Hybrid implicit–explicit XFEM simulation of injection-induced seismicity: resolving multi-scale rupture nucleation and dynamics

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Abstract. This study presents a novel hybrid implicit–explicit time-integration strategy for simulating injection-induced fault reactivation within a fully coupled poroelastodynamic extended finite element (XFEM) framework. Modeling these systems is inherently challenging due to the extreme separation of temporal scales—ranging from (more than) days for fluid diffusion to (less than) milliseconds for dynamic rupture. Although the fully implicit formulation can provide enhanced numerical stability, the accuracy of the simulated rupture propagation is governed by the prescribed minimum time-step size, that is, the smallest allowable time step reached during adaptive time-step reduction. Under-resolving these rapid transients leads to delayed instability, reduced slip rates, and underestimated seismic moments. Notably, the temporal resolution required to accurately capture rupture propagation is found to be of the same order as the critical time step defined by the Courant–Friedrichs–Lewy (CFL) condition. To address this, the proposed solver strategy relies on an implicit solver for quasi-static loading and nucleation stages, and seamlessly transitions to a fully dynamic explicit update once a slip-velocity threshold is exceeded. This hybrid time-integration leads to a significant reduction in computational cost, ranging from 60–70% relative to fully implicit simulations, while accurately reproducing fault slip, stress evolution, and seismic magnitudes. Fully implicit solutions predict slightly higher peak slip velocities and rupture speeds; however, event timing and final deformation remain broadly consistent between the two approaches. Furthermore, we demonstrate that the computational efficiency of the proposed hybrid solution algorithm scales independently of the computation of the complex slip tangent operator. These findings demonstrate the potential of the proposed approach for computationally efficient simulation of dynamic reactivation of prescribed faults.

1 Introduction

The increasing reliance on subsurface technologies such as enhanced geothermal systems (EGS) and carbon capture and storage (CCS) has become a cornerstone of modern energy transition and climate-mitigation strategies. Despite their societal benefits, these operations inevitably involve large-scale fluid injection into the crust, which has been repeatedly associated with injection-induced seismicity (Cao et al., 2022; Grigoratos et al., 2022; Cheng et al., 2023; Templeton et al., 2023; Zhou et al., 2024; Sun et al., 2026). Induced seismicity poses significant operational, regulatory, and societal challenges (Grigoli et al., 2017; Trutnevyte and Ejderyan, 2018; Bommer, 2022), particularly when events exceed acceptable risk thresholds. Consequently, accurately simulating the mechanisms controlling the timing, magnitude, and spatial evolution of induced earthquakes remains a key scientific and engineering objective.

Injection-induced seismicity arises from the coupled interactions of multiple physical processes, including pore-pressure diffusion, thermoporoelastic stress redistribution, and frictional weakening along pre-existing faults (Boyet et al., 2023; Kivi et al., 2023). Capturing this complexity in numerical models requires to fully resolve the details of the coupling among these physical processes while also incorporating advanced fracture constitutive laws that govern fault strength and deformation

(Cueto-Felgueroso et al., 2018; Riga and Turuntaev, 2021; Heimisson et al., 2022; Ito et al., 2023; Gerardi et al., 2024; Ito et al., 2024; Im and Avouac, 2025; Hosseini et al., 2025). While significant progress has been made in describing frictional fault behavior, its practical application to realistic injection scenarios remains computationally demanding.

A key challenge stems from the extreme separation of temporal scales inherent to injection-driven fault reactivation. Reservoir pressurization and fluid diffusion typically evolve over hours to days, whereas fault nucleation and dynamic rupture unfold over less than milliseconds to seconds (Han and Younis, 2023). To accurately resolve both regimes within a single simulation places stringent demands on time integration schemes. Fully implicit methods are commonly adopted because of their unconditional stability and efficiency during long periods of quasi-static loading (Jin and Zoback, 2018; Romanet et al., 2025). However, once rapid fault acceleration begins, these schemes require very small time steps and repeated global Newton iterations to maintain accuracy, leading to prohibitive computational costs. In contrast, explicit schemes are well-suited for dynamic rupture due to their low per-step cost and straightforward treatment of inertia, but their strict Courant–Friedrichs–Lewy (CFL) stability constraint (Alkhimenkov et al., 2021) renders them impractical for simulating long-term fluid injection and slow nucleation phases.

This fundamental trade-off between stability, accuracy, and efficiency highlights a methodological gap: no single time-integration strategy is optimal across the full spectrum of processes governing injection-induced seismicity. Addressing this limitation is essential to advance numerical simulations that are both accurate during rupture and tractable over operational time scales. While transitions from slow quasi-static or quasi-dynamic deformation to fully dynamic rupture have been previously addressed in earthquake-cycle and rupture simulations (Jin and Zoback, 2018; Pampillon et al., 2018; Katsman and Ben-Avraham, 2025), here we develop and implement a hybrid time-integration framework within a fully coupled poroelastodynamic XFEM formulation to reduce the computational cost of injection-induced fault reactivation simulations. The proposed approach advances the system implicitly during quasi-static loading and nucleation, and automatically transitions to a fully dynamic explicit formulation once a slip-velocity threshold indicative of unstable rupture is exceeded. This switching parameter enables a consistent and adaptive treatment of multi-scale processes within a single computational framework. The implicit formulation developed here serves as a robust reference solution and represents an improvement to our previous effort in modeling fault rupture process (Sabah et al., 2026). Building on this foundation, we demonstrate that the hybrid implicit–explicit solver-switching strategy preserves the accuracy of fault slip, stress evolution, and seismic source parameters, while substantially reducing computational costs under evolving subsurface conditions.

The remainder of the paper is organized as follows. Section 2 details the methodological framework, including the coupled hydromechanical equations, rate-and-state friction model, and XFEM-based spatial discretization. This section also introduces the reference fully implicit scheme, a modified version of our previous work, and the hybrid solution strategy with its switching criteria. Section 3 discusses the model validation, including benchmark problems used to verify the coupled formulation and numerical implementation, and outlines the numerical model setup, including geometry, material properties, and injection protocols. Section 4 presents numerical experiments that assess the influence of time-stepping on the implicit solver and compare the hybrid formulation against the reference in terms of accuracy and performance. Section 5 summarizes the main conclusions, while Section 6 discusses the limitations of the present study and possible directions for future work.

2 Mathematical formulations

2.1 Governing equations

The partial differential equations governing the isothermal deformation of faults in a porous medium comprise the balance of linear momentum for the solid–fluid system and the mass-conservation equations governing fluid flow through the faults and surrounding porous matrix. The dynamic balance of linear momentum is written as (Jaeger et al., 2009):

$$\nabla \cdot \boldsymbol{\sigma} - \rho \ddot{\mathbf{u}} + \rho \mathbf{b} = 0 \tag{1}$$

where $\boldsymbol{\sigma}$ is total stress tensor, $\ddot{\mathbf{u}}$ is the solid acceleration, $\mathbf{b}$ is the body force vector, and ρ is density of saturated rock. Fluid flow in the porous matrix is described by Darcy's law, assuming a constant intrinsic permeability of the pore space. By coupling Darcy's law with the mass conservation principle, the governing equation for fluid flow in a poroelastic medium is obtained as follows (Wang, 2000):

$$\nabla \cdot \left[\boldsymbol{k}\left(-\nabla p - \rho_f \ddot{\mathbf{u}} + \rho_f \mathbf{b}\right)\right] + \alpha \nabla \cdot \dot{\mathbf{u}} + S_s \dot{p} = 0 \tag{2}$$

where p denotes the pore pressure, α is the Biot coefficient, $\boldsymbol{k}$ is the hydraulic mobility tensor, ρ_f is fluid density, and $S_s = \left(\phi C_f + \frac{(\alpha-\phi)(1-\alpha)}{K}\right)$ denotes the specific storage coefficient with and K as porosity and drained bulk modulus, respectively, and C_f as fluid compressibility. Fluid flow within a fault obeys the same governing physical principles as flow in the surrounding porous medium; however, it is formulated over domain Ω' (Fig. 1). In contrast to the porous matrix, fault permeability is not assumed constant but evolves dynamically in response to variations in its hydraulic aperture. This behavior is represented using the cubic law, which links laminar flow along the fault to the cube of its aperture. Accordingly, fluid flow along the fault interface is governed by (Zimmerman and Paluszny, 2024):

$$\nabla \cdot \left[\frac{w^2}{12\mu_f}\left(-\nabla p - \rho_f \ddot{\mathbf{u}} + \rho_f \mathbf{b}\right)\right] + \nabla \cdot \dot{\mathbf{u}} + C_f \dot{p} = 0, \quad on\ \Omega' \tag{3}$$

where w represents the hydraulic aperture of the fault and μ_f is fluid viscosity. It should be noted that Eqs. (1-3) do not represent the full dynamic Biot poroelastic formulation. Instead, they correspond to a coupled $\boldsymbol{u} - p$ formulation, in which dynamic effects are introduced through the solid momentum balance and the acceleration term appearing in the hydraulic driving force. The full two-field inertial structure of dynamic Biot theory, including added-mass effects, tortuosity, and relative fluid acceleration terms, is not considered (Biot, 1956; Zienkiewicz and Shiomi, 1984). This approximation is appropriate for the present study, where the main objective is to resolve pore-pressure diffusion coupled with poroelastodynamic deformation, fault slip, and mechanical wave propagation in the porous skeleton, rather than to model the full dynamic Biot wave modes associated with relative fluid–solid acceleration in the saturated porous medium.

