Supplementary material

Supplement S1: Cone of influence

Physical signals are inherently finite in length. For data points located sufficiently close to the boundaries of a signal, the wavelet may extend beyond the length of the signal, producing unreliable coefficients. The larger the scale of the wavelet, or the larger the period analyzed, the more data points are concerned. To tackle this issue, practitioners employ the concept of the cone of influence (COI). For each data point, the COI refers to the boundary of the time-frequency plane region that encompasses all wavelet coefficients affected by the value of that data point (Mallat, 2009). By representing the COI at the edges of a signal on a scalogram, regions deemed affected by border effects can be delineated. An example of this procedure is provided for the Morlet wavelet in Fig. 1d.

The COI allows some latitude in the definition of the affected region. Torrence & Compo (1998) outlined this region based on the attenuation of the wavelet's amplitude from its peak value. They identified affected areas as those where the value of the wavelet's envelope decays to a fraction $1/e^2$ of its peak value. In this study, we applied a slightly more stringent definition, excluding coefficients for which more than 2 % of the wavelet extends beyond the signal border. The exceeding part of the wavelet was defined as the portion of the area under the wavelet curve that lies outside the signal.

Supplement S2: Wavelet choice for period bands

Wavelet choice influences the structure of period-band peaks and should therefore be considered during coefficient selection. The DOG2 wavelet provides a robust starting point for ecological signals, as it reliably highlights trend discontinuities and seasonal peaks (Mallat, 2009). While the DOG1 wavelet has the same power for discontinuities, it is less effective for peaks or drops in the signal if they are not pronounced enough. For signals dominated by oscillatory behavior, the Morlet wavelet is appropriate, although its complex nature requires separating real and imaginary parts during WAI interpretation (Fig. S4). Higher-order derivatives of the Gaussian (e.g. DOG6) can serve as real-valued alternatives to the Morlet wavelet.

Supplement S3: WT adjustment (third step)

Since many wavelets share common oscillatory behavior, patterns tend to be detected by a variety of wavelets (Torrence & Compo, 1998). However, inadequately suited wavelets might result in substantial undesired area contributions (e.g. the left and central lobe of the wavelet in Fig. 2), or inversely, might fail to encapsulate the entire pattern present in the signal (e.g. the right lobe of Fig. 2, if we are interested in the entire growing season). Both scenarios introduce uncertainties into interpretations given to wavelet coefficients. To mitigate this, the WT can be adjusted to align coefficient values with interpretations. Four such adjustments are suggested:

1. **Modifying the wavelet used** provides a means for matching the wavelet filter to the pattern sought in the signal, as visualized using the WAI. Non-admissible wavelets, such as functions with non-zero mean (Grossmann & Morlet, 1984), can also be selected. In these cases, the operation becomes a filtering process using the selected function rather than a wavelet transform, but interpretations given using the WAI remain unchanged. As a general guideline, the Gaussian function is suitable for evaluating smoothness and values reached by peaks, the DOG1 wavelet for detecting discontinuities and describing rates of change, and the DOG2 wavelet for detecting and assessing the importance of drops in values.
2. **Adjusting the period of analysis** allows the wavelet to encapsulate solely relevant signal segments. Although adjacent periods yield only slight variations in coefficient values, careful consideration should be given to defining the period of analysis to ensure confidence in interpretations. This can be achieved by aligning the support of the wavelet, or directly the period for sinusoidal wavelets, with the duration of the event under study.
3. **Normalizing signals** eliminates the influence of absolute values achieved across different periods. Several normalization options are available, such as dividing the signal by its amplitude or by its sum of values for each period. The former can be harnessed to describe rates of change at the start and end of a trend, provided that sufficiently low periods are used, while the latter expresses coefficients as proportions of the signal's total value.
4. **Combining coefficients** through simple mathematical operations can expand the range of motions studied without adding complexity to the method. One of the most straightforward operations is division, as it allows for the comparison of two sets of values. This is particularly useful for analyzing relative drops or growths in the signal.

