



# Modeling the refractive index profile $n(z)$ of polar ice for ultra-high energy neutrino experiments

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**Abstract.** We have developed an *in-situ* index of refraction profile  $n(z)$  for cold polar ice, using the transit times of radio signals broadcast from an englacial transmitter to 1-5 km distant radio-frequency receivers, deployed at depths up to 200 m. For propagation through a non-uniform medium, Maxwell's equations generally admit two ray propagation solutions from a given transmitter, corresponding to a direct path (D) and a refracted or reflected path (R); the measured D vs. R timing differences (dt(D,R)) are determined by the refractive index profile. We constrain  $n(z)$  near the South Pole, where the Askaryan Radio Array (ARA) neutrino observatory is located, by simulating D and R ray paths via ray tracing and comparing simulations to measured dt(D,R) values. We demonstrate that our dt(D,R) timing data strongly favors a glaciologically-motivated three-phase densification model rather than the single exponential scale height models typically employed by in-ice radio neutrino detectors. Effective volume simulations for a detector of ARA station antenna depths yield a 14 % increase in neutrino sensitivity over a neutrino energy range of  $10^{18} - 10^{21}$  eV using the three-phase model compared to the single exponential.

## 1 Introduction

Ultra-High Energy Neutrino (UHEN) experiments such as the Radio Neutrino Observatory in Greenland (RNO-G), the Askaryan Radio Array (ARA) at the South Pole, and the proposed IceCube Gen-2 experiment seek to extend the energy window of observed neutrinos beyond the MeV (typical of solar neutrinos) and PeV scales (astrophysical, as observed by IceCube) to  $>PeV$  ('cosmogenic') energy scales (Aguilar et al., 2022; Allison et al., 2016; Aartsen et al., 2021). Radio detection provides a cost-effective method for achieving large sensitive neutrino target volume, as radio signals propagate significantly farther in ice compared to optical signals. A major motivation of UHEN experiments is to complement observations of ultra-high energy charged cosmic rays (UHECR) from distant astronomical sources. UHEN are emitted following collisions of UHECR with matter or the Cosmic Microwave Background. Due to their lack of charge and small cross-section, neutrinos are able to propagate undeflected through obstacles otherwise opaque to gamma or cosmic rays. However, these same weakly interacting characteristics render observation difficult.

Radio propagation in ice, over kilometer-long distance scales, is essential for the radio neutrino experiments. Since the expected neutrino flux sharply decreases with increasing energy, radio neutrino experiments must scan large volumes for long exposure times to achieve measurable neutrino event rates. Simulations which incorporate models of the complex-valued radio-



25 frequency ice permittivity are used to estimate the sensitivity of UHEN experiments. The real part of the complex permittivity dictates the ray path followed by radio signals, from interaction point to receiver, while the imaginary part quantifies signal absorption in-ice. Given the non-magnetic nature of ice, the permittivity relates directly to the refractive index profile through  $n = \sqrt{\epsilon_r}$  where  $\epsilon_r$  is the real part of the ice permittivity. Since the overburden in the upper  $\sim 100$ - $150$  m increases with depth, UHEN experiments assume a depth-dependent refractive index. For a given receiver, the depth-dependent refractive index profile generates, by Fermat's Least Time Principle, curved rather than rectilinear ray trajectories. This ray-bending also results in a 'shadowed' volume within which neutrino interactions will be inaccessible to a given receiver - this loss of sensitivity is increasingly important for large horizontal displacements and shallow receivers. As the depth of deployed receivers decrease, the extent of the shadowed zone is an increasingly important determinant of the visible neutrino interaction volume, and therefore the number of detected neutrinos.

35 The simplest ansatz for the refractive index profile is one that follows a single exponential dependence on depth, as expected for a self-gravitating fluid. Glaciological studies of ice density as a function of depth, however, suggest that densification occurs in multiple stages (Herron and Langway, 1980; Stevens et al., 2020; Salamatin et al., 1997). Using ARA receiver timing data obtained in response to a pulser lowered into the ice, we have tested a piece-wise function separated into these 3 stages against a simpler one-stage or two-stage exponential model.

## 40 2 Ice Densification and the Refractive Index

Our model assumes a linear dependence of refractive index on density (Robin et al., 1969). According to Sorge's Law (Bader, 1954), density is constant over time at a given depth, assuming constant snow accumulation and temperature. (In reality, the snow accumulation and temperature conditions are not constant, which leads to density fluctuations; in what follows, we neglect such effects, as well as the possible effect of impurities.) In a scale-height model, the densification rate of snow is taken to be proportional to the change in pressure due to the weight of the snow overburden, leading to the exponential form

$$\rho(z) = \rho_f - b_0 e^{Cz} \quad (1),$$

where  $\rho_f$  is the density of deep ice,  $\rho_f - b_0$  is the density of snow at the surface, and  $C$  is a proportionality constant prescribed by the densification rate.

Theoretical and empirical models of ice density as a function of depth, however, prescribe two boundaries that affect the ice densification rate as  $\rho(z)$  crosses density thresholds (Salamatin et al., 1997; Lipenkov et al., 1997). The (better-studied) first boundary at  $\sim 550$  kg/m<sup>3</sup> (Herron and Langway, 1980) separates the snow and firn regions, and occurs at depths of  $\sim 15$ m and  $\sim 20$ m for Summit and South Polar ice, respectively (Stevens et al., 2020). A second boundary separates the firn and bubbly ice regions. The ice is considered to be fully formed in the third region, but there still exist air pockets within the ice that reduce the overall density relative to  $\rho_f$ . These air pockets are crushed under greater pressure as depth increases, asymptotically reaching the  $\rho_f \sim 920$  kg/m<sup>3</sup> density of typical pure, deep ice at  $T = -30^\circ$  C (Welling and Collaboration, 2024a). To account for changing densification rates in the separate regions, the form of Eqn. (1) is modified to a piecewise function where:



$$\rho < \rho_1 : \rho(z) = \rho_f - b_0 e^{c_1 z},$$

$$\rho_1 \leq \rho < \rho_2 : \rho(z) = \rho_f - b_0 e^{z_1(c_1 - c_2)} e^{c_2 z},$$

$$\text{and } \rho \geq \rho_2 : \rho(z) = \rho_f - b_0 e^{z_1(c_1 - c_2)} e^{z_2(c_2 - c_3)} e^{c_3 z} \quad (2)$$

60 to describe the 3 layers. In this parameterization,  $\rho_1$  and  $\rho_2$  are density boundary conditions with corresponding depths  $z_1$  and  $z_2$ ,  $b_0$  is determined by the surface density and continuity at each boundary, and  $c_1, c_2$  and  $c_3$  are constants describing the densification rate in each depth region.

