Retrieving raindrop size distribution (DSD) using disdrometers has become a research hotspot in the field of weather radar in recent years, as DSD is crucial for understanding cloud and precipitation microphysical processes. This study combines disdrometer measurements with X-Ka band vertically pointing radar to perform DSD retrieval. To address the ambiguity problem in X-Ka band retrieval, the Ka-W band is further introduced as a joint constraint to reduce the ambiguous region between the dual-frequency ratio (DFR) and the mass-weighted diameter (Dm), thereby improving DSD retrieval accuracy. Multi-frequency radar retrieval is an important research direction, and the DFR-Dm ambiguity is a key issue in DSD retrieval. Although this study has practical significance, the methodology section suffers from unclear descriptions and logical issues related to circular reasoning.

Major issues:

1. *The radar used in this study is an X-Ka dual-frequency radar. The proposed method employs the Ka-W dual-band to constrain the retrieval process of the X-Ka dual-frequency radar in order to mitigate the ambiguity problem inherent in X-Ka dual-frequency observations. However, no measured data from W-band radar are provided in the paper. It remains unclear how this constraint can be implemented in practical applications.*

Response: We thank the reviewer for raising this critical point regarding the practical implementation of the W-band constraint.

Under the current X/Ka dual-frequency observational configuration, DFR(Ka,W) cannot be directly obtained. Therefore, the Ka-W constraint is not operationally applied in the retrieval itself, but is used here as a theoretical framework to evaluate the potential of a future three-frequency radar system for resolving the DFR-Dm ambiguity.

The W-band reflectivities employed in this study were obtained entirely through T-matrix scattering simulations based on measured raindrop size distributions (Section 5.1). Therefore, the DFR(Ka, W) constraint should currently be regarded as a theoretical assessment of its potential to mitigate the DFR(X, Ka)-Dm ambiguity, rather than a fully operational retrieval constraint that has been validated with real three-frequency radar data.

In the proposed retrieval framework (Figure 1 and Section 2.1), the W-band constraint is introduced through the scattering simulation branch (blue boxes). The procedure is as follows:

1. Step 2 (Section 2.2): T-matrix scattering calculations are performed to simulate theoretical radar reflectivities at X-, Ka-, and W-band frequencies based on 2DVD-measured DSDs.

2. Step 3: The DFR(Ka, W)-Dm retrieval relationship is established via fitting using

these simulated reflectivities (Figure 13b).

3. Step 6: When actual X/Ka dual-frequency radar observations yield DFR(X, Ka) ≤ 0 (the ambiguous zone), the pre-established simulated DFR(Ka, W)-Dm relationship is invoked to resolve the ambiguity and select the correct Dm solution.

Under the current observational conditions where only X/Ka dual-frequency radar data are available, the W-band constraint cannot be directly measured and must rely on the theoretical relationships derived from disdrometer data. This is explicitly why we characterize this study as a proof-of-concept demonstration (Section 5.1 and Section 6).

For operational application, this approach would require a three-frequency radar system with X/Ka/W bands (Section 5.1). With such a system deployed, the W-band reflectivity would be directly measured, enabling real-time computation of DFR(Ka, W) to constrain the retrieval without dependence on scattering simulations. However, the actual performance may be affected by several practical factors:

- W-band radar calibration accuracy;

- Atmospheric attenuation at W-band (which is significantly stronger than at X- or Ka-band);

- Signal-to-noise ratio limitations in weak precipitation regions.

This study provides a proof-of-concept framework that motivates the development of future three-frequency radar systems. The simulated DFR(Ka, W) constraint demonstrates the theoretical potential to increase the proportion of data permitting a unique Dm solution from 35.38% to 87.5% (Section 3.5 and Table 6). The practical applicability of this Ka-W constraint for operational systems requires further validation with real W-band observations and more extensive datasets covering diverse precipitation regimes (Section 6). We will clarify this distinction more explicitly in the revised manuscript to ensure readers understand the current simulation-based nature of the W-band constraint and the requirements for its future operational implementation.