2.2 Variational formulation of flow and deformation in a fractured porous medium

The variational (weak) formulation is derived by multiplying the governing equations with suitable test functions and integrating over the computational domain. By invoking the Gauss–Green theorem in the presence of discontinuities, enforcing traction continuity across the contact interface, and incorporating Biot's theory to account for poromechanical effects at the interface, the weak form of the mechanical equilibrium (Eq. 1) includes contributions from both the bulk domain and the interface as:

$$\begin{aligned}&\int_\Omega \nabla\delta\mathbf{v}\colon \boldsymbol{\sigma}' d\Omega - \int_\Omega \nabla\delta\mathbf{v}\colon (\alpha p \boldsymbol{m}) d\Omega + \int_\Omega \delta\mathbf{v} \cdot \rho\ddot{\mathbf{u}} d\Omega + \int_{\Gamma_d} [\![\delta\mathbf{v}]\!]\mathbf{t} d\Gamma - \int_{\Gamma_d} [\![\delta\mathbf{v}]\!]\left(p \cdot \mathbf{n}_{\Gamma_d}\right) d\Gamma - \int_\Omega \delta\mathbf{v} \cdot \rho\mathbf{b} d\Omega \\ &- \int_{\Gamma_t} \delta\mathbf{v} \cdot \left(\boldsymbol{\sigma}\mathbf{n}_{\Gamma_t}\right) d\Gamma = 0\end{aligned} \tag{4}$$

where $\boldsymbol{\sigma}'$ is effective stress tensor, $\delta\mathbf{v}$ is the test function for displacement, $\mathbf{t}$ is the traction vector on fault interface, $\mathbf{m} = [1 \quad 1 \quad 0]^T$ is the Voigt representation of the second-order identity tensor, and $\mathbf{n}_{\Gamma_d}$ and $\mathbf{n}_{\Gamma_t}$ are the unit normal vector to the fault interface Γ_d and the mechanical Neumann boundary Γ_t, respectively. In an analogous manner, the weak formulation of the fluid flow equation in the porous matrix is derived by multiplying the governing equation (Eq. 2) by the pressure test function $\delta\psi$ and integrating over the domain:

$$\int_{\Omega} \nabla\delta\psi \boldsymbol{k}\nabla p d\Omega + \int_{\Omega} \nabla\delta\psi \boldsymbol{k}\cdot\rho_f \ddot{\mathbf{u}} d\Omega - \int_{\Gamma_d} \delta\psi \bar{q}_f d\Gamma + \int_{\Omega} \delta\psi\alpha\nabla\cdot\dot{\mathbf{u}} d\Omega + \int_{\Omega} \delta\psi S_s \dot{p} d\Omega$$
$$- \int_{\Omega} \nabla\delta\psi \boldsymbol{k}\cdot\rho_f \mathbf{b} d\Omega + \int_{\Gamma_w} \delta\psi(\boldsymbol{q}\cdot\mathbf{n}_{\Gamma_w}) d\Gamma = 0 \quad (5)$$

where $\boldsymbol{q} = \boldsymbol{k}(-\nabla p - \rho_f\ddot{\mathbf{u}} + \rho_f\mathbf{b})$ is Darcy flux vector field in porous media, Γ_w is hydraulic Neumann boundary, and $\bar{q}_f$ represents the fluid flux component normal to the fault interface, describing the exchange of fluid between the fault and the rock matrix. The weak form of the flow continuity equation within the fault is derived by multiplying Eq. (3) by the test function $\delta\psi(x,t)$ and integrating over fault domain Ω' (as illustrated in Fig. 1), resulting in (Khoei, 2015):

$$\int_{\Omega'} \nabla\delta\psi \frac{w^2}{12\mu_f} \nabla p d\Omega + \int_{\Omega'} \nabla\delta\psi \frac{w^2}{12\mu_f} \rho_f \ddot{\mathbf{u}} d\Omega - \int_{\Gamma_d} \delta\psi \bar{q}_f d\Gamma + \int_{\Omega'} \delta\psi \nabla\cdot\dot{\mathbf{u}} d\Omega$$
$$+ \int_{\Omega'} \delta\psi C_f \dot{p} d\Omega - \int_{\Omega'} \nabla\delta\psi \frac{w^2}{12\mu_f} \rho_f \mathbf{b} d\Omega = 0 \quad (6)$$

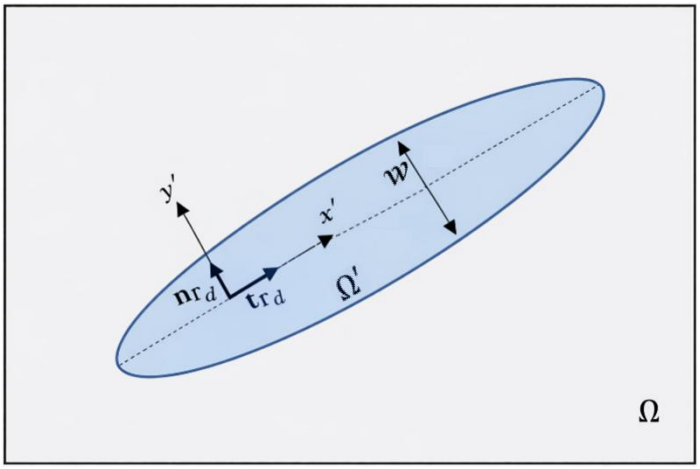


Figure 1: Schematic representation of the local coordinate system and fluid flow geometry within the fault. The blue elliptical-shaped region is used only to illustrate the hydraulic aperture w in cross-sectional view and should not be interpreted as the actual computational fault geometry. In the XFEM formulation, the fault domain Ω' is represented as a one-dimensional line embedded in the two-dimensional porous domain Ω. This is because the aperture is very small relative to the fault length, the pressure gradient across the aperture is neglected, and flow is modeled only along the fault direction x'.

Equation (6) presents a unified variational formulation for hydro-mechanic coupled fluid flow within faults, enabling consistent fluid exchange between the fault and the surrounding porous matrix. The formulation is evaluated over domain Ω', using a fault-aligned local coordinate system (Fig. 1). Assuming negligible pressure gradients across the fault aperture, justified by the small fault width relative to its length, the cross-sectional integration is simplified by treating pressure as uniform across the aperture. Accordingly, a lower-dimensional representation of flow along the interface can be written as follows (Khoei, 2015):

$$\int_{\Gamma_d} \nabla\delta\psi\cdot\mathbf{t}_{\Gamma_d}\left(\frac{w^3}{12\mu_f}\right)\nabla p\cdot\mathbf{t}_{\Gamma_d} d\Gamma + \int_{\Gamma_d} \nabla\delta\psi\cdot\mathbf{t}_{\Gamma_d}\left(\frac{w^3}{12\mu_f}\rho_f\right)\langle\ddot{\mathbf{u}}\rangle\cdot\mathbf{t}_{\Gamma_d} d\Gamma + \int_{\Gamma_d} \delta\psi w\mathbf{t}_{\Gamma_d}\cdot\langle\nabla\dot{\mathbf{u}}\rangle\cdot\mathbf{t}_{\Gamma_d} d\Gamma$$
$$+ \int_{\Gamma_d} \delta\psi\,[\![\dot{\mathbf{u}}]\!]\cdot\mathbf{n}_{\Gamma_d} d\Gamma + \int_{\Gamma_d} \delta\psi w C_f \dot{p} d\Gamma - \int_{\Gamma_d} \nabla\delta\psi\cdot\mathbf{t}_{\Gamma_d}\left(\frac{w^3}{12\mu_f}\rho_f\right)\mathbf{b}\cdot\mathbf{t}_{\Gamma_d} d\Gamma - \int_{\Gamma_d} \delta\psi\bar{q}_f d\Gamma = 0 \quad (7)$$

where $\mathbf{t}_{\Gamma_d}$ denotes the unit tangent vector along the interface, $\langle\ddot{\mathbf{u}}\rangle$ represents the average acceleration of the two opposing fault surfaces in $\boldsymbol{x}'$ direction, $\langle\nabla\dot{\mathbf{u}}\rangle$ denotes the averaged velocity-gradient tensor, and $[\![\dot{\mathbf{u}}]\!]$ is velocity jump across the fault.

The weak formulations for flow in the porous matrix (Eq. 5) and along the fault (Eq. 7) are coupled through the exchange flux $\bar{q}_f$, which is also flow normal to fault interface. In the present formulation, Eq. (7) can be rearranged to isolate the exchange-flux contribution $\bar{q}_f$, with all remaining terms placed on the right-hand side. The resulting exchange flux is then directly

incorporated into the porous-medium flow equation (Eq. 5) as a source/sink term. Therefore, flow perpendicular to the fault plane is represented through the matrix–fault exchange flux, rather prescribing an independent fault-normal permeability. In contrast, conventional zero-thickness interface-element formulations often require a transversal conductivity or leak-off law to relate the matrix–fault pressure difference to the normal exchange flux (Segura and Carol, 2008; Cerfontaine et al., 2015).

2.3 Variational treatment for enforcing contact constraints

The relative normal and tangential displacement between the opposing fault faces, i.e., $\Gamma_d{}^+$ and $\Gamma_d{}^-$, can be defined as follows:

$$g_N = [\![\mathbf{u}]\!] \cdot \mathbf{n}_{\Gamma_d} \tag{8}$$

$$g_T = [\![\mathbf{u}]\!] \cdot \mathbf{t}_{\Gamma_d} \tag{9}$$

where $[\![\mathbf{u}]\!]$ represents the displacement jump across the discontinuity**.** To enforce the unilateral contact condition, the penalty method is commonly employed (Maddio et al., 2025). In this approach, the impenetrability constraint is approximately satisfied by introducing stiff virtual springs at the contact interface. The normal contact force (P_N) is computed as the product of the normal penalty parameter (k_N) and the relative normal displacement (g_N). Likewise, the elastic (stick) component of the tangential contact force (P_T) is obtained by multiplying the tangential penalty parameter (k_T) with the elastic part of the relative tangential displacement (g_T). The total traction vector on the contact interface can then be given by (Jaeger et al., 2009; Liu and Borja, 2009):

$$\mathbf{t} = P_N \mathbf{n}_{\Gamma_d} + P_T \mathbf{t}_{\Gamma_d} = \left(k_N [\![\mathbf{u}]\!] \cdot \mathbf{n}_{\Gamma_d}\right)\mathbf{n}_{\Gamma_d} + \left(k_T [\![\mathbf{u}]\!] \cdot \mathbf{t}_{\Gamma_d}\right)\mathbf{t}_{\Gamma_d} \tag{10}$$

