This manuscript “Asymptotic Behavior of Lidar Scattering Properties of Absorbing Dust Aerosols Across Rayleigh and Geometrical-Optics Regimes: Theory and Implications” by Luna et al. presents a systematic theoretical investigation of how particle size and iron-oxide mineralogy control the asymptotic behavior of lidar observables- specifically the lidar ratio, linear depolarization ratio, and single-scattering albedo for nonspherical mineral dust. Using the TAMUdust2020 irregular hexahedral database and triaxial ellipsoid calculations, the authors derive limiting behaviors in the Rayleigh regime and the geometrical-optics regime. A central claim is that these asymptotic limits are fundamentally driven by absorption and size parameter rather than particle shape. The authors then integrate single-particle properties over idealized and in-situ measured size distributions (FENNEC, AER-D, AERONET) to explain multiwavelength spectral features, including the frequently observed "inverse-v" depolarization pattern. The paper concludes by identifying an "optical ambiguity" whereby super-coarse, iron-rich dust can mimic fine-mode aerosol signatures in standard lidar retrievals, with important implications for spaceborne missions.

The topic is timely and highly relevant to the aerosol physics and remote sensing communities. The theoretical framework connecting single-particle physics to bulk, multiwavelength lidar observables is interesting, and the mineralogical focus aligns well with new constraints from imaging spectroscopy (e.g., EMIT). Overall, the science is sound, and the conclusions are well-written. However certain claims, particularly regarding the exact GO limit of SSA, the practical degeneracy of super-coarse dust, and the shape-insensitivity assertion would benefit from clarification. I recommend the authors address the points below:

1. In section 3.2, the authors derive results that for large, strongly absorbing particles w ~ 1/2 (Eq. 16), based on the assumption that $C^{D}_{sca} \approx C_{geo}$ and $C^{R}_{sca} \ll C_{geo}$. However, in section 4.1 around line 284, the text states that “..that asymptote remains above 0.5 and reflects both refractive-index and shape effects.” Furthermore, Figure 2 (right column) and Figure 4 (right column) show bulk SSA values at large sizes/radii approaching ~0.55-0.65 rather than 0.5, even for high iron-oxide fractions. This raises the question is w ~ 1/2 a strict theoretical limit only for perfectly absorbing particles or idealized geometries, whereas finite absorption and realistic shapes may yield a higher asymptote? Or does the 0.5 limit apply only to the single particle level while PSD averaging shifts the bulk value? Please clarify under which conditions the 1/2 limit is exact versus approximate, and please discuss why the numerical results converge to value > 0.5. Also please clarify the mathematical condition (e.g., is it k·x >> 1, or a threshold on k itself)?. For a given iron-oxide fraction, does the limit strictly require x tends to infinity, or is there a finite x where the asymptote is effectively reached? Clarifying this would help readers know when the 1/2 limit is physically relevant versus merely mathematical.

Thank you for the thoughtful question.

The strict limit of $\omega=1/2$ mathematically assumes that the entire ray-tracing contribution to scattering is zero ($C^R_{sca}=0$). While strong absorption effectively eliminates the internally transmitted rays, it does not eliminate first-surface Fresnel reflection. Because mineral dust possesses a real refractive index significantly different from air (e.g., $n\approx1.5$), a fraction of the incident light will always scatter off the exterior surface. Therefore, the numerical results converge to a value of $\omega=0.5+$surface reflection contribution (yielding values around 0.55–0.65). The exact magnitude of this reflection heavily depends on the distribution of incidence angles, which is why this higher asymptote reflects particle shape and random orientation, and is present even at the single-particle level prior to PSD averaging.

Regarding the mathematical conditions: the asymptote is approached when the product of the imaginary refractive index and size parameter is sufficiently large ($k\cdot x\gg1$). This dictates the attenuation of internal rays. The limit does not require x to tend strictly to infinity; the asymptote is effectively reached at a finite x once the internal optical depth is large enough to fully extinguish the transmitted wave. The strict $\omega=1/2$ limit would only be physically exact for a hypothetically perfectly index-matched particle ($n=1$) with no surface reflection. We have updated Section 3.2 and Section 4.1 to clarify the role of Fresnel reflection in elevating the realistic asymptote above 0.5.