2. *This study uses DSD data observed by disdrometers during three rainfall events to fit the attenuation coefficients for the X-band and Ka-band radars. These coefficients are then used to correct attenuation in the X-band and Ka-band radar observations from the same three rainfall events. Subsequently, the DFR-Dm relationship is fitted, and DSD retrieval is performed. This algorithm may suffer from logical issues of self-consistency bias.*

Response: We thank the reviewer for raising this critical concern regarding potential self-consistency bias in the validation procedure.

We agree that the use of the same DSD dataset for both relationship development and preliminary evaluation may lead to optimistic estimates of retrieval performance, regardless of whether the underlying relationships are physically based or empirically fitted. Therefore, an independent event in 2025 was introduced, which was not included in the relationship development. The independent validation results (Table 7) demonstrate comparable accuracy to the training-period evaluation, suggesting that the proposed relationships possess a degree of generalizability beyond the training data.

Despite the independent validation for the DSD retrieval, we recognize that the attenuation correction coefficients and the DFR-Dm fitting were both derived from the same 2022 events used for the initial evaluation. This represents a limitation of the current proof-of-concept study. As discussed in Section 5.4, future studies will employ event-independent validation using leave-one-event-out cross-validation with a larger and more diverse dataset to further quantify the robustness and generalization capability of the proposed method. We will also explicitly separate the datasets used for establishing attenuation relationships from those used for validating retrieval results to completely avoid any self-consistency issues.

Thank you for this valuable suggestion, which has prompted us to more carefully consider the validation strategy and its limitations in the revised manuscript.

Specific comments:

1. *The flowchart shown in Figure 1, along with its corresponding steps and the formulas contained therein, requires additional explanatory clarification.*

Response: We acknowledge that Figure 1 and its corresponding workflow require more detailed explanatory clarification to ensure readers can fully understand the retrieval framework. In the revised manuscript, we have enhanced the explanatory content as follows:

(1) A new Section 2.1 (Retrieval Framework) is included to systematically elaborate on the three core components of the overall retrieval framework: observational inputs, scattering simulations and multi-constraint retrieval. The corresponding retrieval flowchart has also been redrawn. Blue boxes denote the results from scattering simulations, and green boxes represent those derived from radar observation data.

(2) A new Section 2.2 (Retrieval Procedure) presents the complete workflow from 2DVD observations to the final retrieval of Dm, LWC and Nw in eight steps. The input, processing and output of each step are clearly specified.

We have clarified that blue arrows represent the scattering simulation workflow (physics-based theoretical derivation using 2DVD data), while green arrows represent the operational retrieval workflow applied to real radar observation data. This distinction is critical because the W-band constraint (DFR(Ka, W)) is currently derived from the simulation branch (blue), whereas the actual retrieval for radar observations (green) relies on X/Ka-band measurements augmented by this pre-established theoretical constraint.

These revisions ensure that Figure 1 serves as a self-contained visual guide to the retrieval framework, with clear traceability between the flowchart, the step-by-step procedure in Section 2.2.

Thank you for this constructive comment, which significantly improve the accessibility and reproducibility of our retrieval method.

2. *Table 1 indicates that the X-Ka dual-frequency radar has the capability to observe polarimetric parameters. If more polarimetric observation parameters could be used to constrain the DSD retrieval, the retrieval accuracy would theoretically be higher than that achieved using only dual-frequency parameters.*

Response: We thank the reviewer for this insightful suggestion regarding the potential use of polarimetric parameters to further improve DSD retrieval accuracy.