Assuming the active contact surface $\widetilde{\Gamma_d} \subset \Gamma_d$ is known, the contact pressure and its variation satisfy $\{P_N, \delta P_N\} < 0$ on $\widetilde{\Gamma_d}$ and vanish on the inactive portion $\Gamma_d \setminus \widetilde{\Gamma_d}$. The weak form contribution from the interface integral in Eq. (4) then simplifies to (Liu and Borja, 2010):

$$\int_{\Gamma_d} [\![\delta\mathbf{v}]\!]\mathbf{t} d\Gamma = \int_{\widetilde{\Gamma_d}} [\![\delta\mathbf{v}]\!]\left(k_N [\![\mathbf{u}]\!] \cdot \mathbf{n}_{\Gamma_d}\right)\mathbf{n}_{\Gamma_d} d\Gamma + \int_{\widetilde{\Gamma_d}} [\![\delta\mathbf{v}]\!]\left(k_T [\![\mathbf{u}]\!] \cdot \mathbf{t}_{\Gamma_d}\right)\mathbf{t}_{\Gamma_d} d\Gamma \tag{11}$$

To simulate tangential fault deformation during the slip phase, whether seismic or aseismic, a distinct variational formulation is adopted. One of the most widely used criteria for slip, commonly adopted to model frictional contact behavior, is Coulomb's friction law, which is expressed in terms of the contact forces as:

$$F_c = |P_T| + f P_N, \;\; P_N \leq 0 \tag{12}$$

where f is friction coefficient, which evolves as a function of slip rate and state variables consistent with rate-and-state friction theory. The interface remains in the stick state when $F_c < 0$, whereas sliding occurs when $F_c = 0$. Once slip is detected, the tangential traction magnitude is no longer governed by an elastic resistance but by the interface frictional strength. To efficiently capture the stick-slip transition, a predictor–corrector scheme based on classical plasticity theory is adopted and implemented via a return-mapping algorithm (Fig. 2). During the slip phase, the tangential traction is governed by a rate-dependent constitutive law (Liu and Borja, 2009):

$$\dot{P_T} = k_T([\![\dot{\boldsymbol{u}}]\!] \cdot \mathbf{t}_{\Gamma_d} - \mathrm{sgn}(P_T)\dot{\zeta}) \tag{13}$$

where $\dot{\zeta}$ represents the rate of irreversible (plastic) slip. The ζ variable thus quantifies the accumulated tangential plastic slip along the interface. Time integration of Eq. (13) leads to the incremental update of the tangential traction:

$$P_T{}^{n+1} = P_T{}^{n} + k_T([\![\mathbf{u}]\!]^{n+1} - [\![\mathbf{u}]\!]^{n}) \cdot \mathbf{t}_{\Gamma_d} - \mathrm{sgn}(P_T) k_T(\zeta^{n+1} - \zeta^{n}) \tag{14}$$

where superscripts n and $n+1$ denote quantities at the previous and current time steps, respectively. Substituting the updated tangential traction into the sliding condition of Coulomb's friction law (Eq. 12) leads to the nonlinear residual equation:

$$F_c = \left| {P_T}^n + k_T([\![\boldsymbol{u}]\!]^{n+1} - [\![\boldsymbol{u}]\!]^n) \cdot \boldsymbol{t}_{\Gamma_d} - \mathrm{sgn}(P_T) k_T(\zeta^{n+1} - \zeta^n) \right| + f^{n+1} k_N [\![\boldsymbol{u}]\!] \cdot \boldsymbol{n}_{\Gamma_d} = \boldsymbol{0} \tag{15}$$

To enforce the active sliding condition, the nonlinear consistency equation $F_c(\zeta^{n+1}) = 0$ is solved using a Newton–Raphson iteration. During this local update, the displacement jump is kept fixed, and the Jacobian is obtained by differentiating Eq. (15) with respect to the plastic slip variable ζ:

$$\frac{\partial F_c}{\partial \zeta} = -k_T + \frac{\partial f^{n+1}}{\partial \zeta} k_N [\![\mathbf{u}]\!] \cdot \boldsymbol{n}_{\Gamma_d} \tag{16}$$

At each iteration, the derivative of the friction coefficient with respect to the plastic slip must be evaluated. The friction coefficient evolves according to a rate-and-state formulation (Dieterich, 1979; Ruina, 1983), which is a phenomenological constitutive relation widely used to model velocity- and state-dependent fault friction:

$$f = f_0 + a\, ln\left(\frac{V^{n+1}}{V^*}\right) + b\, ln\left(\frac{\theta^{n+1}}{\theta^*}\right) \tag{17}$$

where V^* denotes the reference slip velocity, while f_0 represents the steady-state friction coefficient. The state variable, θ, characterizes the dependence of friction on both slip time and slip history, whereas θ^* corresponds to its steady-state value when the fault interface is sliding at the reference velocity V^*. The direct effect and friction evolution are denoted by the parameters a and b, respectively. In this work, the evolution of the state variable is governed by the aging law (Dieterich, 1979; Ruina, 1983):

$$\dot{\theta} = 1 - \frac{V^{n+1}\theta^{n+1}}{D_c} \tag{18}$$

where D_c is the characteristic slip distance. The aging law is commonly used in the Dieterich–Ruina rate-and-state friction framework to describe contact aging and frictional healing during quasi-stationary periods (Dieterich, 1979; Ruina, 1983). Applying a backward Euler discretization yield:

$$\theta^{n+1} = \frac{\theta^n + \Delta t}{1 + \frac{\Delta t V^{n+1}}{D_c}} \tag{19}$$

The derivative of the friction coefficient with respect to slip is subsequently obtained using the chain rule:

$$\frac{\partial f}{\partial \zeta} = \frac{\partial f}{\partial V}\frac{\partial V}{\partial \zeta} + \frac{\partial f}{\partial \theta}\frac{\partial \theta}{\partial \zeta} = \frac{1}{\Delta t}\left(\frac{a}{V^{n+1}} - \frac{b}{\theta^{n+1}}\frac{D_c \Delta t(\theta^n + \Delta t)}{(D_c + \Delta t V^{n+1})^2}\right), \quad V^{n+1} = \frac{\zeta^{n+1} - \zeta^n}{\Delta t} \tag{20}$$

The slip variable ζ^{n+1} is iteratively updated until Eq. (15) is satisfied. At each iteration, the slip velocity, state variable, friction coefficient, and its derivative with respect to slip are updated accordingly. Upon convergence, the tangential traction and all internal variables are finalized, ensuring a consistent and physically sound representation of the interface response under evolving frictional conditions. It should be noted that the fault is represented as a one-dimensional interface with a single tangential slip direction; therefore, V is a scalar slip speed. For a higher-dimensional fault surface, the tangential slip velocity would generally be vector-valued, and the quantity V entering the rate-and-state friction law would correspond to the magnitude of that tangential slip-velocity vector.

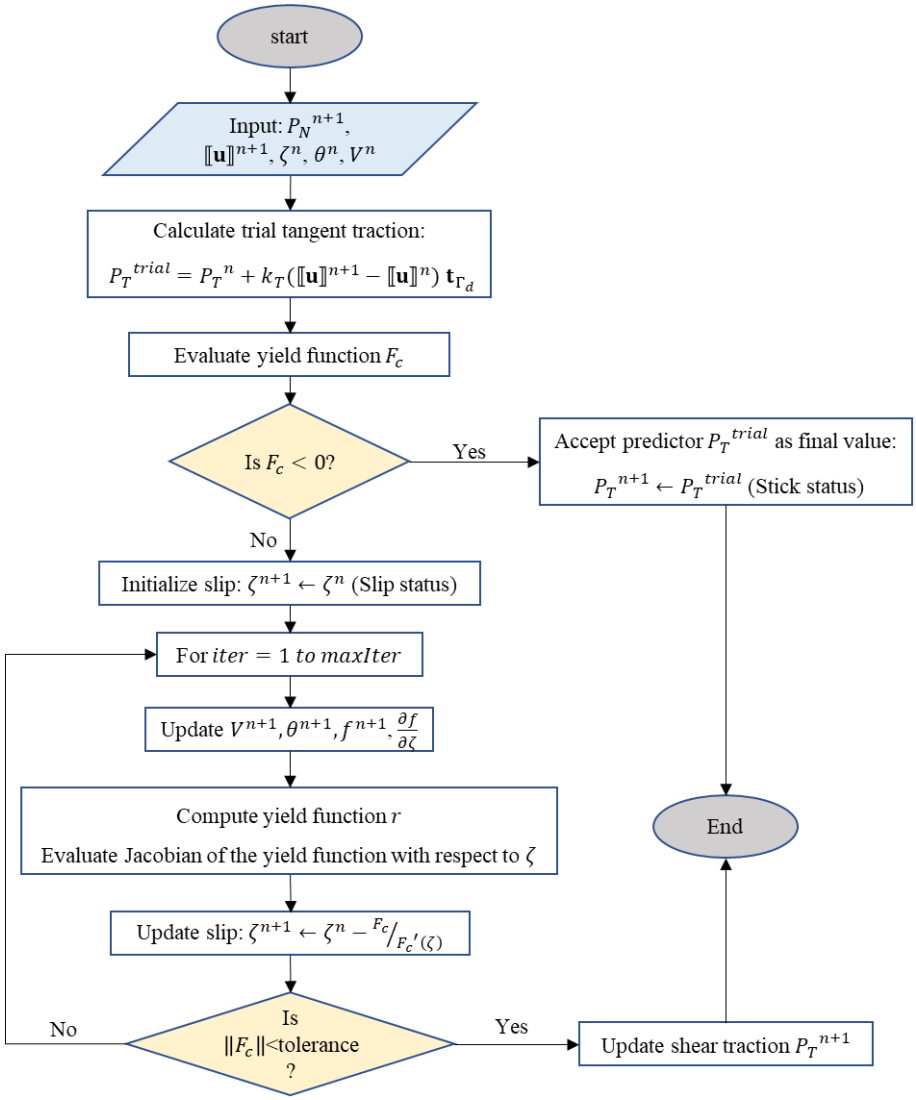


Figure 2: Flowchart of the predictor–corrector return-mapping algorithm used to update tangential traction and slip at the fault interface under rate-and-state friction. The procedure evaluates stick or slip conditions based on a yield criterion, with iterative correction of slip and frictional variables during the slip phase until convergence is achieved.