2. The manuscript correctly emphasizes that the asymptotic GO and Rayleigh limits are insensitive to particle shape, and the comparison between hexahedra (Figure 2) and spheroids (Figure 3) indicate this. However the intermediate size regime ($x\sim10$-100), which are highly relevant for lidar observations at 355-1064 nm of coarse dust), the depolarization ratios differ substantially in magnitude between the two models (for e.g. $\delta\sim0.5$ for hexahedra vs. 0.3-0.4 for spheroids at 355 nm). This absolute difference will translate to around 20-40% relative difference, which is large enough to shift aerosol-typing thresholds in algorithms like CALIPSO or EarthCare. Since the operational lidar retrievals often sample this transition region rather than the true asymptotic limit, the practical retrieval uncertainty due to shape choice maybe larger than the "weakly sensitive" framing suggested. Could the authors provide a discussion that while functional form (peak-and-decline for absorbing particles) is robust, the absolute magnitude of δ and the exact location of the peak in x-space remain shape dependent. Could the authors show a relative difference between the two models as a function of x? This would further help the community assess where shape uncertainty matters most.

We thank the reviewer for this highly insightful comment. The reviewer is absolutely correct: while the asymptotic limits and the functional form (the absorption-driven peak-and-decline) are robust across shape models, the absolute magnitude of δ and the exact location of the peak in x-space are highly shape-dependent in the intermediate transition regime ($x\sim10$–100).

We agree that our original "weakly sensitive" framing overly prioritized the theoretical limits at the expense of the operational reality. Because operational lidars (355–1064 nm) often sample coarse dust exactly within this intermediate size regime, a 20–40% relative difference in δ is practically significant and can indeed impact aerosol typing thresholds for missions like CALIPSO or EarthCARE.

To address this, we have generated the requested relative difference plot comparing the hexahedral and spheroidal models as a function of size parameter, which we have added to the Supplementary Material (Figure S2). Furthermore, we have revised Section 3.3 to explicitly delineate the shape-insensitive asymptotic bounds from the highly shape-dependent intermediate regime.

"While the asymptotic limits ($x \ll 1$ and $x \gg 1$) and the overarching functional form of the backscattering optical properties are robust across shape models, we emphasize that the intermediate size regime ($x \sim 10$–100) remains highly shape-dependent. The absolute magnitude of the depolarization ratio and the exact location of the peak in *x*-space vary significantly between the two geometries. As shown in our supplementary relative difference analysis (Figure S2), the absolute magnitude of δ can differ by 0.1 to 0.4 between the hexahedral and spheroidal models in this transition zone. Because operational lidars at wavelengths between 355 nm and 1064 nm frequently sample coarse dust within this exact size-parameter range, this shape-induced variance is practically significant. Such absolute differences are large enough to shift aerosol-typing thresholds in standard retrieval algorithms (e.g., CALIOP, EarthCARE). The importance of shape selection in this intermediate regime aligns with recent findings that the choice between hexahedral and spheroidal models substantially impacts combined lidar-polarimeter dust retrievals and that spheroids saturate at a lower maximum depolarization ratio (Regmi et al., 2026 Fig. C1-C3)."

3. Can the authors either overlay their modeled spectral depolarization curves on Haarig et al. observations or provide a quantitative discussion comparing modeled ranges (e.g., δ355, δ532, δ1064) against reported observational values from SALTRACE or Barbados?

We thank the reviewer for this excellent suggestion. We completely agree that a direct, quantitative visual comparison against field data is highly valuable for contextualizing our theoretical bounds.As requested, we have extracted the triple-wavelength SALTRACE Barbados observations from Haarig et al. (2017) and overlaid their means and standard deviations directly onto our modeled spectral curves in Figures 6 and 8. Furthermore, we calculated the corresponding depolarization color ratios ($\delta_{1064}/\delta_{355}$ and $\delta_{1064}/\delta_{355}$) with standard error propagation from these observations, and overlaid them on Figure 10. We believe these additions successfully ground our geometric optics framework in real-world measurements and clearly demonstrate how our models align with the reported observational values.

4. The manuscript reports S > 300 sr at 355 nm (Figure 8a-c). However, elastic-backscatter lidars like CALIOP and CATS assume S ≈ 40-44 sr for dust. Are these modeled extreme values within the physically realizable range for atmospheric dust, or would they be filtered as non-physical outliers in operational retrievals? Is it possible that the extinction is associated with other particle types mixed together, such as absorbing carbon particles, which have been labeled together with dust by the existing lidar algorithms?

We appreciate the reviewer raising this critical point.

To answer the first question: as we have now clarified in Section 4.3), S>300 sr represents a theoretical upper bound for a pristine, homogeneous plume composed entirely of super-coarse, highly absorptive iron-rich dust. In the real atmosphere, we agree these absolute values are rarely physically realized. This is partly due to the known challenges idealized shape models face in accurately simulating exact backscatter (P_{11}), but largely because natural plumes are heterogeneous mixtures. The blending of super-coarse iron-rich dust with finer, less absorptive dust naturally dilutes the macroscopic lidar ratio down closer to observational realities (e.g., 50–80 sr).