Figure 1 shows ice density data taken from a number of sites in Greenland. This density data approaches the asymptotic value and therefore includes measurements well into the expected bubbly ice region. We use a  $550 \text{ kg/m}^3$  boundary between the snow and firm regions based on previous studies (Herron and Langway, 1980; Stevens et al., 2020), while the boundary between firm and bubbly ice is determined empirically by fitting the  $\rho(z)$  data for the inflection point separating the intermediate and highest density regions, using the Levenberg–Marquardt fitting algorithm (Gavin, 2019). In this fitting procedure, the surface density,  $c_1, c_2, c_3$ , and the firm/bubbly ice boundary are taken as free parameters while the snow/firm boundary and asymptotic density are fixed at  $550 \text{ kg/m}^3$  and  $920 \text{ kg/m}^3$ , respectively. Each data point from the compiled density data is weighted equally in this fit. Applying the fitting algorithm gives a  $758 \text{ kg/m}^3$  boundary between the firm and bubbly ice regions which is used to determine the second depth boundary in Eqn. (2). Past studies of the snow/bubbly ice density boundary have used values from  $800\text{-}830 \text{ kg/m}^3$  (Herron and Langway, 1980; Lipenkov et al., 1997). For the single exponential, the same fitting algorithm is used and only  $b_0$  and  $c$  in Equation (1) are taken to be free parameters. This gives a best fit single exponential to the compiled data:

$$75 \quad \rho(z) = 920 - 548.5 e^{-0.0241z} \quad (3).$$

A 3 stage exponential fit using the form of Equation (2) gives:

$$\rho < 550 \text{ kg/m}^3 : \rho(z) = 920 - 551.7 e^{-0.0262z},$$

$$550 \text{ kg/m}^3 \leq \rho < 758 \text{ kg/m}^3 : \rho(z) = 920 - 373.4 e^{-0.0193(z-14.9)}, \text{ and}$$

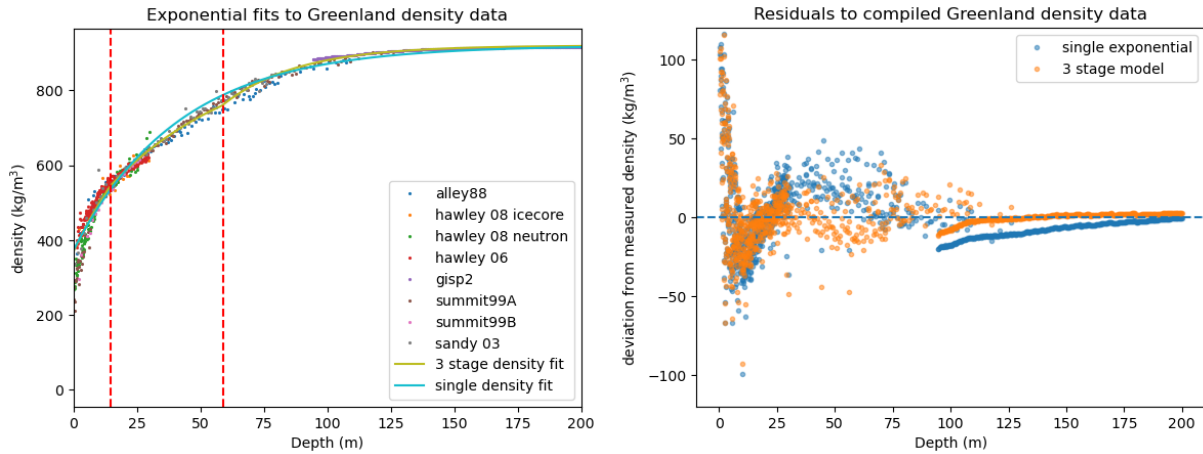
$$\rho \geq 758 \text{ kg/m}^3 : \rho(z) = 920 - 159.0 e^{-0.0339(z-58.9)} \quad (4).$$

## 80 2.1 Calculating refractive index from density

While there exists variation in the parameters for different sets of data, the linear relation assumed for  $n(\rho(z))$  serves as a useful ansatz for the dielectric–specific-gravity relation (Kovacs et al., 1995), and is supported by studies at the Maudheim and McMurdo ice shelves as well as studies of snow in Canada and Japan (Schytt, 1958; Evans, 1965; Cumming, 1952):

$$n(z) = 1 + A\rho(z) \quad (5),$$

85 where  $\rho$  is the pure ice specific-gravity and  $A$  is a proportionality constant with units  $\text{cm}^3/\text{g}$ , and can be estimated from the constraint that deep, bulk ice has a refractive index corresponding to the measured value at a given site. At Summit Station, Greenland,  $n=1.778 \pm 0.006$ , (Welling and Collaboration, 2024a, b) for deep bulk ice. Different fits to McMurdo ice shelf data give  $A$  values ranging from  $0.840\text{-}0.858$ . The  $A$  value of  $0.845$  determined from the most recent McMurdo ice shelf study (Kovacs et al., 1995) also matches the conversion of the asymptotic ice density  $920 \text{ kg/m}^3$  to the refractive index value  $n=1.778$ .  
90 We therefore use  $A=0.845$  to convert SPICE core density to  $n(z)$  (see Section VI), to give:  $n(z) = 1.778 - B_0 e^{Cz}$  (6), where



**Figure 1.** Equations (3) (single) and (4) (3 stage) parameterization fits to compilation of Greenland ice density data. Fixing the snow/firn boundary at  $550 \text{ kg/m}^3$ , the fit implies a firn/bubbly ice boundary of  $758 \text{ kg/m}^3$ . Red dotted lines (14.9 m, 58.9 m) indicate corresponding density boundaries. Density data have been taken from (Gow et al., 1997; Alley and Koci, 1988; Hawley and Morris, 2006; Hawley et al., 2008).