Supplement S4: Additional indicators

1. **Yearly values**: 1-year peak of the Mexican Hat wavelet (or the Gaussian function). It serves as a proxy for an annual summation, applying a bell-shaped weighting factor. This approach could prove useful, for instance, if the signal follows a Gaussian shape.
2. **Steepness of decrease**: Same approach described for IRise but applied at the end of the GS.
3. **End of season values**: 6-month November peak (Fig. 5c). The Haar wavelet (Fig. S2) can be used to avoid applying Gaussian weighting. Wavelet adjustments akin to those introduced for IPeak can be incorporated here.
4. **Proportion of values at the onset of the GS**: same procedure as for IPeak on a previously normalized signal by dividing each year's GPP values by its annual sum. Alternatively, the indicator can be defined by the ratio of the 6-month April peak (Fig. 5a) and the annual GPP values (1 year coefficient or yearly sum). This indicator is proportional to the mid-season decline in GPP values, resulting in a strong positive correlation ($r = 0.74$, p-value < 0.001) with indicator IDrop.

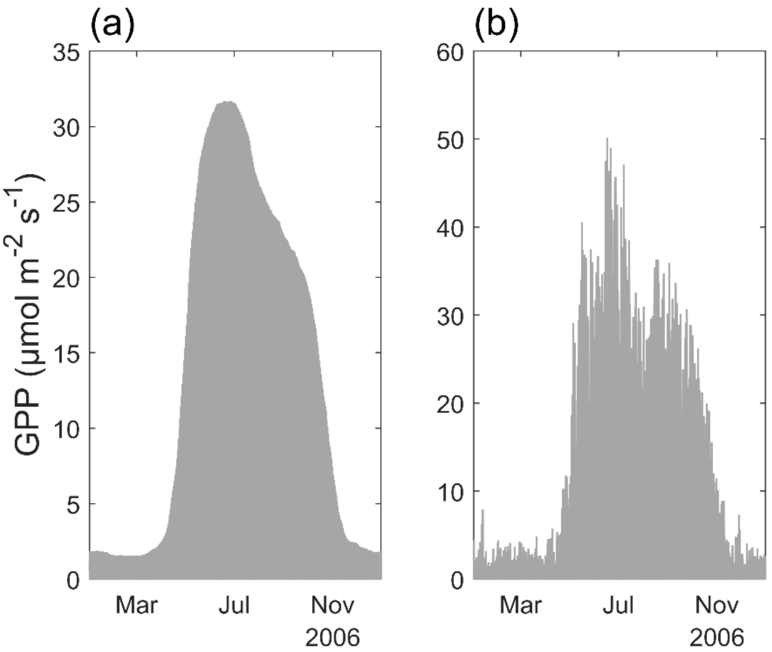

Figure S1: Example of the FR-Hes GPP signal for the year 2006 (a) as modelled according to incoming radiation and phenology (using 30-day sliding windows), (b) as measured on the eddy-covariance tower (restricted to positive values only for readability). Beech is a deciduous species, producing new leaves in spring that last all summer and fall in autumn. As such, its photosynthetic activity (translated by the GPP signal) follows seasonal trends, reaching near-zero values during winter and maximal values at the beginning of summer.