However, to the reviewer's second point regarding operational retrievals: if an elastic-backscatter lidar algorithm (like CALIOP or CATS) encounters a dust layer with heavily suppressed backscatter, it would indeed likely misclassify it. Both absorbing carbon (smoke) and iron-rich super-coarse dust act to suppress backscatter relative to extinction. Consequently, standard typing algorithms observing this suppressed signal will naturally assume the extinction is associated with a mixture of dust and carbon (e.g., classifying it as "polluted dust" or "dust and smoke"). Our theoretical framework demonstrates that iron-oxide absorption in coarse dust can produce an overlapping optical signature, creating a fundamental ambiguity where pure, iron-rich coarse dust mimics a polluted mixture. We have ensured this mechanism is clearly emphasized in the text.

5. On page 18, the authors state that an automated classification algorithm observing high S with low δ (≈ 0.18) may misclassify iron-rich, super-coarse dust as a mixture of dust and smoke or pure pollution. However, Figure 8e shows that the FENNEC depolarization ratio spans a broad

range (~0.1-0.45 at 355 nm depending on iron-oxide fraction). Could the authors specify the exact combination of particle size distribution parameters (e.g., median radius, geometric standard deviation) and iron-oxide volume fraction that produces $\delta \sim 0.18$? Tying the optical ambiguity claim to a concrete point in the parameter space would strengthen the argument and allow readers to assess whether this scenario is representative of typical Saharan dust conditions or an extreme edge case. Given that there are also extensive wildfire sources which may mix with Saharan dust, it is possible that the non-iron component has substantial amounts of carbon? If so, based on specific size distributions of this mixed fraction, how would this lead to different ranges specifically at 355 nm? Would including data at 532 nm and 1064 nm help clarify these mixtures?

We thank the reviewer for pushing us to make this optical ambiguity claim more concrete, as doing so actually highlights how common this scenario may be.

To the first point: in our model, a depolarization ratio of approximately 0.18 (0.18139834) at 355 nm is achieved by combining the super-coarse FENNEC size distribution with an iron-oxide volume fraction of 3%. We have updated the text in Section 4.3 to explicitly define these parameters. Since 3% iron oxide is a highly representative value for typical Saharan dust, this means the suppressed depolarization signature may in fact be a common optical state for pristine, super-coarse plumes near the source.

This leads directly to the reviewer's excellent point regarding wildfire emissions and carbon mixing. Carbon strongly absorbs in the UV, suppressing the depolarization ratio and inflating the lidar ratio in a manner very similar to iron oxide. This is the "optical ambiguity" our paper seeks to highlight. Our results demonstrate that pure, standard Saharan dust (at 3% iron oxide) with an intact super-coarse mode mimics a polluted signature without any carbon present.

While explicitly modeling the optical properties of external dust-carbon mixtures falls outside the scope of this theoretical study, we completely agree that multiwavelength data is the key to separating them. As we discuss in Section 4.4, utilizing data at 532 nm and 1064 nm to calculate the depolarization color ratio (e.g., 1064/355) is specifically designed to help disentangle these ambiguities, as carbon and iron oxide possess fundamentally different spectral absorption slopes.

The manuscript by Anthony La Luna and co-authors investigates the influence of absorption on the aerosol optical properties like the linear depolarization ratio, the extinction-to-backscatter ratio (so-called lidar ratio) and the single scattering albedo. Their work focuses on active remote sensing observations with lidar (180 degree scattering angle) for the non-spherical dust particles. The topic is relevant and the manuscript makes a significant contribution. Unfortunately, the authors were not using a correct citation key that messed up the in-text references. An issue which should have been eliminated before submission. Besides of this, some major revisions are required before publication.

Major points

1. The citation style (probably originating from a citation style using numbers), does not fit to ACP. In latex, you have two comments

   \citep{LaLuna2025} – (La Luna et al., 2025)

   and \citet{LaLuna2025} or \cite{LaLuna2025} – La Luna et al. (2025)

   Especially, for multiple citations as at the beginning of your introduction, you should use the first comment and all citations will be combined in one. It really disturbs reading, if the citations are not set properly, especially in long lists of refences like in the introduction and in case of duplications as in lines 80-83.

Fixed.