2.4 Interpolation of pressure and displacement fields

Within the XFEM framework, the displacement field $\mathbf{u}(\boldsymbol{x}, t)$ is approximated using an enriched formulation that augments the standard finite element interpolation with additional functions to represent displacement discontinuities and near-tip singularities (Dolbow et al., 2000). The resulting approximation is written as:

$$\boldsymbol{u}^e(\boldsymbol{x},t) = \sum_{i=1}^{\mathcal{N}} N_{ui}(\boldsymbol{x})\overline{\mathbf{u}}_i(t) + \sum_{i=1}^{\mathcal{N}^{Hev}} N_{ui}(\boldsymbol{x})\left(H\big(\varphi(\boldsymbol{x})\big) - H\big(\varphi(\boldsymbol{x}_i)\big)\right)\overline{\mathbf{a}}_i(t)$$
$$+ \sum_{i=1}^{\mathcal{N}^{tip}} N_{ui}(\boldsymbol{x}) \sum_{j=1}^{4} \left(T_j(\boldsymbol{x}) - T_j(\boldsymbol{x}_i)\right)\overline{\mathbf{b}}_{ji}(t) = \mathbf{N}_u^{std}(\boldsymbol{x})\overline{\mathbf{u}}(t) + \mathbf{N}_u^{Hev}(\boldsymbol{x})\overline{\mathbf{a}}(t) + \mathbf{N}_u^{tip}(\boldsymbol{x})\overline{\mathbf{b}}(t) \quad (21)$$

where $\mathcal{N}$ denotes the number of all nodal points within element e, and $\mathcal{N}^{Hev}$ and $\mathcal{N}^{tip}$ represent the number of nodes enriched by the Heaviside function $H(\varphi(\boldsymbol{x})) - H(\varphi(\boldsymbol{x}_i))$ and crack-tip asymptotic functions $\sum_{j=1}^{4}\left(T_j(\boldsymbol{x}) - T_j(\boldsymbol{x}_i)\right)$, respectively. Here, $\varphi(\boldsymbol{x})$ denotes the signed distance function defining the crack geometry. The vectors $\overline{\mathbf{u}}_i(t)$, $\overline{\mathbf{a}}_i(t)$, and $\overline{\mathbf{b}}_{ji}(t)$ correspond to the standard, Heaviside-enriched, and tip-enriched degrees of freedom. In this formulation, Heaviside enrichment means that the approximation space of nodes whose support is intersected by the crack is augmented by discontinuous basis functions of the form $N_{ui}(\boldsymbol{x})\big(H\big(\varphi(\boldsymbol{x})\big) - H(\varphi(\boldsymbol{x}_i))\big)$, which introduces additional nodal degrees of freedom $\overline{\boldsymbol{a}}_i(t)$. Since the Heaviside

function takes different constant values on the two sides of the crack, this enrichment allows the displacement field to have two independent traces across the crack faces. Therefore, a displacement jump can be represented inside an element without requiring the finite element mesh to conform to the crack geometry. The shifted form $H(\varphi(\boldsymbol{x})) - H(\varphi(\boldsymbol{x}_i)$ is used to preserve the standard nodal interpolation property. In other words, the Heaviside enrichment enables the representation of strong discontinuities, where the displacement field itself is allowed to be discontinuous across the contact surface. The Heaviside function is given by

$$H(\varphi(x)) = \begin{cases} +1 & \varphi(x) > 0 \\ -1 & \varphi(x) < 0 \end{cases} \tag{22}$$

In contrast, the crack-tip enrichment improves the approximation of the near-tip singular displacement field. The crack-tip enrichment functions are defined as (Moës et al., 1999):

$$T(r,\theta) = \{T_1, T_2, T_3, T_4\} = \left\{\sqrt{r}\sin\frac{\beta_f}{2}, \sqrt{r}\cos\frac{\beta_f}{2}, \sqrt{r}\sin\frac{\beta_f}{2}\sin\theta, \sqrt{r}\cos\frac{\beta_f}{2}\sin\theta\right\} \tag{23}$$

where r is the distance from the crack tip, and β_f is the polar angle measured from the crack line. The displacement jump across the contact surface can be written as:

$$[\![\mathbf{u}]\!] = \left[\!\left[\mathbf{N}_u^{std}(\boldsymbol{x})\overline{\mathbf{u}}(t) + \mathbf{N}_u^{Hev}(\boldsymbol{x})\overline{\mathbf{a}}(t) + \mathbf{N}_u^{tip}(\boldsymbol{x})\overline{\mathbf{b}}(t)\right]\!\right] = [\![\mathbf{N}_u^{std}(\boldsymbol{x})\overline{\mathbf{u}}(t)]\!] + [\![\mathbf{N}_u^{Hev}(\boldsymbol{x})\overline{\mathbf{a}}(t)]\!] + \left[\!\left[\mathbf{N}_u^{tip}(\boldsymbol{x})\overline{\mathbf{b}}(t)\right]\!\right] \tag{24}$$

The term involving the standard shape functions $\mathbf{N}_u^{std}$ vanishes since these functions are continuous across the interface. Consequently, the displacement jump induced by the Heaviside and tip enrichment reduces to:

$$[\![\mathbf{N}_u^{Hev}(\boldsymbol{x})\overline{\mathbf{a}}(t)]\!] = 2\sum_{i=1}^{\mathcal{N}^{Hev}} N_{ui}(\boldsymbol{x})\overline{\mathbf{a}}_i(t) \tag{25}$$

$$\left[\!\left[\mathbf{N}_u^{tip}(\boldsymbol{x})\overline{\mathbf{b}}(t)\right]\!\right] = 2\sum_{i=1}^{\mathcal{N}^{tip}} N_{ui}(\boldsymbol{x})\ \sqrt{r}\,\overline{\boldsymbol{b}}_{1i}(t) \tag{26}$$

Among the four crack-tip asymptotic enrichment functions, only the first function generates a displacement discontinuity across the crack faces (Khoei, 2015). The remaining functions primarily enhance the approximation quality in the vicinity of the crack tip and therefore do not contribute to the jump. Based on the XFEM displacement enrichment, the strain field is decomposed into standard, discontinuous, and crack-tip contributions. Accordingly, the strain at position $\boldsymbol{x}$ and time t can be written as:

$$\boldsymbol{\varepsilon}^e(\boldsymbol{x},t) = \mathbf{B}^{std}(\boldsymbol{x})\overline{\mathbf{u}}(t) + \mathbf{B}^{Hev}(\boldsymbol{x})\overline{\mathbf{a}}(t) + \mathbf{B}^{tip}(\boldsymbol{x})\overline{\mathbf{b}}(t) \tag{27}$$

where $\mathbf{B}^{std}$ is spatial derivatives of standard shape functions, and $\mathbf{B}^{Hev}$ and $\mathbf{B}^{tip}$ are spatial derivatives of enriched shape functions associated with Heaviside enrichment function and crack-tip asymptotic functions, respectively. The evaluation of the spatial derivatives of the crack-tip enrichment functions requires a transformation from the local crack-tip polar coordinate system to the global Cartesian coordinates (x, y) (Khoei, 2015). This transformation can be expressed as:

$$\nabla T_j = \boldsymbol{R}(\beta_f)\nabla' T_j = \boldsymbol{R}(\beta_f)\mathbf{J}_{r\beta_f}\begin{bmatrix} T_{j,r} \\ T_{j,\beta_f} \end{bmatrix} \tag{28}$$

where $\boldsymbol{R}$ is rotation matrix with β_f denoting the angle between the local crack-tip coordinate system (x', y') and the global coordinate axes (x, y), and $\mathbf{J}_{r\beta_f}$ is the Jacobian relating polar coordinates (r, β_f) to the local Cartesian coordinates.

The enriched approximation of the XFEM for the pressure field $p(\boldsymbol{x}, t)$ can be written as:

$$p^e(\boldsymbol{x}, t) = \sum_{i=1}^{\mathcal{N}} N_{p_i}(\boldsymbol{x})\bar{p}_i(t) + \sum_{i=1}^{\mathcal{N}^{mod}} N_{p_i}(\boldsymbol{x})\big(\Psi(\boldsymbol{x}) - \Psi(\boldsymbol{x}_i)\big)\bar{d}_i(t) = \mathbf{N}_{\mathrm{p}}^{\mathrm{std}}(\mathbf{x})\overline{\mathbf{p}}(\mathrm{t}) + \mathbf{N}_{\mathrm{p}}^{\mathrm{mod}}(\mathbf{x})\overline{\mathbf{d}}(\mathrm{t}) \tag{29}$$

where $\mathcal{N}^{mod}$ represent the number of nodes enriched by the modified level-set function $\Psi(\boldsymbol{x}) - \Psi(\boldsymbol{x}_i)$, $\bar{p}_i(t)$ denotes the nodal pressure degrees of freedom associated with the standard interpolation, whereas $\bar{d}_i(t)$ denote the additional enriched pressure degrees of freedom introduced to capture non-smooth pressure variations near the interface. The matrix $\mathbf{N}_p^{mod}$ collects the enriched pressure shape functions with i-th entry of $N_{p_i}(\boldsymbol{x})\big(\Psi(\boldsymbol{x}) - \Psi(\boldsymbol{x}_i)\big)$. The shifted form $\Psi(\boldsymbol{x}) - \Psi(\boldsymbol{x}_i)$ ensures that the enriched contribution vanishes at the enriched node $\boldsymbol{x}_i$, thereby preserving the standard nodal pressure interpolation. In this study, the modified level-set function is defined following Fries & Belytschko (2010) as:

$$\Psi(\boldsymbol{x}) = \sum_{i=1}^{\mathcal{N}^{mod}} N_{p_i}(\boldsymbol{x})|\varphi(\boldsymbol{x}_i)| - \left|\sum_{i=1}^{\mathcal{N}^{mod}} N_{pi}(\boldsymbol{x})\varphi(\boldsymbol{x}_i)\right| \tag{30}$$

This formulation produces an enrichment function that forms a ridge aligned with the interface. As a result, the pressure field remains continuous across the interface, while its spatial gradient is allowed to exhibit a jump. An additional advantage of this construction is that the enrichment naturally vanishes in elements that are not intersected by the interface, thereby eliminating the need for special blending treatments. The pressure gradient inside an enriched element can therefore be evaluated as:

$$\nabla p^e(\boldsymbol{x}, t) = \sum_{i=1}^{\mathcal{N}} \nabla N_{p_i}(\boldsymbol{x})\bar{p}_i(t) + \sum_{i=1}^{\mathcal{N}^{mod}} \big[\nabla N_{p_i}(\boldsymbol{x})\big(\Psi(\boldsymbol{x}) - \Psi(\boldsymbol{x}_i)\big) + N_{p_i}(\boldsymbol{x})\nabla\Psi(\boldsymbol{x})\big]\bar{d}_i(t) \tag{31}$$