$B_0$  is  $A \times b_0$  from Eqns. (5) and (1), respectively and  $C$  is constant, as in Eqn. (1). Applying the same density to refractive index conversion to Eqn. (2) gives a 3 stage refractive index model with the piecewise form:

$$\begin{aligned}
 z < z_1 : n(z) &= 1.778 - B_0 e^{c_1 z}, \\
 z_1 \leq z < z_2 : n(z) &= 1.778 - B_0 e^{z_1(c_1 - c_2)} e^{c_2 z}, \text{ and} \\
 z \geq z_2 : n(z) &= 1.778 - B_0 e^{z_1(c_1 - c_2)} e^{z_2(c_2 - c_3)} e^{c_3 z} \quad (7).
 \end{aligned}$$

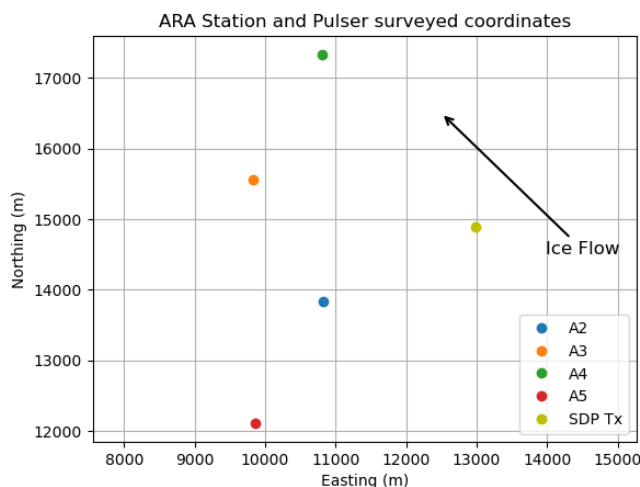
In Eqn. (7),  $c_1, c_2, c_3$  are exponential parameters to be determined by fitting the model to timing data.

## 2.2 Experimental Layout

Analyzed ARA data discussed below are drawn from multiple (and often overlapping) combinations of transmitters and receivers, spanning in-ice transmitter depths of up to 1.5 km and horizontal trajectories of up to 5 km over a wide range of incidence angles. A map of the relevant radio-frequency instrumentation in the vicinity of South Pole is presented in Figure 2, showing the receiver stations relative to the radio-frequency transmitters.

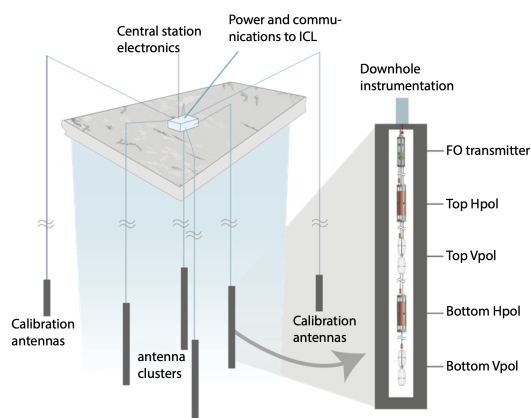
## 3 Deep Pulser Data

A typical ARA station is shown in Figure 3; ARA consists of 5 such stations, denoted as 'A1'-'A5'. Each station consists of 4 boreholes at the vertices of a horizontal square, with an inter-string lateral separation of order 20 meters; each borehole contains a vertical string of radio receiver antennas. Each string consists of 2 horizontally polarized (Hpol) and 2 vertically polarized (Vpol) antennas, deployed at depths ranging from 170-200 m. This allows the stations to record both Hpol and Vpol



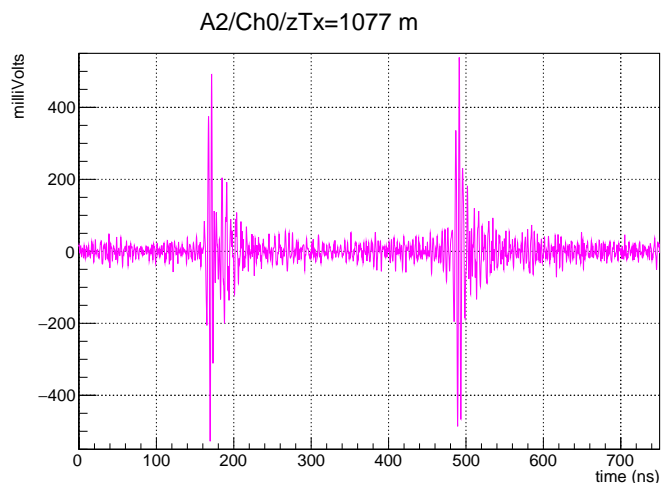
**Figure 2.** Surveyed coordinates (Easting, Northing) for ARA stations A2, A3, A4 and A5 for which timing data from the SPICE Deep Pulser (SDP) are analyzed.

signals, and thereby infer the polarization of a signal based on the relative amplitudes registered on the Hpol vs. Vpol antennas. Reconstruction of a source location is based on relative signal timings received by multiple antennas. Nearby (30–50 m away) englacial pulsers are used to calibrate channel positions. In our case, we are interested in not only the direct (D) signal arrival time, but also the refracted/reflected (R) signal arrival time from a distant transmitter.