2. The asymptotic behavior discussed in Sect. 3 widely seems to be textbook-like. You conclude, that for the asymptotic behavior the shape model does not matter. And in fact, the conclusions hold for spherical particles as well. For very low size parameters, the depolarization ratio is zero and for very large ones as well. Similar conclusions can be drawn for the lidar ratio. In between, the shape matters as you show with the two non-spherical particle models. Epecially, the comparison to spherical particles shows that the shape matters for in between size parameters. Thus, you should include spherical particles in your considerations as well. Therefore, this asymptotic behavior is not new, but very general and should hold for all particles shapes. Actually, this behavior does not help us very much in finding the best description for mineral dust, but sets a frame for all particle shape models what they should reach in the asymptotic behavior. By including spherical particles, it gets even more obvious that the asymptotic behavior alone is not concessive for the correct description of the optical properties of mineral dust.

We sincerely thank the reviewer for this feedback. We appreciate the suggestion to include spherical particles to help illustrate this baseline. Because perfect spheres exhibit exactly zero depolarization across all size parameters due to their complete symmetry, adding them to the plots results in a flat line at zero. For this reason, we opted to use spheroids to represent a smooth, non-spherical baseline that still exhibits a transition.

However, the reviewer's comment helped us realize that we did not adequately highlight why our specific findings diverge from standard textbook cases. We completely agree with the reviewer that the geometrical-optics limits of extinction ($Q_{ext} \rightarrow 2$) and single-scattering albedo ($\omega \rightarrow 1/2$) for absorbing particles are classical, well-documented textbook results.

To the best of our knowledge, the corresponding exact-backscatter limits of the linear depolarization ratio ($\delta \rightarrow 0$) and the lidar ratio ($S \rightarrow$ large) for large absorbing particles have not previously been derived in prior classical treatises or textbooks. While the suppression of depolarization by absorption has been observed numerically for ice crystals at 2 µm (Kustova et al., 2022) and measured in the laboratory for hematite at 180° (Miffre et al., 2023), these works did not provide a theoretical explanation for the phenomenon.

The central novelty of our work is providing this explicit analytic derivation. We demonstrate that at exact backscatter, diffraction vanishes, and strong volumetric absorption attenuates internally transmitted rays. This leaves only the first-surface Fresnel reflection, which produces no cross-polarization at near-normal incidence, ultimately yielding $\delta \rightarrow 0$, a suppressed $P_{11}(\pi)$, and a correspondingly large S.

We have revised Section 3.3 to explicitly acknowledge the established textbook baselines the reviewer pointed out, cite the previous numerical and laboratory observations, and cleanly state the specific novelty of our derivation.

"It is important to place these findings within the context of established scattering theory. The geometrical-optics limits of extinction and single-scattering albedo for absorbing particles are classical results (e.g., van de Hulst, 1957; Bohren and Huffman, 1983; Mishchenko et al., 2002). To our knowledge, however, the corresponding exact-backscatter limits of the linear depolarization ratio and lidar ratio for large absorbing particles have not previously been derived. The suppression of depolarization by absorption has been observed numerically for ice crystals at 2 µm (Kustova et al., 2022) and measured in the laboratory for hematite at 180.0° (Miffre et al., 2023), in both cases without theoretical explanation.

The vanishing of diffraction at exact backscatter and the absorption of internally transmitted rays leave only the first-surface Fresnel reflection. Because this reflection produces no cross-polarization at near-normal incidence, the result is $\delta \rightarrow 0$, suppressed $P_{11}(\pi)$, and correspondingly large S. The convergence of $\delta \rightarrow 0$ observed here at large size parameters is therefore not a general size rule for all nonspherical particles, but strictly an absorption-driven phenomenon that effectively overrides the expected shape-dependent plateau seen in non-absorbing limits.

3. Tabular values of the optical properties shown in Fig 6 and 8 at the 3 lidar wavelengths for absorbing and non-absorbing aerosols are recommended. Only in this way, you are able to compare it lidar field observations. Furthermore, you might add some lidar field observations to your respective plots (Fig. 6, 8, 9, 10).

We thank the reviewer for this suggestion and completely agree that facilitating direct comparisons with lidar field observations is a primary goal of this work.

Providing the full tabular data for Figures 6 and 8 would require listing over 1,000 individual data points, which we felt would be unwieldy and difficult for readers to utilize within the static format of a manuscript or supplementary PDF. To ensure the community can easily access, plot, and compare our exact results against field data, we have made the complete, machine-readable dataset publicly available as a NetCDF file (cited in the Data Availability statement).