The gradient of the modified level set function $\Psi(\boldsymbol{x})$ is computed as follows:

$$\nabla\Psi(\boldsymbol{x}) = \sum_{i=1}^{\mathcal{N}^{mod}} \nabla N_{p_i}(\boldsymbol{x})|\varphi(\boldsymbol{x}_i)| - sgn\left(\sum_{i=1}^{\mathcal{N}^{mod}} N_{pi}(\boldsymbol{x})\varphi(\boldsymbol{x}_i)\right)\sum_{i=1}^{\mathcal{N}^{mod}} \nabla N_{p_i}(\boldsymbol{x})\varphi(\boldsymbol{x}_i) \tag{32}$$

2.5 The XFEM spatial and temporal discretization

By introducing the enriched trial functions for the displacement field (Eq. 21) and the pressure field (Eq. 29), together with their corresponding test functions, and substituting them into the weak formulations given in Eqs. (4) and (5), as well as the relevant interface integral relations, the governing equations can be rearranged into a global matrix form, leading to the following semi-discrete system governing the displacement field:

$$\boldsymbol{M}_{l\times l}\ddot{\boldsymbol{U}}_{l\times 1} + \boldsymbol{D}_{l\times l}\dot{\boldsymbol{U}}_{l\times 1} + \boldsymbol{K}_{l\times l}\boldsymbol{U}_{l\times 1} - \boldsymbol{Q}_{l\times p}\boldsymbol{P}_{p\times 1} + \boldsymbol{F}_{\boldsymbol{int}} - \boldsymbol{F}_{\boldsymbol{ext}} = \boldsymbol{0} \tag{33}$$

where l denotes the total number of degrees of freedom (DOFs) for displacement, and p is the total number of DOFs associated with pressure. In this formulation, $\boldsymbol{M}$ represents the global mass matrix, $\boldsymbol{D}$ **the** damping matrix, $\boldsymbol{K}$ the global stiffness matrix, and $\boldsymbol{Q}$ the coupling matrix linking displacement field $\boldsymbol{U}$ to pressure $\boldsymbol{P}$ field. The vectors $\boldsymbol{F}_{\boldsymbol{ext}}$ and $\boldsymbol{F}_{\boldsymbol{int}}$ represent the external and internal force vectors, respectively**.** The individual matrices are defined as follows:

$$\begin{aligned} \boldsymbol{M} &= \int_{\Omega} (\boldsymbol{N}_u)^T \rho \boldsymbol{N}_u d\Omega \\ \boldsymbol{K} &= \int_{\Omega} \boldsymbol{B}_u{}^T \mathbb{C} \boldsymbol{B}_u d\Omega \end{aligned} \tag{34}$$

$$\boldsymbol{D} = a_{Ray}\boldsymbol{M} + b_{Ray}\boldsymbol{K} + \int_{\Gamma_{abs}} \boldsymbol{N}_u{}^T \rho \boldsymbol{V} \boldsymbol{N}_u d\Gamma$$
$$\boldsymbol{Q} = \int_{\Omega} \boldsymbol{B}_u{}^T \alpha \boldsymbol{m} \boldsymbol{N}_p d\Omega + \int_{\Gamma_d} [\![\boldsymbol{N}_u]\!]^T . \mathbf{n}_{\Gamma_d} \boldsymbol{N}_p d\Gamma$$
$$\boldsymbol{F_{ext}} = \int_{\Omega} (\boldsymbol{N}_u)^T \rho \boldsymbol{b} d\Omega + \int_{\Gamma_t} (\boldsymbol{N}_u)^T \bar{\mathbf{t}} d\Gamma$$
$$\boldsymbol{F_{int}} = \int_{\widetilde{\Gamma_d}} [\![\boldsymbol{N}_u]\!]^T \left(P_N \mathbf{n}_{\Gamma_d} + P_T \mathbf{t}_{\Gamma_d}\right) d\Gamma$$

where $\mathbb{C}$ is the fourth-order elastic constitutive tensor and Γ_{abs} denotes the absorbing portion of the boundary. The global damping matrix $\boldsymbol{D}$ comprises two contributions: Rayleigh damping and the Lysmer–Kuhlemeyer absorbing boundary formulation. Rayleigh damping is introduced at the semi-discrete level as a linear combination of the global mass and stiffness matrices to control spurious oscillatory responses (Liu & Gorman, 1995), whereas the Lysmer–Kuhlemeyer formulation is applied at the external boundaries to minimize artificial wave reflections (Lysmer & Kuhlemeyer, 1969). Material viscosity is therefore not introduced explicitly through the continuous constitutive relation; instead, energy dissipation is represented after XFEM spatial discretization through the Rayleigh damping matrix. In this formulation, a_{Ray} and b_{Ray} denote the mass- and stiffness-proportional Rayleigh damping coefficients, respectively. These coefficients are determined by prescribing a target damping ratio of $\xi = 0.04$ at two selected natural angular frequencies, ω_1 and ω_3, and are defined as (Kontoe et al., 2011):

$$a_{Ray} = \frac{2\omega_1\omega_3\xi}{\omega_1 + \omega_3},\ b_{Ray} = \frac{2\xi}{\omega_1 + \omega_3} \tag{35}$$

In the second term in damping matrix, the parameter $\bar{\rho}$ denotes the spatially averaged density of the mixture, and $\boldsymbol{V}$ is a wave-speed matrix defined as:

$$\boldsymbol{V} = \begin{cases} \begin{bmatrix} a_p \bar{V}_p & 0 \\ 0 & a_s \bar{V}_s \end{bmatrix}, & \text{if } \boldsymbol{n}_\Gamma \| x \\ \begin{bmatrix} a_s \bar{V}_s & 0 \\ 0 & a_p \bar{V}_p \end{bmatrix}, & \text{if } \boldsymbol{n}_\Gamma \| y \end{cases} \tag{36}$$

where a_p and a_s are scaling coefficients, here both taken equal to unity, and $\bar{V}_p$ and $\bar{V}_s$ are the average compressional and shear velocities. For the pressure field, the semi-discrete governing equation can be written as:

$$\boldsymbol{C}_{p\times l}\ddot{\boldsymbol{U}}_{l\times 1} + \boldsymbol{Q}^p{}_{p\times l}\dot{\boldsymbol{U}}_{l\times 1} + \boldsymbol{H}_{p\times p}\boldsymbol{P}_{p\times 1} + \boldsymbol{S}_{p\times p}\dot{\boldsymbol{P}}_{p\times 1} - \boldsymbol{q_{ext}} = 0 \tag{37}$$

where $\boldsymbol{C}$ represents the inertial coupling between the fluid and the solid skeleton, $\boldsymbol{H}$ is the hydraulic conductivity matrix, $\boldsymbol{S}$ denotes the storage matrix, and $\boldsymbol{q_{ext}}$ is an external source term. These matrices are defined as follows:

$$\boldsymbol{C} = \int_{\Omega} \left(\nabla \boldsymbol{N}_p\right)^T \boldsymbol{k} \rho_f \boldsymbol{N}_u d\Omega + \int_{\Gamma_d} \left(\nabla \boldsymbol{N}_p\right)^T \cdot \mathbf{t}_{\Gamma_d} \left(\frac{w^3}{12\mu_f}\rho_f\right) \langle \boldsymbol{N}_u \rangle \cdot \mathbf{t}_{\Gamma_d} d\Gamma$$
$$\boldsymbol{H} = \int_{\Omega} \left(\nabla \boldsymbol{N}_p\right)^T \boldsymbol{k} \left(\nabla \boldsymbol{N}_p\right) d\Omega + \int_{\Gamma_d} \left(\nabla \boldsymbol{N}_p\right)^T . \mathbf{t}_{\Gamma_d} \left(\frac{w^3}{12\mu_f}\right) \left(\nabla \boldsymbol{N}_p\right) . \mathbf{t}_{\Gamma_d} d\Gamma$$
$$\boldsymbol{S} = \int_{\Omega} \left(\boldsymbol{N}_p\right)^T S_s \boldsymbol{N}_p d\Omega + \int_{\Gamma_d} \left(\boldsymbol{N}_p\right)^T w C_f \boldsymbol{N}_p d\Gamma \tag{38}$$
$$\boldsymbol{Q}^p = \boldsymbol{Q}^T + \int_{\Gamma_d} \left(\boldsymbol{N}_p\right)^T w\, \mathbf{t}_{\Gamma_d} \cdot \langle \nabla \boldsymbol{N}_u \rangle \cdot\ \mathbf{t}_{\Gamma_d} d\Gamma$$
$$\boldsymbol{q_{ext}} = \int_{\Omega} \left(\nabla \boldsymbol{N}_p\right)^T \boldsymbol{k} \rho_f \mathbf{b} d\Omega - \int_{\Gamma_w} \left(\boldsymbol{N}_p\right)^T \bar{q} d\Gamma - \int_{\Gamma_d} \left(\nabla \boldsymbol{N}_p\right)^T . \mathbf{t}_{\Gamma_d} \left(\frac{w^3}{12\mu_f}\rho_f\right) \mathbf{b} . \mathbf{t}_{\Gamma_d} d\Gamma$$

2.5.1 Fully implicit time integration strategy

The semi-discrete XFEM governing equations are first integrated in time using a fully implicit solution strategy, which serves as the reference formulation throughout this work. Displacement is advanced with a generalized Newmark GN22 method, while pressure is updated using a GN11 scheme. The unknown fields at time $t_{n+1} = t_n + \Delta t$ are related to known values at t_n through the following formulas:

$$\begin{aligned}
\ddot{\boldsymbol{U}}_{n+1} &= \tfrac{1}{\beta\Delta t^2}(\boldsymbol{U}_{n+1} - \boldsymbol{U}_n) - \tfrac{1}{\beta\Delta t}\dot{\boldsymbol{U}}_n - \left(\tfrac{1}{2\beta} - 1\right)\ddot{\boldsymbol{U}}_n \\
\dot{\boldsymbol{U}}_{n+1} &= \tfrac{\gamma}{\beta\Delta t}(\boldsymbol{U}_{n+1} - \boldsymbol{U}_n) - \left(\tfrac{\gamma}{\beta} - 1\right)\dot{\boldsymbol{U}}_n - \Delta t\left(\tfrac{\gamma}{2\beta} - 1\right)\ddot{\boldsymbol{U}}_n \\
\dot{\boldsymbol{P}}_{n+1} &= \frac{1}{\Delta t}(\boldsymbol{P}_{n+1} - \boldsymbol{P}_n)
\end{aligned} \tag{39}$$