The reviewer is correct that the X/Ka dual-frequency radar system listed in Table 1 has the capability to observe polarimetric parameters, including differential reflectivity (Z_{DR}), differential propagation phase (Φ_{DP}), specific differential phase (K_{DP}), and co-polar correlation coefficient (ρ_{HV}). However, the radar was operated in a vertically pointing mode throughout this study (Section 3.1: "vertical pointing mode"). As explicitly noted in Section 5.4, this observation geometry largely eliminates polarimetric differences because the radar beam is aligned with the principal axis of symmetry of the raindrops (near-zenith incidence). Under vertically pointing conditions, the differential reflectivity (Z_{DR}) approaches 0 dB, and the specific differential phase (K_{DP}) becomes negligible, rendering these parameters ineffective as discriminators for DSD retrieval. Consequently, polarimetric constraints could not be meaningfully applied in this specific observational configuration.

We fully agree with the reviewer that incorporating polarimetric parameters would theoretically enhance retrieval accuracy beyond what is achievable with dual-frequency parameters alone. In scanning or slant-path observations, polarimetric variables provide complementary microphysical information:

Z_{DR} is sensitive to the median volume diameter (D_0) and drop shape, offering an independent constraint on Dm.

K_{DP} is proportional to the liquid water content and is immune to absolute calibration errors, providing a robust constraint for Nw retrieval.

ρ_{HV} can help identify non-meteorological echoes, mixed-phase regions, and DSD broadening.

These properties have been widely demonstrated in the literature for improving DSD retrievals and attenuation correction.

As stated in Section 5.4, incorporating polarimetric constraints will be one of the priorities for future work. When the radar operates in scanning mode (non-zenith angles), the full polarimetric capability of the system will be leveraged to:

1. Provide additional independent constraints on Dm and Nw, reducing the reliance on the DFR-based retrieval alone.

2. Improve attenuation correction through the use of the Φ ~DP~ constraint, which is particularly valuable at Ka-band where attenuation is significant.

3. Enhance the discrimination of hydrometeor types and melting layer identification.

Thank you for this valuable suggestion, which reinforces the importance of utilizing the full polarimetric capabilities of the instrument in future observational campaigns.

3. *Figure 3 shows the radar reflectivity factors observed by the X-band and Ka-band radars below the melting layer. The paper does not specify how the position of the melting layer is determined. Furthermore, does the retrieved DSD correspond to the entire rainfall region below the melting layer or only to the near-surface layer? Since the algorithm is based on ground-based disdrometer observations, it is inferred that the retrieval results should represent near-surface conditions. However, the radar observations are presented as corresponding to the rainfall region below the melting layer, indicating an inconsistency.*

Response: We thank the reviewer for raising these important points regarding the melting layer identification and the vertical representativeness of the retrieved DSDs.

The melting layer position was identified from the radar reflectivity observations based on the bright band signature. As noted in the caption of Figure 3, "The height of the melting layer bright band indicates that the zero-degree layer is approximately 5 km." The bright band, characterized by enhanced reflectivity near the 0°C isotherm due to melting hydrometeors, provides a clear marker for the melting layer top. Below this height, the radar observations are considered to be in the rain region. We acknowledge that the manuscript did not explicitly detail the specific algorithm used for bright band detection (e.g., peak reflectivity identification or temperature profile constraints), and we have added this clarification in the revised manuscript.

The retrieved DSDs correspond to the entire rainfall region below the melting layer, not just the near-surface layer. As shown in Figure 16, the retrieval framework produces time-height profiles of Dm and Nw throughout the rain column below the melting layer, capturing the vertical microphysical structure of precipitation (Section 4.3). The radar-based retrievals reveal distinct characteristics at different heights, such as the local enhancement of Dm and Nw near the melting layer due to melting hydrometeors.

The reviewer correctly identifies a potential inconsistency. The retrieval relationships (DFR-Dm, LWC-Z_{Ka}, and attenuation correction coefficients) were established using ground-based 2DVD measurements (Sections 3.2 – 3.5), yet they are applied to radar observations at all heights below the melting layer. This procedure implicitly assumes that the microphysical relationships derived from surface DSDs are representative of the entire rain column below the melting layer.