However, we recognize the value of having quick reference points directly in the text. To accommodate this, we have added a new condensed summary table (Table S1) to the supplementary section of the manuscript. This table provides the exact optical properties (S, δ, ω) at the three lidar wavelengths for a few representative anchor points discussed in the text: 0% iron oxide, 3% iron oxide, and 5% iron oxide at specific, representative median radii (1 μm, 3 μm and 5 μm). We believe this strikes the right balance between providing immediate comparative values for the reader and keeping the manuscript streamlined.

We agree with the reviewer that grounding our theoretical curves in field observations is crucial for contextualizing the results. To ensure the most accurate comparison for our modeling of Saharan dust, we have utilized the SALTRACE campaign observations over Barbados (Haarig et al., 2017), which represent a well-characterized, purely transported Saharan dust layer. We have overlaid the mean and standard deviation of these triple-wavelength measurements directly onto our spectral analysis plots. Specifically, we have added these observational bounds to Figures 6 and 8, where the axes directly align with the spectral lidar measurements. We opted not to overlay these single-point field observations on the remaining figures (e.g., the conceptual framework in Fig. 9 or the PSD curves in Fig. 7), as it would not be physically meaningful in those specific parameter spaces. We thank the reviewer for this suggestion, as plotting the SALTRACE data directly over our theoretical curves substantially strengthens the bridge between our geometric optics framework and real-world remote sensing.

4. Please provide more details about the in situ size distributions, e.g., sampling location, sampling altitude etc.

   Sect 4.3 and Fig 7: Which radius is provided by the in situ observations? And how it is converted to serve as input for the calculations? For non-spherical particles, the size definitions might be ambiguous as some literature report volume equivalent diameter, others use the geometrical diameter or even other definitions. Please describe which size definition was used and how – if necessary – it was converted.

   It puzzled me a lot to see the AERONET PSD in Fig 7 which goes well beyond 50 µm in radius. In your text, you state that it is truncated at 10 – 15 µm, which would be shortly after the peak. How can this behavior be explained?

We thank the reviewer for their careful attention to detail.

First, regarding the AERONET distribution in Figure 7: AERONET inversions are indeed algorithmically truncated at a radius of 15 µm. The extended tail beyond 50 µm in our original Figure 7 was an artifact of plotting the continuous, unbounded Dubovik bimodal lognormal equation, rather than plotting the strict retrieval limits. We have corrected Figure 7 and the corresponding calculations to strictly truncate the AERONET distribution at 15 µm to accurately reflect the operational retrieval.

Additionally, applying this strict mathematical cutoff resulted in virtually no changes to the calculated bulk optical properties, but we did update Figure 8 to reflect the values of the truncated AERONET PSD. This is because the standard AERONET bimodal fit inherently lacks any meaningful super-coarse volume fraction to begin with.

Second, we have updated Section 4.3 to explicitly define the size metrics used. To interface correctly with the TAMUdust2020 database, all particle sizes are defined by their volume-equivalent radius. The FENNEC and AER-D in situ distributions utilized in this study were previously processed into volume-equivalent radii in their respective foundational papers, requiring no further conversion on our part.

The FENNEC data represents near-source, heavy dust sampled via aircraft at altitudes below 3 km over the Sahara (Mauritania and Mali), while the AER-D data represents gravitationally settled, transported dust sampled via aircraft off the coast of Cabo Verde.

5. Fig 2 is only partly showing the content which is described in the text. Not data at 355 and 1064 nm is shown. Be sure that the figures fit to the text. Such issues make the review process more difficult.

We apologize for the confusion between the text and Figure 2. To prevent the figure from becoming visually cluttered and to best highlight the core single-particle asymptotic behavior, we elected to show only the 532 nm results in the main manuscript. We have corrected the text to accurately reflect this, and we have added the corresponding 355 nm and 1064 nm plots to the Supplementary Material (Figure S1) for completeness.

"Figure 2 shows the single-scattering properties of hexahedral dust particles computed using the TAMUdust2020 database as a function of size parameter ($x = 2\pi r / \lambda$) and iron-oxide content for three lidar wavelengths at 532 nm from Rayleigh $x=10^{-2}$ to GO regime $x=10^{3}$ (results for 355 nm and 1064 nm exhibit similar asymptotic behavior and are provided in Figure S1)."

Minor comments

- As you mentioned, the linkage of dust optical and microphysical properties is an emerging field and the review process serves to update you with some recent relevant literature in the field. I would consider the work of
    - Yuyang Chang et al., 2025 (ACP) and 2026 (JGR) about the spectral modelling of dust particles.
    - Sofia Gomez Maqueo Anaya et al. 2024 (GMD), 2025 (ACP) about mineralogically resolved dust transport modelling and its implications on optical properties.
    - And the recent work of Claudia Di Biagio (2026, ACP) on dust properties at longer wavelengths and Moritz Haarig (2025, ACP) on the spectral slope of the lidar ratio.