This temporal discretization approach provides stable and accurate integration of second-order and first-order time-dependent equations, respectively. Unconditional stability of the displacement integration is ensured by selecting Newmark parameters satisfying $\gamma \geq 0.5, \beta \geq 0.25(0.5+\gamma)^2$. In this work, the constant-average-acceleration scheme is employed with $\gamma = 0.5$ and $\beta = 0.25$ (Newmark, 1959). The vectors $\boldsymbol{U}_n$, $\dot{\boldsymbol{U}}_n$, and $\ddot{\boldsymbol{U}}_n$ denote the displacement, velocity, and acceleration fields— including both standard and enriched degrees of freedom—at time step n, while $\boldsymbol{P}_n$ represents the corresponding pressure field. Substitution of the time discretization into the semi-discrete equations yields a fully discrete nonlinear system at $n+1$ is formulated. The residual form of the coupled system is written as:

$$\begin{aligned}
\boldsymbol{R}_{\boldsymbol{U}_{n+1}} &= \left(\frac{1}{\beta\Delta t^2}\boldsymbol{M} + \frac{\gamma}{\beta\Delta t}\boldsymbol{D} + \boldsymbol{K}\right)\boldsymbol{U}_{n+1} - \boldsymbol{Q}\boldsymbol{P}_{n+1} - \left(\frac{1}{\beta\Delta t^2}\boldsymbol{M} + \frac{\gamma}{\beta\Delta t}\boldsymbol{D}\right)\boldsymbol{U}_n \\
&- \left(\frac{1}{\beta\Delta t}\boldsymbol{M} + \left(\frac{\gamma}{\beta} - 1\right)\boldsymbol{D}\right)\dot{\boldsymbol{U}}_n - \left(\left(\frac{1}{2\beta} - 1\right)\boldsymbol{M} + \Delta t\left(\frac{\gamma}{2\beta} - 1\right)\boldsymbol{D}\right)\ddot{\boldsymbol{U}}_n + \boldsymbol{F}_{\boldsymbol{int}} - \boldsymbol{F}_{\boldsymbol{ext}} = \boldsymbol{0}
\end{aligned} \tag{40}$$

$$\begin{aligned}
\boldsymbol{R}_{\boldsymbol{P}_{n+1}} &= \left(\frac{1}{\beta\Delta t^2}\boldsymbol{C} + \frac{\gamma}{\beta\Delta t}\boldsymbol{Q}^p{}_{n+1}\right)\boldsymbol{U}_{n+1} + \left(\boldsymbol{H}_{n+1} + \frac{1}{\Delta t}\boldsymbol{S}_{\boldsymbol{n+1}}\right)\boldsymbol{P}_{n+1} - \left(\frac{1}{\Delta t}\boldsymbol{S}_{\boldsymbol{n+1}}\right)\boldsymbol{P}_n \\
&- \left(\frac{1}{\beta\Delta t^2}\boldsymbol{C} + \frac{\gamma}{\beta\Delta t}\boldsymbol{Q}^p{}_{n+1}\right)\boldsymbol{U}_n - \left(\frac{1}{\beta\Delta t}\boldsymbol{C} + \left(\frac{\gamma}{\beta} - 1\right)\boldsymbol{Q}^p{}_{n+1}\right)\dot{\boldsymbol{U}}_n \\
&- \left(\left(\frac{1}{2\beta} - 1\right)\boldsymbol{C} + \Delta t\left(\frac{\gamma}{2\beta} - 1\right)\boldsymbol{Q}^p{}_{n+1}\right)\ddot{\boldsymbol{U}}_n - \boldsymbol{q}_{\boldsymbol{ext}} = \boldsymbol{0}
\end{aligned} \tag{41}$$

The resulting coupled nonlinear system is solved using a fully monolithic Newton–Raphson approach, in which displacement and pressure unknowns are updated simultaneously at each time step. At iteration k of time step $n+1$, the residual equations are linearized about the current solution estimate, leading to the following incremental system:

$$\boldsymbol{J}\left({\boldsymbol{U}_{n+1}}^k, {\boldsymbol{P}_{n+1}}^k\right)\begin{Bmatrix}{\delta\boldsymbol{U}_{n+1}}^{k+1} \\ {\delta\boldsymbol{P}_{n+1}}^{k+1}\end{Bmatrix} = -\begin{Bmatrix}{\boldsymbol{R}_{\boldsymbol{U}_{n+1}}}^k \\ {\boldsymbol{R}_{\boldsymbol{P}_{n+1}}}^k\end{Bmatrix} \tag{42}$$

where $\boldsymbol{J}$ is the Jacobian matrix evaluated at the current iteration. The Jacobian has a block structure and is given by:

$$\boldsymbol{J}\left({\boldsymbol{U}_{n+1}}^k, {\boldsymbol{P}_{n+1}}^k\right) = \begin{bmatrix}\frac{\partial{\boldsymbol{R}_{\boldsymbol{U}_{n+1}}}^k}{\partial\boldsymbol{U}} & \frac{\partial{\boldsymbol{R}_{\boldsymbol{U}_{n+1}}}^k}{\partial\boldsymbol{P}} \\ \frac{\partial{\boldsymbol{R}_{\boldsymbol{P}_{n+1}}}^k}{\partial\boldsymbol{U}} & \frac{\partial{\boldsymbol{R}_{\boldsymbol{P}_{n+1}}}^k}{\partial\boldsymbol{P}}\end{bmatrix} = \begin{bmatrix}\frac{1}{\beta\Delta t^2}\boldsymbol{M} + \frac{\gamma}{\beta\Delta t}\boldsymbol{D} + \boldsymbol{K} + \frac{\partial\boldsymbol{F}_{\boldsymbol{int}}}{\partial\boldsymbol{U}} & -\boldsymbol{Q} \\ \frac{1}{\beta\Delta t^2}\boldsymbol{C} + \frac{\gamma}{\beta\Delta t}\boldsymbol{Q}^p{}_{n+1} & \boldsymbol{H}_{n+1} + \frac{1}{\Delta t}\boldsymbol{S}_{\boldsymbol{n+1}}\end{bmatrix} \tag{43}$$

Due to the hydro-mechanical coupling terms and the time discretization, the Jacobian matrix is generally non-symmetric. This non-symmetry can increase both memory requirements and computational cost. To improve the algebraic scaling of the coupled system, the rows associated with the displacement equations are multiplied by the non-zero factor $\frac{-\gamma}{\beta\Delta t}$. This row scaling

yields an algebraically equivalent linearized system, since it does not change the solution vector. Therefore, the scaled Jacobian should still be regarded as a non-symmetric matrix:

$$\boldsymbol{J}\left({\boldsymbol{U}_{n+1}}^{k}, {\boldsymbol{P}_{n+1}}^{k}\right) = \begin{bmatrix} \frac{-\gamma}{\beta\Delta t}\left(\frac{1}{\beta\Delta t^2}\boldsymbol{M} + \frac{\gamma}{\beta\Delta t}\boldsymbol{D} + \boldsymbol{K} + \frac{\partial \boldsymbol{F_{int}}}{\partial \boldsymbol{U}}\right) & \frac{\gamma}{\beta\Delta t}\boldsymbol{Q} \\ \frac{1}{\beta\Delta t^2}\boldsymbol{C} + \frac{\gamma}{\beta\Delta t}{\boldsymbol{Q}^{p}}_{n+1} & \boldsymbol{H}_{n+1} + \frac{1}{\Delta t}\boldsymbol{S_{n+1}} \end{bmatrix} \tag{44}$$

The contribution of the interface to the Jacobian matrix arises from the dependence of the internal force vector on displacement discontinuities. Its linearization with respect to the displacement field is expressed as:

$$\frac{\partial \boldsymbol{F_{int}}}{\partial U} = \int_{\widetilde{\Gamma_d}} [\![\boldsymbol{N}_u]\!]^T \boldsymbol{E} [\![\boldsymbol{N}_u]\!] d\Gamma \tag{45}$$

where $\boldsymbol{E}$ denotes the algorithmic (consistent) tangent operator associated with the fault constitutive response. At the current time step and Newton iteration, it is defined as the derivative of the algorithmically updated interface traction with respect to the displacement jump across the interface:

$$\boldsymbol{E} = \frac{\partial(\mathbf{t})}{\partial [\![\boldsymbol{u}]\!]} = \frac{\partial\left(P_N \boldsymbol{n}_{\Gamma_d} + P_T \boldsymbol{t}_{\Gamma_d}\right)}{\partial [\![\boldsymbol{u}]\!]} \tag{46}$$

In fact, $\boldsymbol{E}$ represents the tangent stiffness of the time-discrete interface constitutive update, including the effects of contact stiffness, frictional slip, and the relevant internal state variables. The tangent operator is called consistent when it is obtained by differentiating the same discrete update equations used to compute the residual traction. In this case, $\boldsymbol{E}$ provides the exact linearization of the interface contribution entering the Newton residual, for a fixed active regime of the fault. This improves the robustness and convergence rate of the Newton–Raphson iterations. Since the interface response may change between stick and slip, $\boldsymbol{E}$ is evaluated according to the current active fault regime. In the stick regime, the corresponding tangent operator associated with fault-plane plasticity is given by:

$$\boldsymbol{E}^{stick} = \boldsymbol{n}_{\Gamma_d} \otimes \frac{\partial P_N}{\partial [\![\boldsymbol{u}]\!]} + \boldsymbol{t}_{\Gamma_d} \otimes \frac{\partial P_T}{\partial [\![\boldsymbol{u}]\!]} = k_N\left(\boldsymbol{n}_{\Gamma_d} \otimes \boldsymbol{n}_{\Gamma_d}\right) + k_T(\boldsymbol{I} - \boldsymbol{n}_{\Gamma_d} \otimes \boldsymbol{n}_{\Gamma_d}) \tag{47}$$