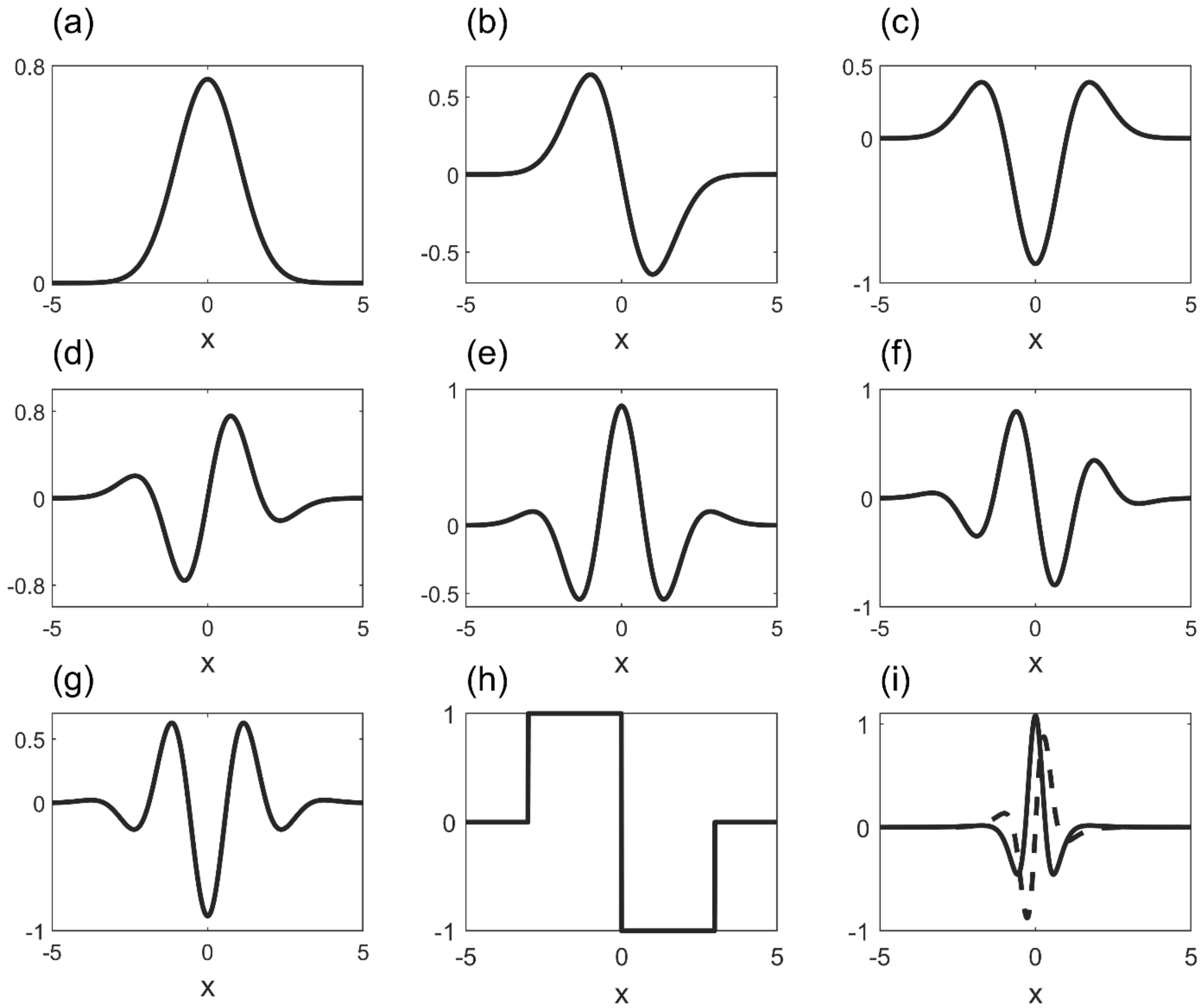

Figure S2: Illustration of useful analyzing functions for continuous wavelet transforms. (a) Gaussian function, derivatives of the Gaussian function, with (b) indicating the first order and each subsequent letter refers to an increased order of derivation by one, (h) Haar wavelet, (i) Paul wavelet of order four.

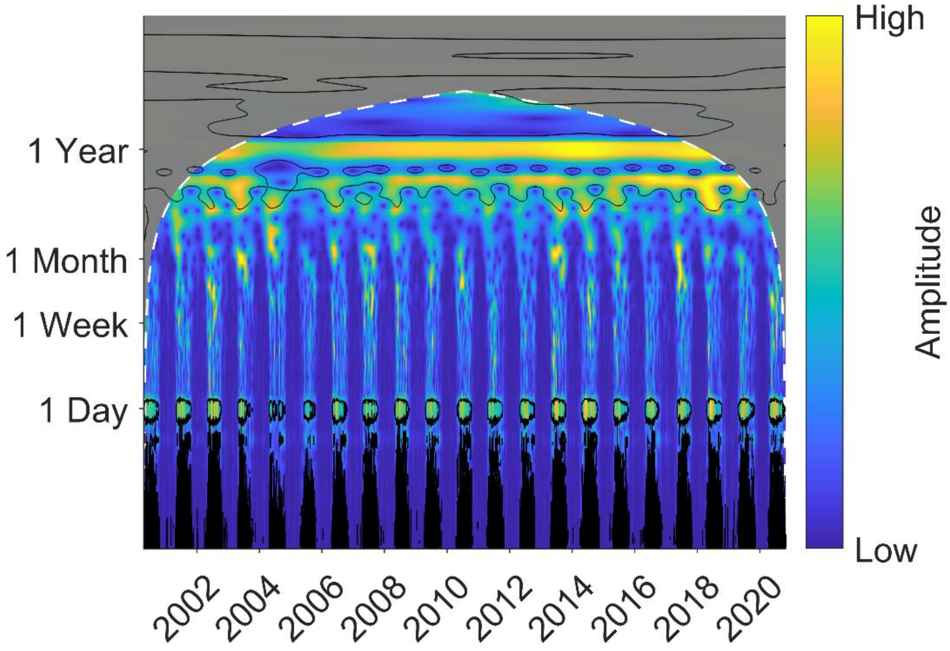

Figure S3: Scalogram depicted in Fig. 1d, for which coefficients are divided by their maximal value inside each period band.