We sincerely thank the reviewer for providing these excellent recent references. We completely agree that ensuring our manuscript reflects the absolute latest advancements is critical.

We have carefully reviewed these papers and incorporated them into the manuscript to better contextualize our work. Specifically, we have added Chang et al. (2025, 2026) and Gomez Maqueo Anaya et al. (2024, 2025) to the Introduction to highlight recent strides in mineralogically resolved modeling and spectral property derivation. We have also integrated Di Biagio (2026) into Section 2.3 to update our discussion of refractive index parameterizations at longer wavelengths, and we have included Haarig (2025) in Section 4.2 to support our analysis of the spectral variation of the lidar ratio.

- Looking at your figures 4+5, one might argue, that the absorption plays a role, but all your conclusions hold also for the non-absorbing particles. After reaching a peak, the depolarization ratio is decaying and the lidar ratio is increasing for increasing size. The only difference is that the non-absorbing particles have to be larger to see the same effect.

Therefore, I would not say that it is an absorption effect alone (e.g., L303). However, in real world, there the PSD is given, the optical properties change for stronger absorption as you show in your next section.

We thank the reviewer for this careful observation, which highlights a need for clearer terminology in our manuscript. The reviewer correctly notes that the curves for our "non-absorbing" case eventually exhibit the same decay in depolarization and increase in lidar ratio at very large sizes.

However, this is because our 0% iron oxide case is not perfectly non-absorbing (k=0). As detailed in Table 1, the host mineralogy still possesses a small background imaginary refractive index (e.g., k=0.0004 at 532 nm). The geometrical-optics attenuation of internal rays is governed by the product of the imaginary refractive index and the size parameter ($k \cdot x$). For very small k, a much larger x is required to achieve the necessary attenuation, which perfectly explains why the decay feature shifts to larger sizes for the 0% iron oxide case but does not disappear.

To verify this, we can look to the fundamental Geometrical Optics (GO) limits established in Section 3.2 of our manuscript. As shown in Equations 15-18, the suppression of depolarization relies on the attenuation of internal cross-polarizing rays. In the true absence of absorption (k=0.0i), internal rays are never attenuated, regardless of how large the size parameter becomes. Consequently, the ray-tracing cross-polarization terms (S_3^R, S_4^R) do not vanish, and the depolarization ratio does not decay, but instead asymptotes to a high, constant value dictated by the particle's macroscopic shape.

Therefore, the decay feature is indeed a fundamentally coupled effect of both size and absorption; it cannot occur without absorption. We have revised the manuscript to replace the term "non-absorbing" with "weakly absorbing" where appropriate, and we have explicitly clarified the $k \cdot x$ coupling mechanism in the text to prevent this confusion.

- Why do you have chosen those specific in situ size distributions? General dust size distribution at different stages of transport are available, see e.g., Formenti & Di Biagio (2024).

We thank the reviewer for directing us to the comprehensive work by Formenti & Di Biagio (2024), which provides excellent context for dust size distributions at various transport stages.

Our primary rationale for selecting the FENNEC, AER-D, and AERONET distributions was to establish a specific comparative framework grounded in campaigns that also featured collocated or highly relevant depolarization ratio measurements. We aimed to contrast a pristine source-region distribution containing extreme super-coarse modes (FENNEC), a mid-range transported plume where gravitational settling has altered the coarse tail (AER-D), and a standard remote-sensing inversion product (AERONET) to highlight the optical ambiguities discussed in the text.

We agree that framing our selected case studies within the broader context of generalized dust transport is important. To address this, we have added a discussion of Formenti & Di Biagio (2024) to Section 4.3 to better contextualize our selected distributions within the wider literature on dust transport stages.

“While generalized size distributions for dust at various stages of transport have been documented in recent literature (e.g., Formenti & Di Biagio, 2024), we specifically selected the FENNEC, AER-D, and AERONET distributions for this analysis. These datasets allow us to contrast a source-region profile, a mid-range transported plume, and a standard remote-sensing retrieval, while leveraging the availability of relevant depolarization measurements necessary to validate our theoretical framework.“

- L287 EarthCARE’s lidar measures the extinction and lidar ratio directly (HSRL) and needs no assumptions about the lidar ratio.