**Figure 3.** ARA station layout, comprising 16 antennas (8 Hpol and 8 Vpol) ranging from 170-200 m depths.

A waveform similar to that seen in Figure 4 is divided into its D and R components. The waveform is converted to a Hilbert envelope, and the timing of each peak is determined by fitting the Hilbert envelope of each peak to a Gaussian.



**Figure 4.** Typical waveform recorded during SDP transmitter run; Direct (earlier) and Refracted (later) pulses are evident in the Figure.

The  $dt(D,R)$  timing difference offers several advantages in calibration. In addition to being immune to single channel data acquisition signal transmission (aka ‘cable delay’) uncertainties, it also offers enhanced discrimination between different  $n(z)$  models due to the differences between the D and R optical paths (illustrated in Figure 5). The R ray path travels through ice well above the receiver antenna which allows for testing a greater range of depths for  $n(z)$ . By contrast, nearby calibration pulsers only test  $n(z)$  over the limited depth range of the deployed ARA antennas. From a science perspective, since, to a first approximation,  $dt(D,R)$  depends linearly on the range to source location (discussed below), it can also be used as a powerful constraint in neutrino reconstruction, independent of conventional interferometric techniques to locate interaction points (Latif, 2020).

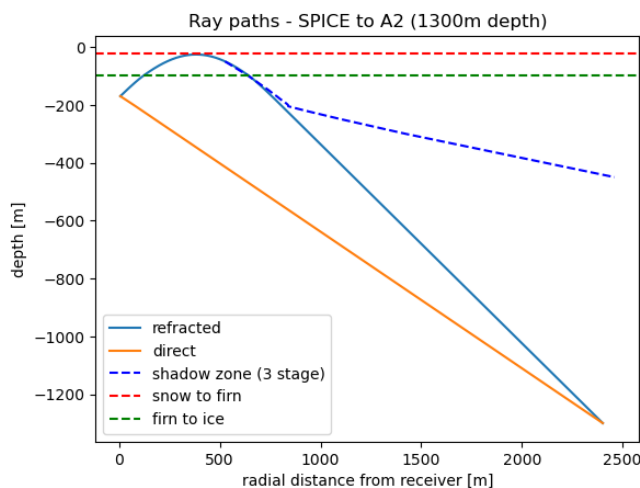
The South Pole Ice Core Experiment (SPICE) borehole was drilled to 1751 m over the 2014-2015 and 2015-2016 austral seasons with the purpose of collecting data to determine changes in atmospheric chemistry, climate, and biogeochemistry since the most recent [40 ka] glacial-interglacial cycle (Casey et al., 2014). Proximal (1-4 km from stations A2-A5 (Allison et al., 2020)) to the ARA stations, SPICE offers both density data and the opportunity for deep pulser timing measurements.

The SPICE deep pulser (‘SDP’) transmitter antenna was deployed into the SPICE borehole, and broadcast 1 pulse per second (pps) signals. Approximately 200 pulses are transmitted at each depth and averaged to calculate timing differences. Figure 4 shows a 64-event average waveform obtained from a deep pulser (‘SDP’) lowered into the SPICE borehole in Dec., 2018. Stations began receiving double pulse (D,R) signals once the transmitter emerged from the shadow zone (around a depth of 500–700m, depending on the receiver station) resulting in a dataset of  $dt(D,R)$  pulses recorded to  $z=1300$  m. ARA stations recorded signal from the SDP over a range of 150-1000 MHz. A more detailed description of the SDP pulsing runs and received signals can be found in the literature (Allison et al., 2020). We use this dataset to constrain  $n(z)$  models.



#### 4 Simulated Deep Pulser Signals

Ray paths originating at the SDP transmitter are simulated using the numerical ray tracer `RadioPropa` (Winchen, 2019), which calculates the two possible solutions (D and R), given source and receiver positions and an input  $n(z)$  model. Figure 5 shows the simulated ray paths from a 1300 m deep source to an ARA antenna. The D ray path travels only through the bubbly ice region while the R path travels through both the bubbly ice and firm regions of the 3 stage model.



**Figure 5.** Simulated (using Eqn. (6)) direct and refracted ray paths from a 1300 m deep pulser in the SPICE borehole to channel 0 of ARA station A2. Horizontal dotted lines depict the boundaries between regions of the 3 stage  $n(z)$  model. Shadow zone refers to the area above and to the right of the dashed blue line where pulses cannot be detected.

The ray tracing code `RadioPropa` is incorporated into `NuRadioMC`, a complete Monte Carlo simulation suite that can be used to simulate signal waveforms and determine the neutrino detection volume for a given radio neutrino detector configuration (Glaser et al., 2020). In addition to relative signal arrival times, the relative amplitudes of the D and R signals as well as frequency content can also be used to compare simulated waveforms to data. In what follows, we consider only timing information; amplitude analysis is currently underway.