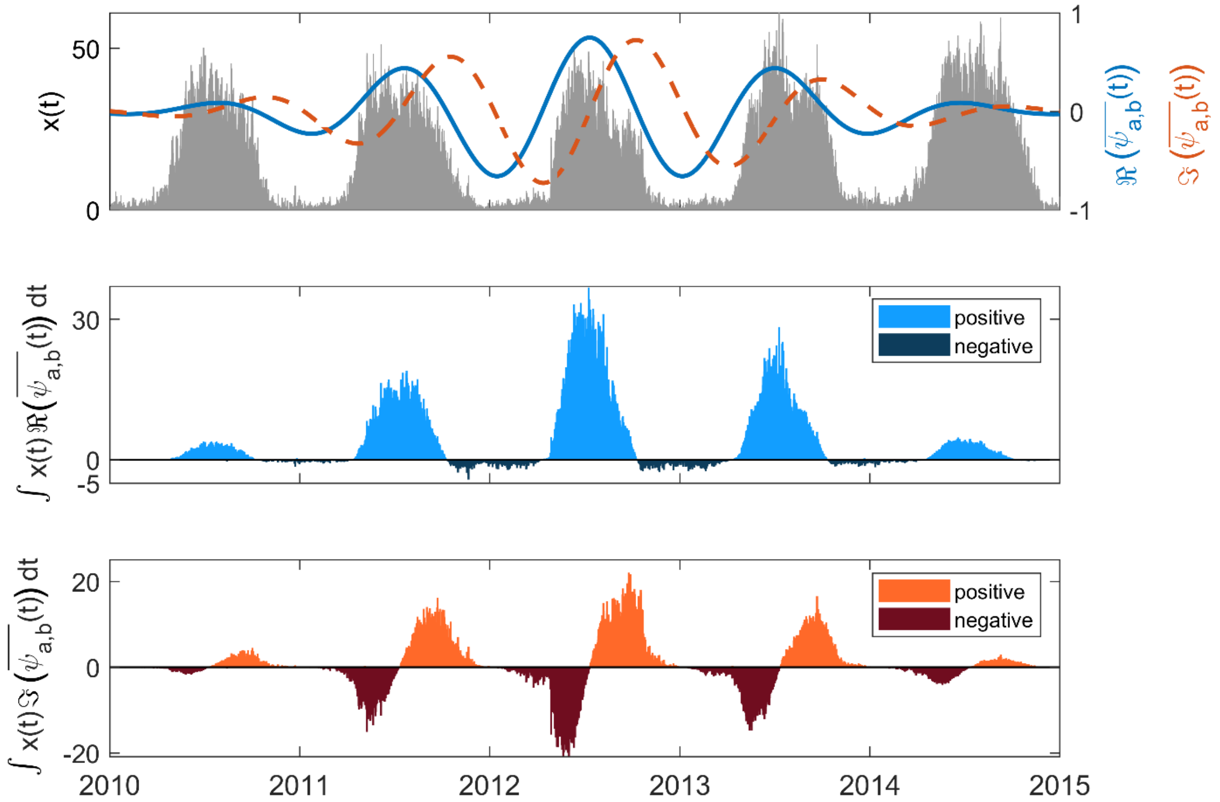

Figure S4: Detailed computation of a wavelet coefficient using the complex Morlet wavelet at a 1-year period. The blue color refers to the real part, while the red color denotes the imaginary part. In this example, the real part of the wavelet extracts the annual trend of the GPP signal. The Imaginary part of the wavelet coefficient is close to zero. As the wavelet is translated, the relationship of information capture is inverted, yielding a consistently smoothed result on the 1-year periodic band throughout the translation process.