This expression reflects the linear elastic resistance to tangential motion during the stick phase, where the amount of plastic slip is zero ($\zeta^{n+1} = \zeta^{n}$). Under slipping conditions ($\zeta^{n+1} \neq 0$), the tangential traction, P_T, is governed by a rate-dependent frictional constitutive law. In this case, the linearization of the tangential traction with respect to the displacement jump is required to construct a consistent Jacobian contribution. Under slipping conditions, the derivative of the tangential traction magnitude with respect to the displacement discontinuity is given by:

$$\frac{\partial P_T}{\partial [\![\boldsymbol{u}]\!]} = k_T\left(\boldsymbol{t}_{\Gamma_d} - \operatorname{sgn}(P_T)\frac{\partial \zeta}{\partial [\![\boldsymbol{u}]\!]}\right) \tag{48}$$

At a locally converged stress state, the derivative of ζ with respect to the displacement jump can be expressed as (Liu and Borja, 2009):

$$\frac{\partial \zeta}{\partial [\![\boldsymbol{u}]\!]} = \frac{\operatorname{sgn}(P_T) k_T \boldsymbol{t}_{\Gamma_d} + k_N f^{n+1} \boldsymbol{n}_{\Gamma_d}}{k_T - \frac{\partial f}{\partial \zeta} k_N [\![\boldsymbol{u}]\!] \cdot \boldsymbol{n}_{\Gamma_d}} \tag{49}$$

Substituting Eq. (20) into Eq. (49), and inserting the result into the traction derivative yields the consistent linearization of the tangential traction with respect to the displacement jump:

$$\begin{aligned}\frac{\partial P_T}{\partial [\![\boldsymbol{u}]\!]} &= k_T \left(\boldsymbol{t}_{\Gamma_d} - \operatorname{sgn}(P_T) \frac{\operatorname{sgn}(P_T) k_T \boldsymbol{t}_{\Gamma_d} + k_N f^{n+1} \boldsymbol{n}_{\Gamma_d}}{k_T - \frac{\partial f}{\partial \zeta} k_N [\![\boldsymbol{u}]\!] \cdot \boldsymbol{n}_{\Gamma_d}} \right) \\ &= k_T \left(\boldsymbol{t}_{\Gamma_d} - \frac{k_T \boldsymbol{t}_{\Gamma_d} + \operatorname{sgn}(P_T) k_N f^{n+1} \boldsymbol{n}_{\Gamma_d}}{k_T - \frac{1}{\Delta t} \left(\frac{a}{V^{n+1}} - \frac{b}{\theta^{n+1}} \frac{D_c \Delta t (\theta^n + \Delta t)}{(D_c + \Delta t V^{n+1})^2} \right) k_N [\![\boldsymbol{u}]\!] \cdot \boldsymbol{n}_{\Gamma_d}} \right)\end{aligned} \tag{50}$$

Finally, the consistent algorithmic tangent operator associated with the slipping regime is obtained as:

$$\begin{aligned}\boldsymbol{E}^{slip} &= \boldsymbol{n}_{\Gamma_d} \otimes \frac{\partial P_N}{\partial [\![\boldsymbol{u}]\!]} + \boldsymbol{t}_{\Gamma_d} \otimes \frac{\partial P_T}{\partial [\![\boldsymbol{u}]\!]} \\ &= k_N \left(\boldsymbol{n}_{\Gamma_d} \otimes \boldsymbol{n}_{\Gamma_d} \right) + \boldsymbol{t}_{\Gamma_d} \otimes k_T \left(\boldsymbol{t}_{\Gamma_d} - \frac{k_T \boldsymbol{t}_{\Gamma_d} + \operatorname{sgn}(P_T) k_N f^{n+1} \boldsymbol{n}_{\Gamma_d}}{k_T - \frac{1}{\Delta t} \left(\frac{a}{V^{n+1}} - \frac{b}{\theta^{n+1}} \frac{D_c \Delta t (\theta^n + \Delta t)}{(D_c + \Delta t V^{n+1})^2} \right) k_N [\![\boldsymbol{u}]\!] \cdot \boldsymbol{n}_{\Gamma_d}} \right)\end{aligned} \tag{51}$$

The convergence of the Newton–Raphson solution procedure is evaluated by tracking scaled residual measures associated with the displacement and pore-pressure unknowns. At every iteration, the residual of the mechanical equilibrium equations is normalized with respect to the applied external forces and expressed as:

$$Re_{\boldsymbol{U}} = \frac{\left\| \boldsymbol{R}_{\boldsymbol{U}_{n+1}} \right\|}{\left\| \boldsymbol{F}_{\boldsymbol{ext}} \right\|} \tag{52}$$

while the fluid-flow residual is scaled by the prescribed external flux and written as:

$$Re_{\boldsymbol{P}} = \frac{\left\| \boldsymbol{R}_{\boldsymbol{P}_{n+1}} \right\|}{\left\| \boldsymbol{q}_{\boldsymbol{ext}} \right\|} \tag{53}$$

To consistently assess convergence across the coupled hydro-mechanical system, a unified error indicator is constructed by combining the normalized residuals into a single Euclidean norm:

$$\|R\| = \sqrt{(Re_{\boldsymbol{U}})^2 + (Re_P)^2} \tag{54}$$

Convergence is achieved once this combined measure in iterations drops below a specified tolerance (set here to 10^{-6}). This formulation enforces simultaneous convergence of both the mechanical and hydraulic subproblems, thus ensuring that force equilibrium and mass conservation are satisfied with comparable accuracy while avoiding preferential convergence of either field.

An adaptive time-stepping scheme proposed by Lapusta & Liu (2009) is employed exclusively within the fully implicit formulation to efficiently resolve the quasi-static loading and nucleation phases preceding dynamic rupture. During periods of stable fault behavior, relatively large time steps are permitted to reduce computational cost. As fault slip begins to accelerate toward nucleation, the time step is automatically reduced to ensure adequate resolution of the evolving frictional response and stress redistribution. The time increment is determined as:

$$\Delta t = min[\Delta t_{max}, max\{\Delta t_{min}, \Delta t_{evol}\}] \tag{55}$$

where Δt_{max} and Δt_{min} denote the maximum allowable time-step size and lower bound required to resolve rapid instabilities, respectively, while Δt_{evol} is a variable step size controlled by the current fault slip rate defined by (Lapusta and Liu, 2009):

$$\Delta t_{evol} = \xi D_c / V \tag{56}$$

where ξ scales the allowable slip increment per time step and is constrained to remain below a prescribed upper limit to ensure numerical stability and accuracy. Increasing ξ increases the allowable slip increment per time step, since $\Delta t_{evol} \propto \xi D_c/V$. Therefore, larger ξ values lead to larger time steps and lower computational cost, but they reduce the temporal resolution of accelerating slip, nucleation, and stress redistribution. Although the fully implicit formulation can often remain numerically stable for relatively large ξ, excessively large values may under-resolve rapid frictional evolution and introduce noticeable differences in slip velocity, rupture timing, and seismic moment. Following Lapusta & Liu (2009), ξ is therefore treated as an accuracy-control parameter rather than only a stability parameter. Their sensitivity analysis showed that values $\xi \leq 1/3$ produced nearly indistinguishable results, whereas $\xi = 1/2$ led to small differences. In this work, we use $\xi = 0.1$, which is more conservative than these limits and ensures enhanced temporal resolution during rapid fault slip.

2.5.2 Hybrid implicit-explicit solver-switching approach

The fully implicit scheme is advantageous during the pre-nucleation stage, where the response evolves slowly and relatively large time steps can be taken efficiently. In contrast, once rapid slip develops, the system becomes dominated by dynamic rupture processes, strong frictional nonlinearity, and rapidly varying inertial response. In this regime, an explicit update is well suited because it efficiently resolves fast dynamic motion without repeated nonlinear iterations at each time step. To exploit the advantages of both time integration schemes, we here introduce a hybrid time integration strategy that builds directly on the fully implicit framework described above. In the initial stage, the system is advanced using a fully implicit dynamic solver. This regime is maintained until the slip velocity on the fault reaches a prescribed threshold V_{th}. Beyond this point, the mechanical equation of motion is advanced using an explicit dynamic update, initialized from the state variables obtained at the final implicit time step. The pressure equation, however, remains implicitly solved using a backward Euler discretization during the dynamic phase.

Other switching parameters may also serve as indicators of dynamic activation, including rupture-front propagation speed, slip-velocity acceleration, kinetic energy, stress-drop rate, and variations in the contribution of inertial forces. However, slip velocity is selected here because it is a standard output of rate-and-state frictional models and directly reflects the onset of dynamic fault motion. In contrast, cumulative slip alone is not an appropriate switching variable, since substantial aseismic slip may accumulate before the onset of seismic or dynamic rupture. The use of V_{th} therefore provides a simple, physically motivated, and numerically practical criterion for detecting when the governing dynamics have shifted from slow fault loading to rapid rupture.

Given the strong nonlinearity of the problem during dynamic slip, the explicit phase employs a central difference time integration scheme. This method approximates the kinematic variables using staggered time levels, with displacements evaluated at integer time steps and velocities at half-time steps. It provides an efficient explicit update for transient dynamic problems and is therefore widely used in finite element dynamic analyses (Koutromanos, 2018). However, its stability remains conditional and requires a sufficiently small time step. The kinematic relations are expressed as (Koutromanos, 2018):

$$\dot{\boldsymbol{U}}_{n+1/2} = \frac{\boldsymbol{U}_{n+1} - \boldsymbol{U}_n}{\Delta t} \rightarrow \boldsymbol{U}_{n+1} = \boldsymbol{U}_n + \Delta t \dot{\boldsymbol{U}}_{n+1/2} \tag{57}$$

$$\ddot{\boldsymbol{U}}_n = \frac{\dot{\boldsymbol{U}}_{n+1/2} - \dot{\boldsymbol{U}}_{n-1/2}}{\Delta t} \rightarrow \dot{\boldsymbol{U}}_{n+1/2} = \dot{\boldsymbol{U}}_{n-1/2} + \Delta t \ddot{\boldsymbol{U}}_n \tag{58}$$