#### 5 Ice model results

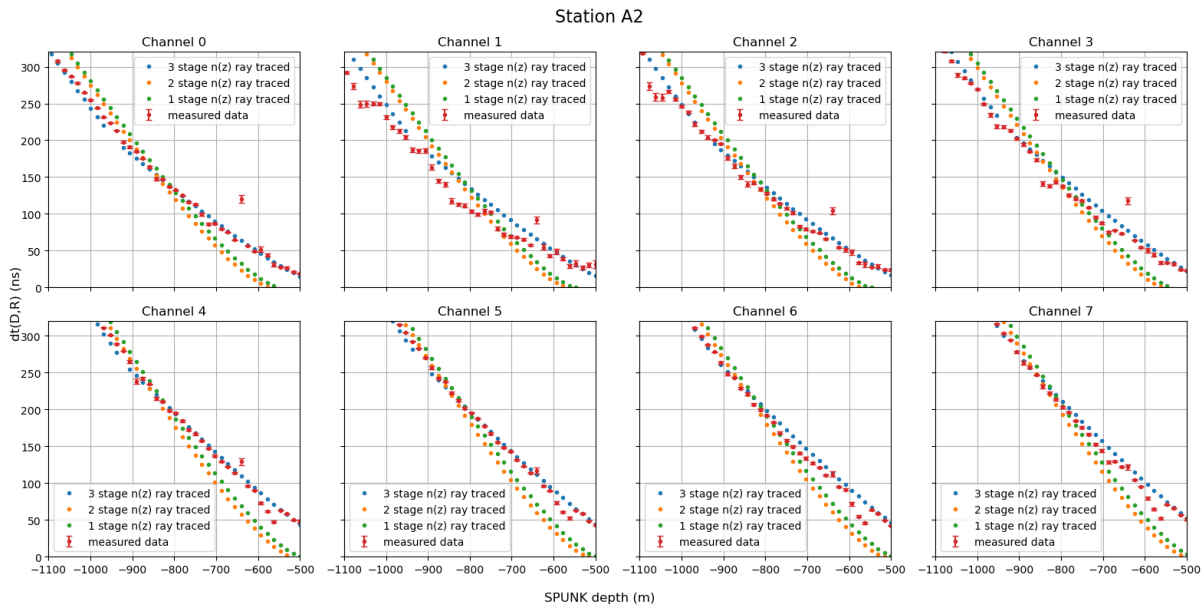
To ensure the transmitter is well beyond the shadow zone, we measure  $dt(D,R)$  over the depth range from 700–1300 m for most stations. Typical horizontal separations are of order 1–5 km - in the case of A3, for example, the SPICE borehole is laterally displaced 3230 m. A sample A2 VPol waveform used for analysis is shown in Figure 4; in this case, the transmitter was located at a depth of 1077 m.



### 5.1 Fitting models to A2 dt(D,R) data

To assess the fidelity of our  $n(z)$  model, we use station A2 dt(D,R) data (selected on the basis of being the most-studied of the current ARA stations) as a ‘training set’ to extract, via  $\chi^2$  fit of simulation to data, numerical values for the required one-stage, two-stage, or three-stage  $n(z)$  constants (Eqns. (6) and (7)) and then fix those values for comparing data vs. simulations for the other stations. For all models, the surface parameter  $B_0$  and the asymptotic refractive index 1.778 are fixed. For the multi-stage models, boundary depths  $z_1$  and  $z_2$  are determined using the SPICE core density data, and then fixed; density boundaries of  $550 \text{ kg/m}^3$  and  $758 \text{ kg/m}^3$  imply phase transition boundary depths of 20.5 m and 96.6 m, respectively. When converted using Eqn. (5), the surface density implies  $B_0=0.45$ .

The minimum  $\chi^2$  estimate is performed on 368 measured dt(D,R) data points from the SDP across the 8 VPol channels. The same minimum  $\chi^2$  estimate is performed for the 2 stage and 3 stage models with 2 and 3 free parameters for the additional  $c_2$  and  $c_3$  parameters for the models, respectively. The minimum  $\chi^2$  parameters for the 1 stage, 2 stage, and 3 stage models are presented in Table 1. The summed  $\chi^2$  per degree of freedom for these models are 406.5, 295.8, and 9.3, respectively. The higher  $\chi^2$  in the 1 stage and 2 stage  $n(z)$  model fits to A2 is a result of an inability of the models to fit dt(D,R) across a range of transmission depths. The resulting fitted 1 stage, 2 stage, and 3 stage model parameters are shown in Table 1.



**Figure 6.** dt(D,R) for Eqn. (5), (6), and (7) compared to measured data for station A2. Models were fit to minimize  $\chi^2$  to data across channels shown, with parameters shown in Table 1.

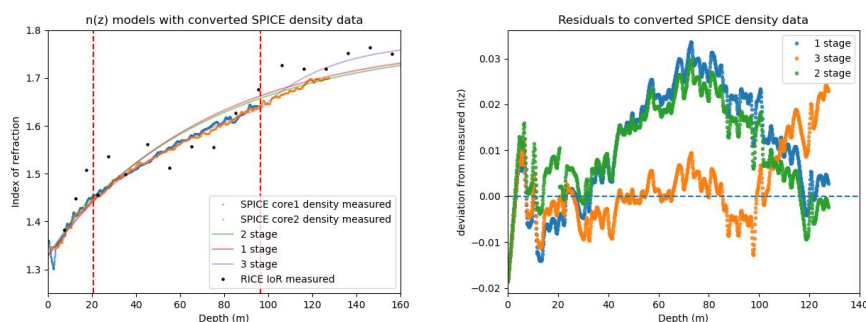


n(z) model	$c_1$	$c_2$	$c_3$
1 stage	0.0146	N/A	N/A
2 stage	0.0165	0.0136	N/A
3 stage	0.0147	0.0112	0.0310