  Changed to: “Similarly, the lidar ratio (S) demonstrates a sensitivity to absorption when r_m is large. For large particles, enhanced absorption suppresses backscattering more effectively than extinction, leading to a substantial increase in S. This effect is most pronounced at 355 nm, where iron-oxide absorption is strongest (Figure 1). Given that S can span multiple orders of magnitude depending on the source region and PSD, the assumption of a uniform lidar ratio in elastic lidar retrievals, particularly for missions such as CALIPSO, may introduce significant uncertainties. Conversely, High Spectral Resolution Lidar (HSRL) missions, such as EarthCARE, measure extinction and backscatter independently, allowing them to directly observe this wide variability in S without relying on assumed values. Furthermore, the single-scattering albedo decreases monotonically with increasing size and iron content, approaching a theoretical asymptote determined by the interplay of particle shape and the imaginary refractive index.”

- L314-315 Again, the HSRL technique is wrongly conceived. A lidar ratio has to be assumed for backscatter lidars (like CALIPSO), but not for HSRL.

  Changed “HSRL” to “backscatter lidars”.

- Sect 3.3: Please consider also the differences between the two models, e.g., for large size parameters the lidar ratio shows a plateau for spheroids and an increase for irregular hexahedra. So, the asymptotic lidar ratio is only present for spheroids, isn’t it? Or do we have to go to even larger size parameters for the irregular hexahedra to see a plateau?

We thank the reviewer for this astute observation regarding the behavior at the extreme upper limits of the size parameter. It is true that spheroids appear to level off while hexahedra continue to climb at the edge of the plotted domain.

However, we are hesitant to definitively characterize the spheroid behavior as a true, permanent "asymptotic plateau," nor can we precisely predict where hexahedra might flatten out. This is due to two factors. First, at these extreme size parameters ($x>10^3$), we approach the computational limits of both the TAMUdust2020 and triaxial databases. Second, in the geometrical-optics limit for strongly absorbing particles, internal refracted rays are entirely attenuated, meaning exact backscatter is governed almost exclusively by first-surface Fresnel reflection. The differing surface symmetries of the two models (smoothly curved spheroids versus faceted, sharp-edged hexahedra) naturally lead to different backscattering efficiencies at this extreme limit.

Furthermore, supplementary tests using perfectly spherical particles indicated that infinite-size behavior is highly sensitive to the exact magnitude of the imaginary refractive index; if absorption is pushed to theoretical extremes, particles begin to act somewhat "metallic," causing reflection (and backscatter) to increase, which drives the lidar ratio back down. Therefore, while the onset of the enhanced lidar ratio is a shape-insensitive feature of absorbing coarse dust, the precise trajectory and potential plateauing at extreme macroscopic sizes remains highly shape-dependent and computationally constrained. We have added a brief discussion of this nuance to Section 3.3:

While the transition into this high-absorption, high-lidar-ratio regime is robust across both models, their exact trajectories at the extreme upper bounds of the size parameter ($x>10^3$) do diverge, with spheroids appearing to level off while hexahedra continue to increase. In the geometrical-optics limit, internal rays are effectively absorbed, rendering exact backscatter highly dependent on first-surface Fresnel reflection. Consequently, the distinct surface geometries, such as smooth curvature for spheroids versus flat facets and sharp edges for hexahedra, yield different backscattering efficiencies. It is difficult to define a universal asymptotic plateau beyond this point due to the computational limits of the underlying databases and the extreme sensitivity of surface reflection to the exact imaginary refractive index. Thus, while the physical mechanism driving the suppression of depolarization and the enhancement of the lidar ratio is independent of shape, the absolute magnitude at extreme macroscopic sizes remains shape-dependent.

- L258-259: Actually, I don't see so much of the difference for absorbing and non-absorbing particles. Even for non-absorbing particles the depolarization ratio decreases for spheroids, the particle has to just a bit larger to bring the depolarization ratio to zero. Please be more careful in your statements about the effect of absorption.

We thank the reviewer for this careful observation of the spheroid data. The reviewer is correct: the depolarization ratio for our 0% iron oxide case does eventually decrease toward zero at very large size parameters.

However, as detailed in our response to a previous comment, this decrease is still fundamentally driven by absorption. Our "0% iron oxide" baseline is not strictly non-absorbing; the host mineralogy retains a small imaginary refractive index (e.g., k = 0.0004 at 532 nm, per Table 1). The attenuation of cross-polarizing internal rays is governed by the particle's optical thickness, which scales proportionally with the product of the size parameter and the imaginary refractive index. Therefore, even a weakly absorbing particle will eventually experience complete internal attenuation if the size parameter becomes large enough, which perfectly explains why the 0% iron oxide curve simply shifts to the right rather than disappearing.