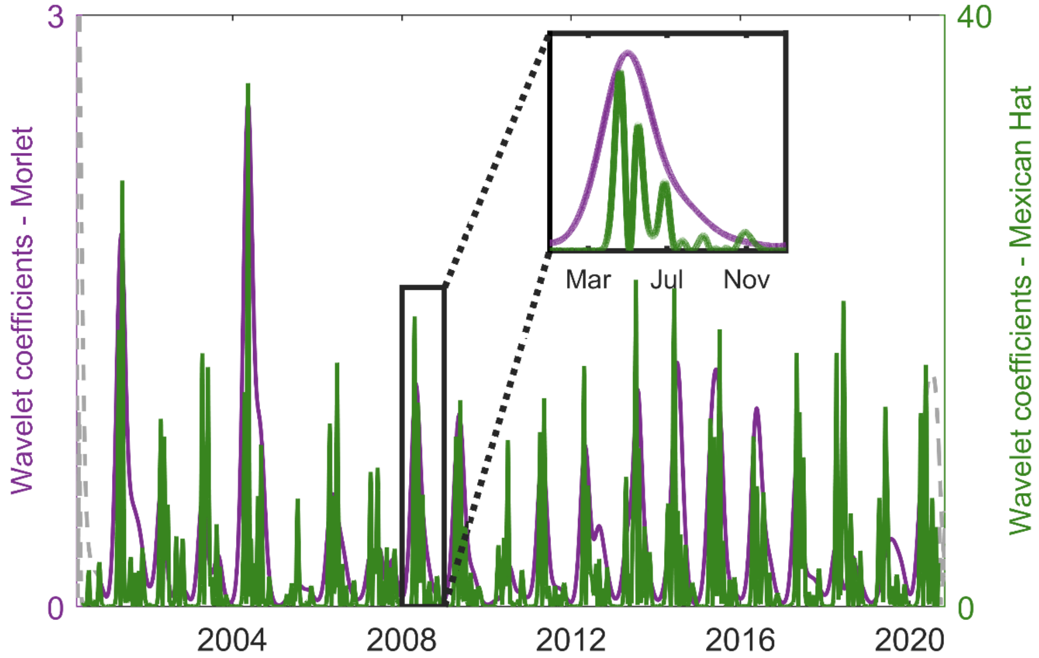

Figure S5: Amplitude of wavelet coefficients using the Morlet wavelet (purple) and the Mexican Hat wavelet (green) at the 2-month periodic band. Dashed lines represent coefficients affected by edge effects. Repeating peaks in March, June and November found at 6 months are still discernable.

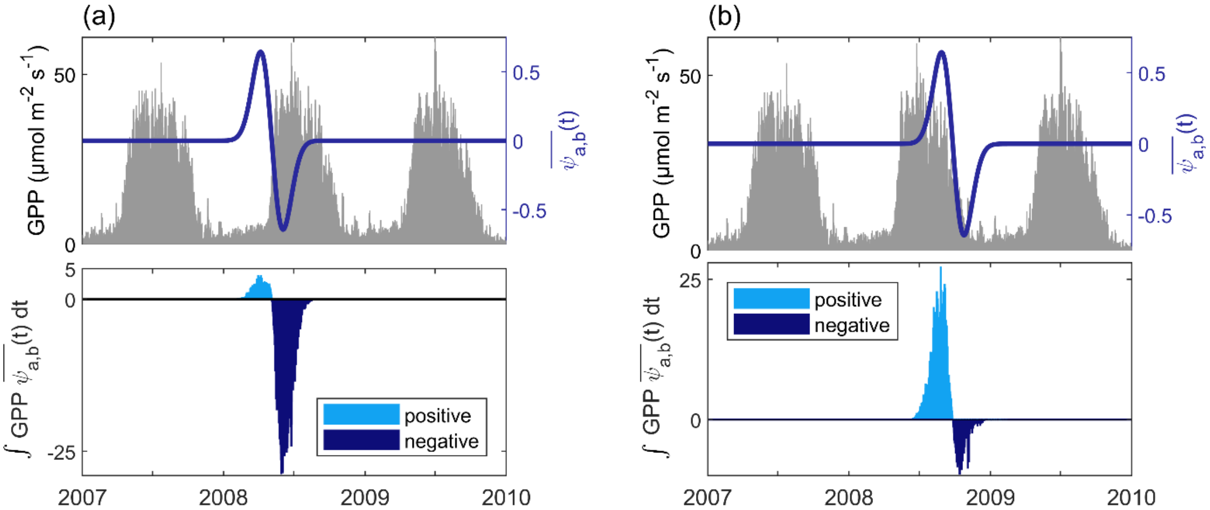

Figure S6: Computation steps for both wavelet coefficients involved in defining IDrop using the DOG1 wavelet. (a) April peak at a 6-month period, and (b) November peak at a 6-month period.