**Table 1.** Parameters of A2 dt(D,R) minimum  $\chi^2$  estimate for 1 stage, 2 stage, and 3 stage models.

## 5.2 Comparison to converted SPICE density data

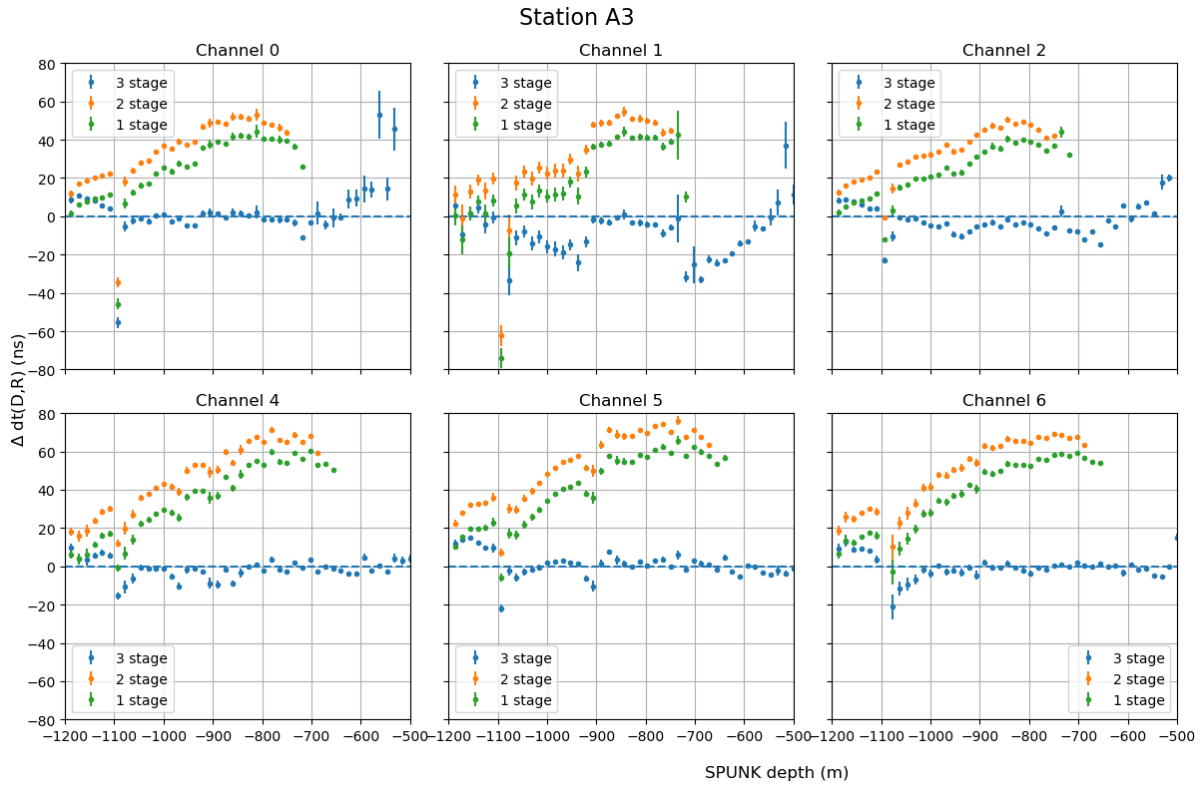
We compare the 3 models to SPICE core density data converted to  $n(z)$  via Eqn. (5), with  $A=0.845$ . Figure 7 shows the 1 stage, 2 stage, and 3 stage A2 dt(D,R) parameterizations shown in Table 1. The residuals show the best-fit 3 stage model to the SPICE density data, particularly over the firm region between 20.5 m and 96.6 m. However, in the bubbly ice region ( $z > 96.6$  m), the 3 stage model typically exceeds the density-implied  $n(z)$ , although this is also the regime for which the data statistics are smallest.



**Figure 7.** Density profiles corresponding to 1 stage, 2 stage, and 3 stage models (see Table 1) compared to converted SPICE core density data (Allison et al., 2020) down to 127 m depth. Deviation between the two SPICE core density measurements suggest an error of  $\pm 0.005$  in density-converted  $n(z)$  values. Residuals are shown relative to converted SPICE core density data.

## 5.3 Timing analysis for other ARA stations

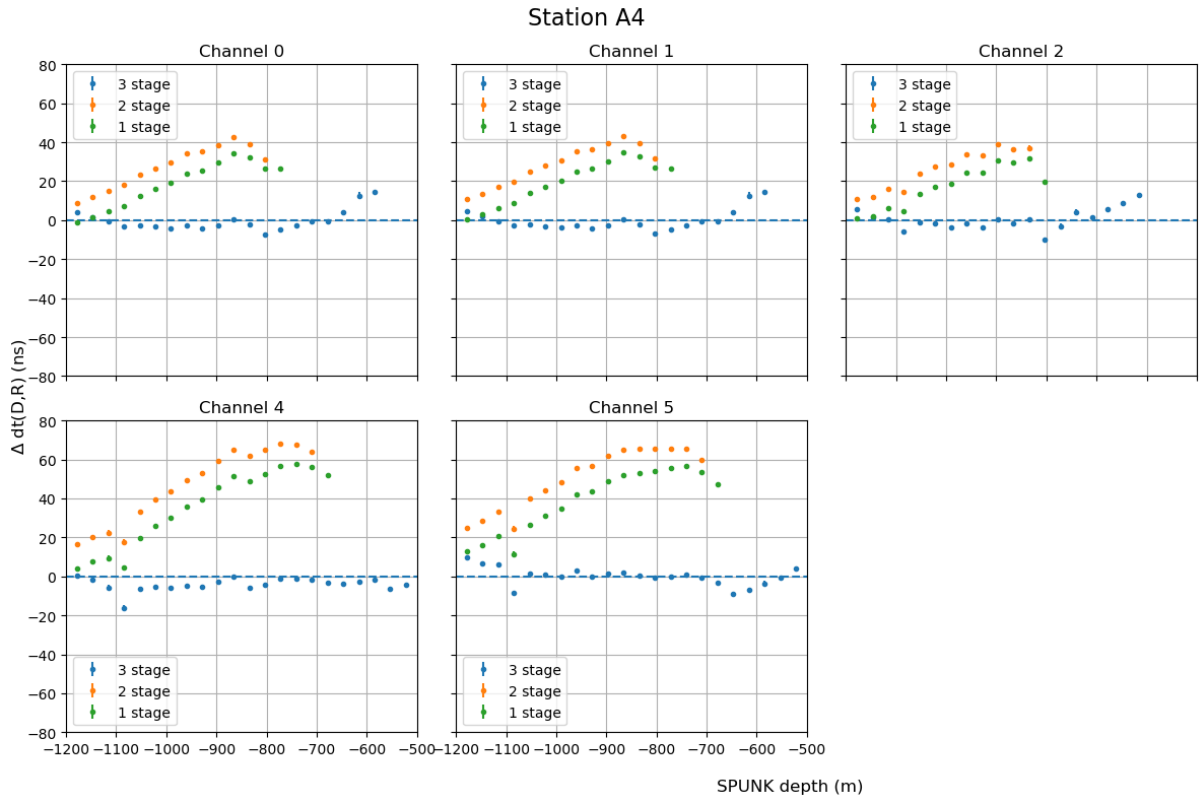
Figure 8 shows the  $\Delta$  dt(D,R) values for various depths of SDP broadcasts to ARA station A3 for the  $n(z)$  models corresponding to the 1 stage, 2 stage, and 3 stage parameterizations extracted from independent fits to A2 data, and shown in Table 1. ‘Measured’ times refer to dt(D,R) calculated using the arrival times of the D and R signals in double pulse waveforms similar to Figure 4; ‘Simulated’ times refer to dt(D,R) calculated from the D and R RadioPropa-prescribed ray paths, for given source and receiver locations. Shown in Figures 8–10 are  $\Delta$ dt(D,R) values as a function of SDP pulsing depth.