If the particle were strictly non-absorbing (k=0), internal rays would persist regardless of size, and the depolarization ratio would asymptote to a constant, shape-dependent value without decaying. We agree, however, that our original phrasing in the text was imprecise and incorrectly implied that the 0% case lacked this decay feature entirely. We have revised the text around Lines 258-259 to clarify this coupled optical thickness mechanism:

“More importantly, the GO regime (x\gg1) exhibits highly shape-insensitive asymptotic behavior when absorption is introduced. For strictly non-absorbing particles (k=0), shape significantly dictates the permanent saturation point of the depolarization ratio; for instance, the absolute magnitude of δ is generally lower for spheroids than for hexahedrals due to differing degrees of symmetry and internal ray-tracing dynamics. However, the introduction of absorptive iron oxides fundamentally overrides these shape-specific differences at large sizes. This is true even with the weak background absorption present in our 0\% iron oxide baseline. As particle size and absorption increase, both shape models demonstrate a distinct peak in δ followed by a sharp, asymptotic decrease. This confirms that the suppression of depolarization in super-coarse dust is a fundamental consequence of volumetric absorption attenuating the internal reflected rays that typically generate cross-polarized signals, completely independent of the particle's external geometric smoothness or sharpness.”

- L294-296 Please check Vesselovskii et al., ACP 2020 for lidar ratios at 355 nm.

We thank the reviewer for directing us to Veselovskii et al. (2020). Their reported lidar ratios at 355 nm (typically 50–80 sr) provide crucial real-world context for our theoretical findings. While our model predicts extreme values (exceeding 300 sr) for purely super-coarse, highly absorptive iron-rich dust, we agree that these values are rarely, if ever, observed in bulk atmospheric measurements. We attribute this discrepancy to two main factors: the inherent difficulty idealized shape models face in accurately simulating exact backscatter (P11) values, and the reality that natural plumes are heterogeneous mixtures that dilute these extreme optical signatures. We have updated the manuscript to explicitly compare our theoretical limits with the Veselovskii et al. (2020) observations and discuss these limitations.

We have added the following to the end of section 4.3:

"It is important to contextualize these modeled values against field observations. For instance, Veselovskii et al. (2020) report typical bulk dust lidar ratios at 355 nm in the range of 50 to 80 sr. The divergence between our theoretical upper limits (>300 sr) and these observational baselines likely stems from two possible factors. First, accurately computing the exact backscatter phase matrix element (P11) remains a notoriously difficult challenge for idealized shape models, which can sometimes underestimate backscatter and artificially inflate the lidar ratio. Second, natural dust plumes are rarely composed entirely of pure, iron-rich super-coarse particles. Rather, they are physically and chemically heterogeneous mixtures. The blending of this extreme absorbing fraction with finer, less absorptive dust and background atmospheric aerosols dilutes the macroscopic lidar ratio down to the ranges commonly observed by operational systems."

- Sect 4.2 & Fig 6: I would recommend to show 1, 3 and 10 µm instead of 1, 5 and 10 µm, because a logarithmic spacing (here approx. factor 3) covers better the relevant processes than the linear spacing. The results at 5 and 10 µm are too similar. Furthermore, I would recommend to highlight the 3 relevant lidar wavelengths in the plot.

Done. Added vertical dashed lines to 355, 532, and 1064 nm wavelengths.

- Please specify the shape model used for your results also in the figure captions of Fig 6 onwards.

  Done.

- Fig 9a is not understandable from the caption and the text. Which size distributions are shown?

Changed caption to "Modeled spectral depolarization features of mineral dust utilizing the hexahedral TAMUdust2020 shape model database. **(a)** Single-particle depolarization ratio as a function of size parameter for non-absorbing (solid lines) and absorbing (dashed lines) particles at 355, 532, and 1064 nm (left vertical axis). Overlaid are lognormal particle size distributions with volume median radii of r_m = 1 µm, 3 µm, and 5 µm (right vertical axis) to illustrate the size-parameter sampling regimes at different wavelengths. **(b)** Resulting bulk spectral depolarization ratios, highlighting the "inverse-v" dependence for the different size distributions and absorption properties."

- L357-359 It highlights the necessity of using multiwavelength depolarization information for aerosol typing as started by Shang et al., AMT 2026.

Added.

- Fig 10: It is sufficient to show the y-axis until a value of approx. 5 to enlarge the relevant part. Higher depolarization color ratios are rarely observed underlying the fact that super coarse dust alone is rarely found in the atmosphere. Again, it would be valuable to compare to lidar field observations to set your modelling into context.

Fixed. Reduced scale to 5 and added SALTRACE observations of DPR to figure 10.

After all my critical points, it is still a very good manuscript which will evolve after the review process to an excellent paper. Well done.