zone of $DFR(Ka, W)$.

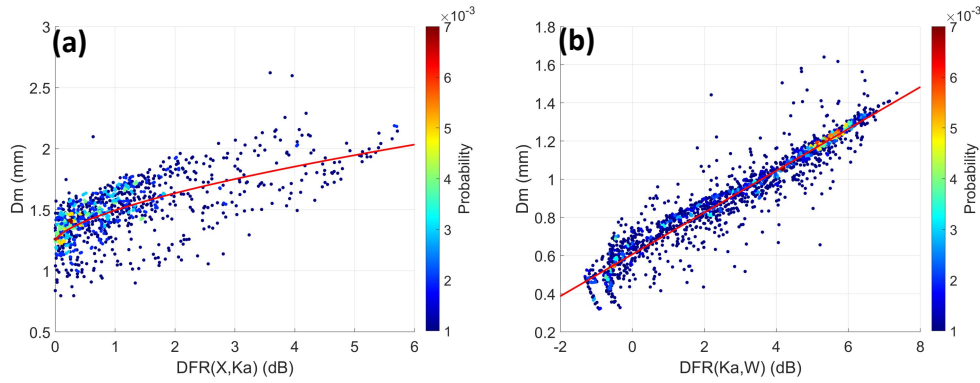


Figure 13. DFR-Dm fitting curve [a: $DFR(X, Ka) > 0$, b: $DFR(Ka, W)$ when $DFR(X, Ka) \leq 0$]

In the revised manuscript (Section 4.1), we present the combined retrieval formula (Eq. 19) as follows:

$$D_m = \begin{cases} 0.251DFR(X, Ka)^{0.6391} + 1.2445 & DFR(X, Ka) > 0 \\ 0.11DFR(Ka, W) + 0.6064 & DFR(X, Ka) \leq 0 \end{cases}$$

This formulation implicitly applies the $DFR(Ka, W)$ - D_m relationship whenever $DFR(X, Ka) \leq 0$, regardless of whether $DFR(Ka, W)$ is positive or negative. We note that Section 4.1 of the revised manuscript includes the following statement: **"Specifically, the $DFR(Ka, W)$ - D_m accurately retrieves larger D_m values when $DFR(Ka, W)$ is less than 0, with retrieval errors remaining only for raindrops smaller than 0.5 mm (Fig. 13b). Furthermore, as indicated in Fig. 14, rain rates do not exceed 0.6 mm h^{-1} when D_m is below 0.5 mm. Therefore, the retrieval error associated with $DFR(Ka, W)$ in this range is almost negligible."**

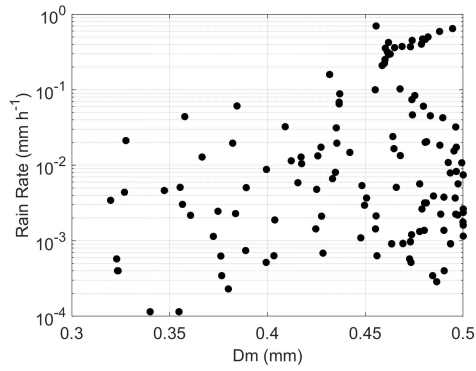


Figure 14. Scatterplots of rain rate versus D_m when D_m is below 0.5 mm based on measured DSDs

3. Quantitative assessment of the inversion errors caused by temperature variations

Provide quantitative error analysis (e.g., D_m bias) for 0°C, 10°C, and 30°C, not just state that errors are small.

Response: We have added error analysis at different temperature. Thanks for your suggestion.

“To evaluate the impact of temperature on raindrop size distribution retrieval, we take $DFR(Ka,W)$ as an example to simulate the retrieved relationship between $DFR(Ka,W)$ and D_m for raindrops at different temperatures (Fig. 2a). As shown in Fig. 2b, the RMSE of D_m retrieval is minimized at 20°C. Therefore, the $DFR-D_m$ retrieval model was established with 20°C adopted as the reference temperature.”

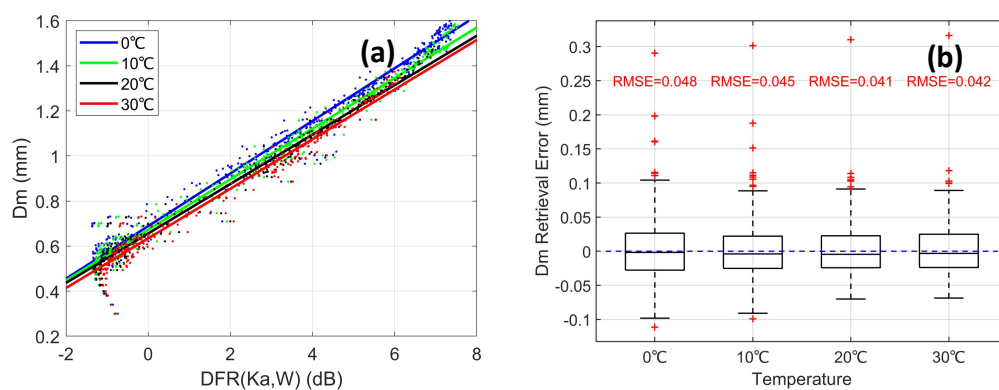


Figure 2. Fitting curves(a) and D_m retrieval error(b) of $DFR(Ka,W)-D_m$ at different temperatures (blue: 0°C; green: 10°C; black: 20°C; red: 30°C).