**Figure 8.**  $\Delta dt(D,R)$  for signals traveling from SDP to station A3 for Vpol channels. Shown are comparisons of simulated vs measured  $dt(D,R)$  for Eqn. (5), Eqn. (6), and Eqn. (7).

As illustrated in Figures 8 and 9, the 1 stage and 2 stage models show a trend for which measured  $dt(D,R)$  increases relative to simulated  $dt(D,R)$  at shallower depths. The discrepancy can be markedly reduced by using a 3 stage model, for which the  $c_3$  parameter is increased in the bubbly ice region relative to the  $c$  parameter of the single exponential.

Station A5 is located 4165 m from the SPICE borehole, approximately 1 km further than the other stations. This larger lateral distance also results in an increased extent of the shadowed zone (see section VII), corresponding to  $dt(D,R)$  data only being measurable over a range of 850-1300 m source transmitter depth. Figure 10 shows the measured – simulated  $dt(D,R)$  results, comparing the different refractive index parameterizations for station A5. As with the A3 and A4 datasets, we use the  $c_3$  parameter determined from A2 data and apply that value to the independent A5 dataset. The 1 stage and 2 stage models deviate further from the A5 data, which is improved using the 3 stage  $n(z)$  model. The 1 stage and 2 stage models also imply a shadow zone boundary deviating from measured data.



**Figure 9.**  $\Delta dt(D,R)$  for signals propagating from SDP to station A3.

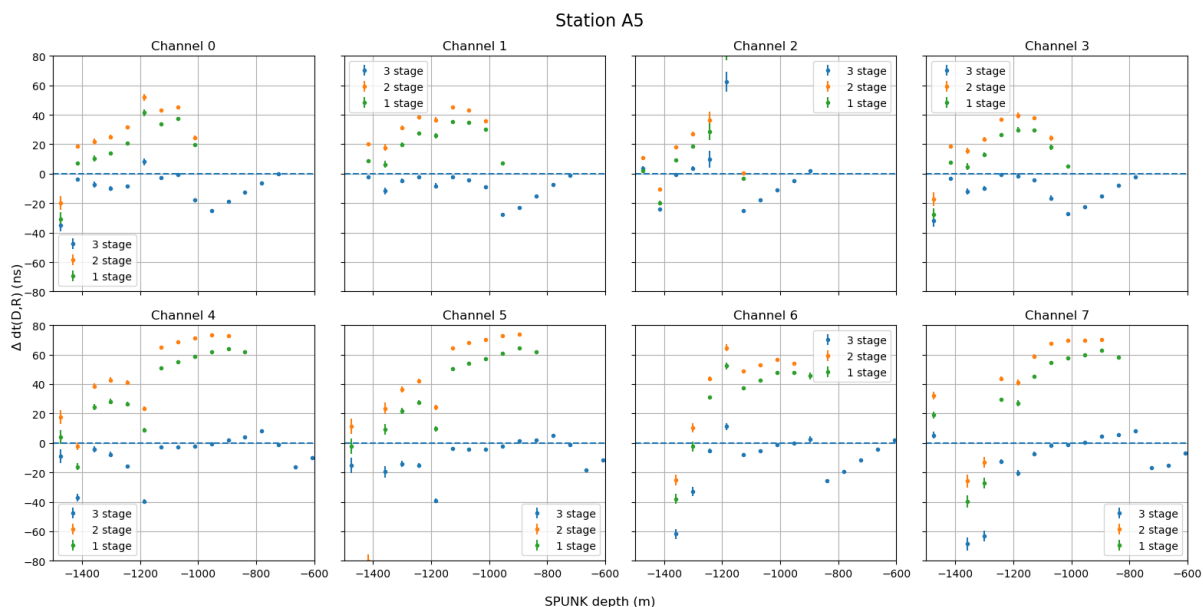
## 6 Shadowed Zone

185 As a source moves further away laterally from a receiver or upwards to a shallower depth,  $dt(D,R)$  decreases. Eventually, as  $dt(D,R)$  approaches 0, the D and R signals seen in Figure 4 coalesce. Initially, this results in focusing that increases signal amplitude. However, beyond a certain point, corresponding to the shadow zone boundary, the bending of possible paths no longer allows signal to reach the receiver from the transmitter. An example of this would be a transmitter that lies above the blue dashed line shown in Figure 5 relative to the receiver. Since refraction is determined by the  $n(z)$  model, the shadowed zone, as well as the detected neutrino rate, both depend on  $n(z)$ .