We acknowledge that this is a simplifying assumption with the following limitations:

1. The DSD may evolve vertically due to processes such as evaporation, coalescence, and breakup, meaning surface-based relationships may not perfectly represent upper-level conditions.

2. The quantitative validation presented in Figures 8, 17 – 20 and Table 7 is indeed limited to near-surface comparisons (reflectivity at 120 m height is used as a proxy for ground-level values, Section 3.3), as the 2DVD only provides ground-truth measurements at the surface.

We have explicitly noted this limitation in Section 5.3 ("Representativeness of 2DVD Validation"), where we discuss that wind drift, fall time, and sampling volume differences between radar and 2DVD limit the accuracy of quantitative comparisons. Furthermore, in Section 6, we state that future work should "incorporate height-dependent DSD models for upper-level retrievals" to address the vertical variability of DSDs and reduce the reliance on surface-derived relationships at higher altitudes.

In summary, while the current framework retrieves DSDs throughout the rain column below the melting layer, the underlying relationships are surface-based, and the quantitative validation is restricted to near-surface levels. We have clarified this distinction more explicitly in the revised manuscript and strengthen the discussion of this limitation. Consequently, the retrievals above the near-surface layer should be interpreted qualitatively rather than as fully validated quantitative estimates.

4. *In Figure 5, the attenuation coefficients are fitted based on disdrometer-observed drop size data and used for radar attenuation correction. Figure 8 then compares the corrected radar retrieval results with the disdrometer observation data from the same several rainfall events. This procedure may suffer from a self-consistency trap. Similar issues exist in the DSD retrieval section.*

Response: We thank the reviewer for raising this critical concern regarding potential self-consistency bias in the validation procedure.

We acknowledge the potential self-consistency bias. The current study is therefore positioned as a proof-of-concept investigation. Independent-event (2025) validation has been added. The independent validation results (Table 7) demonstrate comparable accuracy to the training-period evaluation, suggesting that the proposed relationships possess a degree of generalizability beyond the training data.

Despite the independent validation for the DSD retrieval, we recognize that the attenuation correction coefficients and the DFR-Dm fitting were both derived from the same 2022 events used for the initial evaluation. This represents a limitation of the current proof-of-concept study. As discussed in Section 5.4, future studies will employ event-independent validation using leave-one-event-out cross-validation with a larger and more diverse dataset to further quantify the robustness and generalization capability of the proposed method. We will also explicitly separate the datasets used for establishing attenuation relationships from those used for validating retrieval results to completely avoid any self-consistency issues.

Thank you for this valuable suggestion, which has prompted us to more carefully consider the validation strategy and its limitations in the revised manuscript.

5. *Figure 6 presents DFR results for the Ka-band and W-band obtained through simulations, but no measured W-band data are provided in the paper. In practical applications, how can the W-band be used to constrain the retrieval process?*

Response: We fully acknowledge that this study does not include measured W-band radar observations. The W-band reflectivities used in this study were obtained entirely through T-matrix scattering simulations based on measured raindrop size distributions (Section 5.1). Therefore, the DFR(Ka, W) constraint presented here should currently be regarded as a theoretical assessment of its potential to mitigate the DFR-Dm ambiguity, rather than a fully operational retrieval constraint validated with real three-frequency radar data.

In practical applications, the implementation of this approach would require a three-frequency radar system with X/Ka/W bands (Section 5.1). The physical basis for using W-band as a constraint lies in the fact that the DFR(Ka, W)-Dm relationship exhibits a significantly narrower ambiguous zone compared to DFR(X, Ka)-Dm (Section 3.5 and Figure 12). While the ambiguous zone for DFR(X, Ka) covers a wide Z_{Ka} range of approximately −20 to 45 dBZ (corresponding to Dm from 0.3 to 1.6 mm), the ambiguous zone for DFR(Ka, W) is confined to a much narrower Z_{Ka} range of approximately −20 to 20 dBZ (corresponding to Dm between 0.3 and 0.8 mm). This characteristic allows DFR(Ka, W) to effectively resolve the dual-solution ambiguity when DFR(X, Ka) ≤ 0.