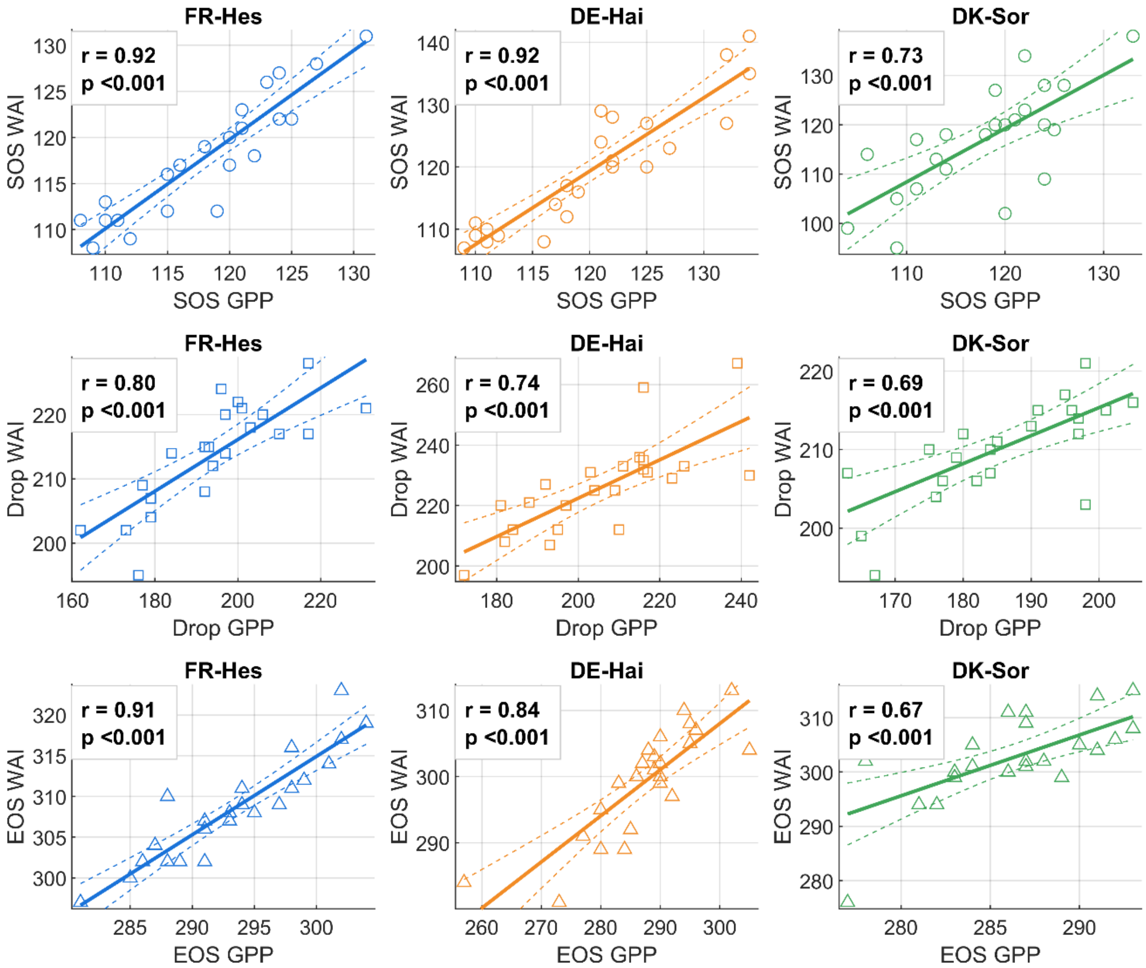

Figure S7: Benchmarking of three phenological metrics: dates of carbon uptake start (CUS), mid-season drop (MSD) and carbon uptake end (CUE). WAI-derived metrics (y-axis) are compared to dates estimated using thresholds on smoothed GPP series (x-axis) for three forest sites (FR-Hes, DE-Hai and DK-Sor). Details are given in the Method section.