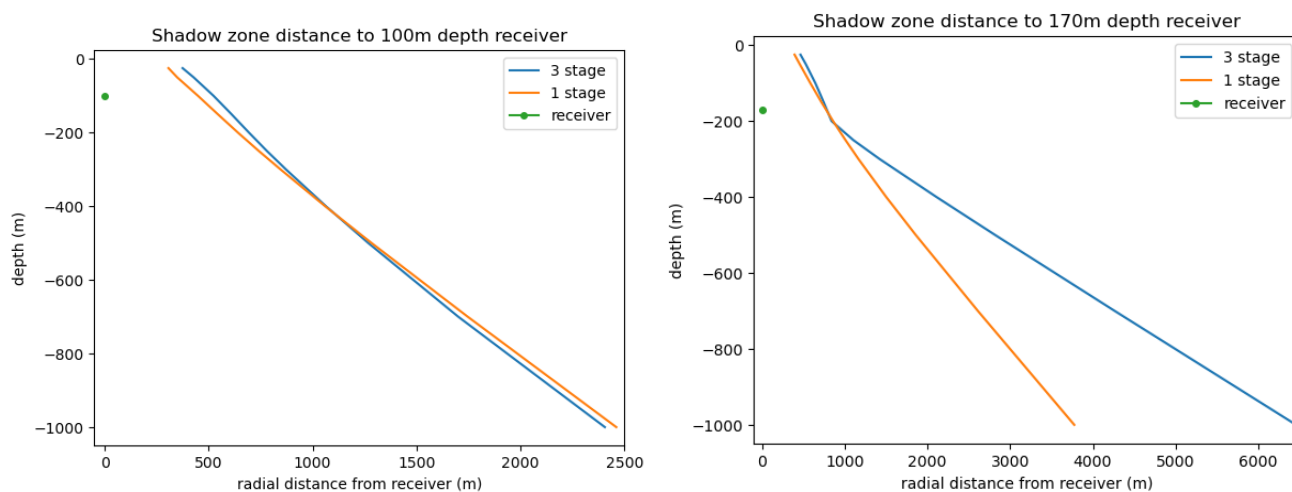
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## 7 Impact on neutrino detection

Figure 11 illustrates how changes in the refractive index model affect the lateral extent of the shadow zone. Overall, we observe that the 3 stage model results in a more restricted accessible target volume for 100 m depth receiver deployments and correspondingly slightly decreased neutrino sensitivity. By contrast, Figure 12 compares effective volume simulations using



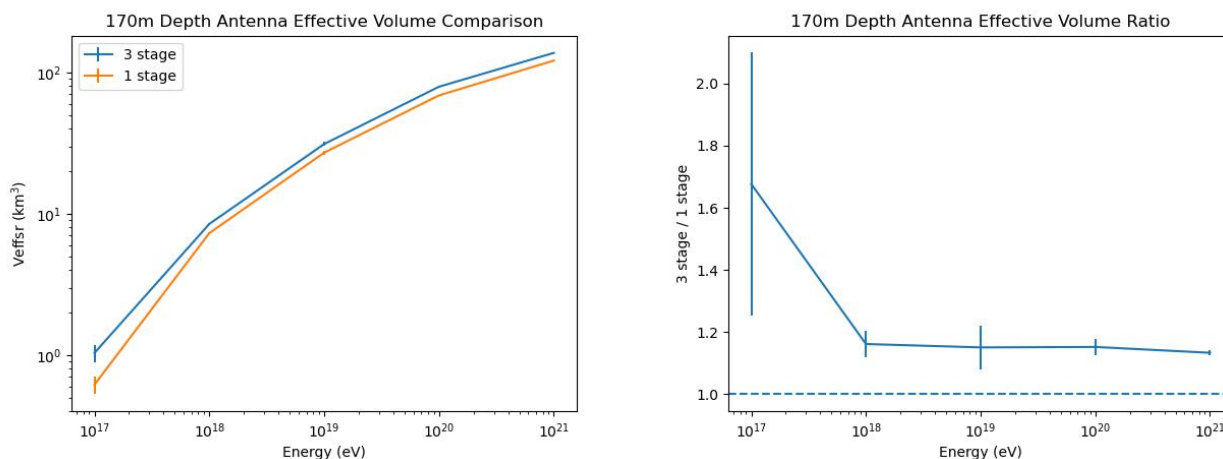
**Figure 10.** ARA station A5  $\Delta dt(D,R)$ , extracted using 1 stage, 2 stage, and 3 stage index of refraction models from Table 1. The refractive index parameterization from the 3 stage model improves the agreement relative to the 1 stage and 2 stage models as a function of SDP depth. A lack of simulated  $dt(D,R)$  solutions from the shadow zone of the models results in some ‘missing’ values at shallow SDP depths for the 1 stage and 2 stage models.



**Figure 11.** Example simulated shadow zone boundary ( $r,z$ ) coordinates for 100 m and 170 m depth receiver antennas, for 1 stage and 3 stage  $n(z)$  profiles (see Table 1). Over the range of the SDP pulsing runs and antenna depths exceeding 100 m, the 3 stage model implies a smaller shadowed volume.



195 the two ice models for neutrino detector stations with 170 m and 100 m deep antenna deployments. The 3 stage model yields a larger effective volume than the 1 stage model at typical ARA station detector depths.



**Figure 12.** Simulated effective volume (left) and ratio (right) plots for refractive index profiles specified by equations (5) and (6), for neutrino energies of 10<sup>17</sup>, 10<sup>18</sup>, 10<sup>19</sup>, 10<sup>20</sup>, 10<sup>21</sup> eV for 170 m depth detector typical of ARA station. Error bars shown are statistical only.

## 8 Conclusion

Measured dt(D,R) time differences from deep radio-frequency transmitters support a glaciologically-motivated 3 stage exponential n(z) model over 1 stage exponential model, whereas the SPICE core density measurements alone give no clear discrimination between the models. The 3 stage model is closer to converted density in the firn region, but exceeds converted density in the bubbly ice region, where the 1 stage model shows improvement. This suggests that the index of refraction may exceed that of the linear proportional relation to density at greater depths. Future analysis of the amplitude and frequency content of D and R signals can help refine the n(z) model, as phenomena such as flux focusing are also sensitive to ray curvature. A comparison of simulated shadowed zone boundaries with those extrapolated from signal amplitudes as a function of depth can also provide an independent check on the refractive index profile. An improved n(z) model should also help to provide a more accurate effective volume estimation and aid in current calibration efforts for UHEN experiments in both Greenland and the South Pole, as well as future planned experiments, such as the radio component of the IceCube-Gen2 Radio experiment.

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