However, for operational implementation, several practical challenges must be addressed (Section 5.1):

1. W-band radar calibration accuracy: Calibration errors at W-band would propagate into DFR(Ka, W) estimates.
2. Atmospheric attenuation at W-band: W-band signals are subject to stronger atmospheric attenuation, which must be carefully corrected.
3. Signal-to-noise ratio limitations: The performance may be constrained by SNR limitations at W-band, particularly in weak precipitation regions.

This study provides a proof-of-concept demonstration that motivates the development of future three-frequency radar systems for operational DSD retrieval. The practical applicability of the Ka-W constraint requires further validation with real W-band observations and more extensive datasets covering diverse precipitation regimes (Section 6). Future work should also quantify the impact of radar calibration and attenuation correction uncertainties on the retrieval accuracy when real W-band data become available (Section 5.2).

6. Figure 12 shows the DFR-Dm relationship curves, indicating that DFR(Ka, W) exhibits a narrower ambiguous region. However, even if DFR(Ka, W) can effectively reduce the ambiguous region, actual observations only include X-band and Ka-band dual-frequency radar data. Under these conditions, how the W-band radar can be introduced to achieve retrieval constraints requires further explanation.

Response: We thank the reviewer for this important question regarding the practical implementation of the W-band constraint under current observational limitations.

It is important to explicitly acknowledge that this study does not include W-band radar observations. The W-band reflectivities used herein were obtained entirely through T-matrix scattering simulations based on measured raindrop size distributions (2DVD data) (Section 5.1). Therefore, the DFR(Ka, W) constraint should currently be regarded as a theoretical assessment of its potential to mitigate the DFR-Dm ambiguity, rather than a fully operational retrieval constraint validated with real three-frequency radar data.

In the proposed retrieval framework (Section 2.1 and Figure 1), the W-band constraint is introduced through the scattering simulation branch (blue boxes). Specifically:

1. The 2DVD-measured DSDs are used as ground-truth inputs (Step 1 in Section 2.2).

2. T-matrix scattering calculations are performed to simulate theoretical radar reflectivities at X-, Ka-, and W-band frequencies (Step 2).

3. The DFR(Ka, W)-Dm retrieval relationship is established via fitting (Step 3).

4. When actual X/Ka dual-frequency radar observations yield DFR(X, Ka) ≤ 0 (the ambiguous zone), the simulated DFR(Ka, W)-Dm relationship is invoked to resolve the ambiguity and retrieve the correct Dm value (Step 6).

Under current conditions where only X/Ka dual-frequency radar data are available, the W-band constraint cannot be directly measured and must rely on the simulated relationships derived from disdrometer data. However, the practical implementation of this approach would require a three-frequency radar system with X/Ka/W bands (Section 5.1). With such a system deployed, the W-band reflectivity would be directly measured, allowing real-time computation of DFR(Ka, W) to constrain the retrieval without dependence on scattering simulations.

As emphasized in Section 5.1 and Section 6, the actual performance of a real three-frequency system may be affected by W-band radar calibration accuracy, atmospheric attenuation at W-band, and signal-to-noise ratio limitations. This study serves as a proof-of-concept demonstration that motivates the development of future three-frequency radar systems. Once real W-band observations become available, the retrieval framework established in this study can be directly applied, with the simulated relationships serving as an initial theoretical foundation until sufficient observational data are accumulated to refine the empirical DFR(Ka, W)-Dm fitting.

Thank you for this valuable comment, which has prompted us to clarify the distinction between the current simulation-based approach and future operational implementation with actual W-band measurements.