The equation of motion is enforced at the discrete time step n. Assuming that the displacement $\boldsymbol{U}_n$, the velocity $\dot{\boldsymbol{U}}_{n-1/2}$, and pressure $\boldsymbol{P}_n$, are known from the previous step, the acceleration is obtained from:

$$\left(\boldsymbol{M}+\frac{\Delta t}{2}\boldsymbol{D}\right)\ddot{\boldsymbol{U}}_n=\boldsymbol{F_{ext}}-\boldsymbol{F_{int}}-\boldsymbol{D}\dot{\boldsymbol{U}}_{n-1/2}-\boldsymbol{K}\boldsymbol{U}_n+\boldsymbol{Q}\boldsymbol{P}_n \tag{59}$$

Since all terms appearing on the right-hand side of Eq. (59) are known at time level n, the nodal acceleration $\ddot{\boldsymbol{U}}_n$ can be directly evaluated. Once the acceleration is obtained, the velocity is advanced using Eq. (58), followed by the update of the displacement field through Eq. (57). With the displacement at time step $n+1$ available, the normal gap is computed according to Eq. (8), allowing the fracture aperture to be updated accordingly. The pressure field is then advanced in time using a backward Euler discretization, yielding:

$$(\boldsymbol{S_{n+1}}+\Delta t\boldsymbol{H}_{n+1})\boldsymbol{P}_{n+1}=\boldsymbol{S_{n+1}}\boldsymbol{P}_n+\Delta t\left[\boldsymbol{q_{ext}}-\boldsymbol{C}\ddot{\boldsymbol{U}}_n-\boldsymbol{Q}^p{}_{n+1}\dot{\boldsymbol{U}}_{n+1/2}\right] \tag{60}$$

Numerical stability during the explicit phase is ensured by enforcing a time-step restriction based on the Courant–Friedrichs–Lewy (CFL) condition, which requires that wave propagation within a single time increment does not exceed the characteristic size of a finite element. The critical time step is therefore estimated as (Koutromanos, 2018):

$$\Delta t_{cr}=0.5\cdot\min_e\left(\sqrt{\frac{\rho^e}{E^e}}l^e\right) \tag{61}$$

where l^e denotes the characteristic element length, taken as the square root of the element area, and E^e is Young's modulus of the element. The explicit dynamic formulation is applied as long as the slip velocity on at least one fault element exceeds the prescribed threshold value V_{th}. Once the slip velocities across all fault elements fall below this threshold, the solution strategy reverts to the fully implicit dynamic formulation. A detailed flowchart illustrating how the adaptive hybrid solution strategy operates is shown in Figure 3. It is worth noting that the implicit scheme uses adaptive time stepping according to Eq. (55), whereas the explicit scheme employs a fixed small time step to satisfy the CFL stability condition. Therefore, the two schemes do not generally use the same time-step size.

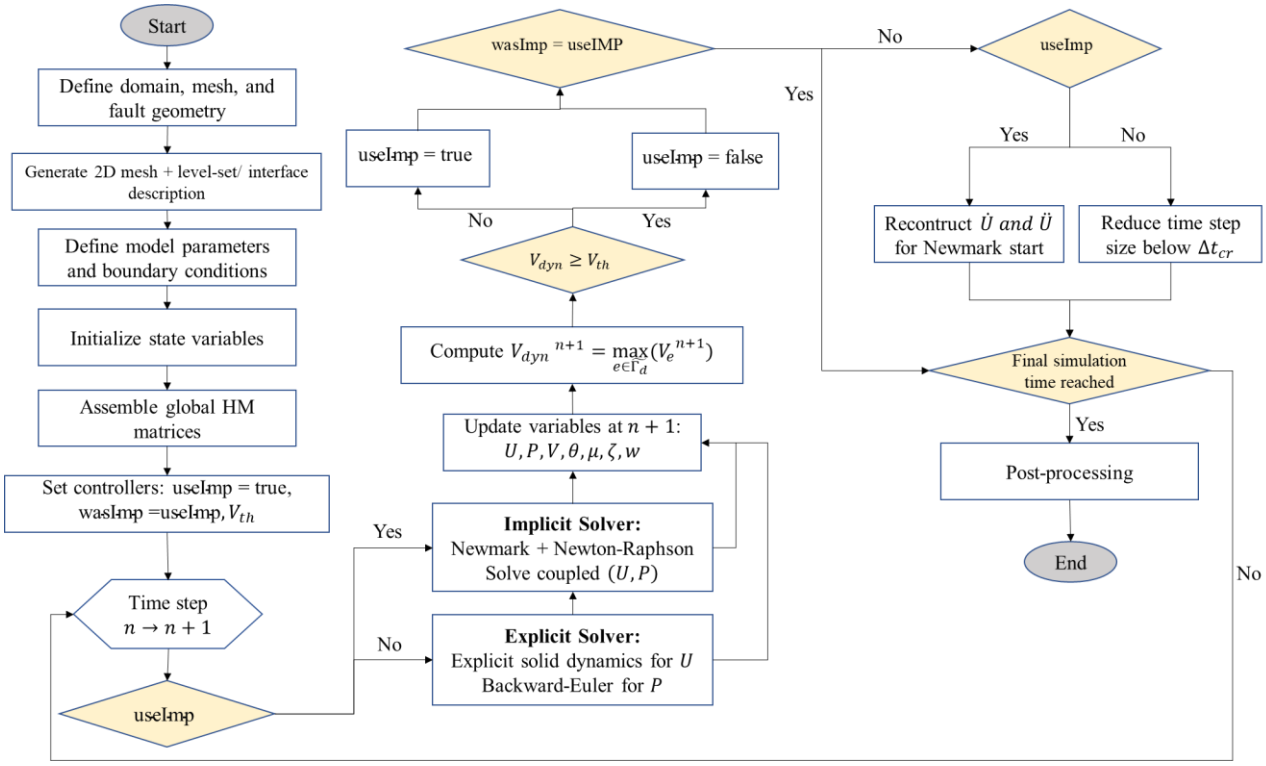


Figure 3: Schematic overview of the adaptive hybrid solution strategy. The model alternates between implicit and explicit solvers based on the fault slip-rate criterion $V_{dyn} \geq V_{th}$, allowing stable treatment of quasi-static deformation and efficient resolution of dynamic rupture.

2.6 Variation of fault permeability due to fault aperture change

In the present formulation, the hydraulic properties of the fault are allowed to evolve with deformation. We distinguish between the mechanical normal gap and the hydraulic aperture because directly using the mechanically computed normal gap may lead to complete hydraulic closure under compressive normal stress. This is inconsistent with natural faults, where fault residual transmissivity can remain due to surface roughness, asperity-supported contact, gouge heterogeneity, and unresolved flow channels. To account for this effect, we relate the hydraulic aperture to the effective normal stress through Barton–Bandis-type hyperbolic closure law (Bandis et al., 1983). The aperture evolution is therefore defined as:

$$w = \begin{cases} w_0 + \dfrac{P_N D_{n,max}}{k_{Ni} D_{n,max} + P_N}, & P_N < 0 \\ w_0 + g_N, & P_N \geq 0 \end{cases} \tag{62}$$

where w_0 is initial aperture, D_{n-max} is the maximum normal displacement under compressive normal stress, and k_{Ni} is initial normal stiffness of the fault. The parameter $D_{n,max}$ controls the residual hydraulic aperture under large compression; as $P_N \to -\infty$, the hydraulic aperture approaches $w_0 + D_{n,max}$, thereby preventing nonphysical negative or zero apertures, provided that $w_0 \geq |D_{n,max}|$. Here, $D_{n,max}$ is negative because it represents compressive normal closure. Under zero or tensile effective normal stress ($P_N \geq 0$), contact is inactive and the hydraulic aperture is taken as the current mechanical opening. Similar stress- and aperture-dependent hydraulic formulations have been widely used in hydromechanical models of fractured rocks and fault reactivation (Cappa et al., 2018; Kivi et al., 2024; Sabah et al., 2025). The hydraulic aperture obtained from this closure law is then used to compute the intrinsic fault permeability according to the parallel-plate relation:

$$k_f = \frac{w^2}{12} \tag{63}$$

where k_f is intrinsic fault permeability. Although Eq. (63) is quadratic in aperture, the hydraulic mobility coefficient governing along-fault flow in the lower-dimensional formulation follows the cubic law. Thus, the fault permeability is not prescribed as a constant material property; instead, it evolves with the current effective normal stress applied on fault interface. This stress-dependent permeability introduces an additional hydromechanical nonlinearity into the coupled system of equations described in section 2.5. Consequently, the inertial coupling matrix $\boldsymbol{C}$, hydraulic conductivity matrix $\boldsymbol{H}$, the storage matrix $\boldsymbol{S}$, external source term $\boldsymbol{q_{ext}}$, and coupling matrix $\boldsymbol{Q}^p$ in Eq. (38) are updated consistently using the current values of either w or k_f at each Newton-Raphson iteration. This, in turn, affects the pore-pressure field and, consequently, the poroelastic stress redistribution within the domain.

The main limitation of this formulation is that the hydraulic aperture is described by a local stress-dependent closure law. Thus, it does not explicitly resolve aperture heterogeneity, roughness-induced channeling, gouge migration, damage, chemical effects, or shear-induced permeability enhancement beyond the computed normal opening. Moreover, the cubic law assumes laminar flow between locally parallel plates, so the resulting permeability should be interpreted as an effective transmissivity rather than a fully resolved fault-pore structure. Despite these simplifications, the formulation consistently incorporates stress-dependent fault transmissivity while preventing nonphysical hydraulic closure under compressive contact.

3 Verification and model setup

The fully implicit hydromechanical formulation employed in this study has been previously verified and validated in detail in Sabah et al. (2026). That work included comprehensive verification against analytical solutions for saturated porous media (Wang, 2000), established numerical benchmarks for fractured porous systems (Damirchi et al., 2022), and the SEAS BP3-

QD benchmark for assessing the rate-and-state friction implementation and the nucleation-stage slip evolution (Erickson et al., 2020). In the present study, the fully implicit formulation is adopted with modifications to the enforcement of contact constraints and is used as the reference solver. Building on this verified framework, the proposed hybrid solution strategy is introduced and systematically compared with the fully implicit reference solution in terms of fault response and numerical performance. To further assess the explicit elastodynamic component of the proposed hybrid framework, we also used the TPV205-2D dynamic rupture benchmark from the SCEC/USGS Earthquake Code Verification Exercise (Harris et al., 2009). It should be noted that TPV205-2D is based on a slip-weakening friction law rather than a rate-and-state friction law. Therefore, this benchmark is used specifically to verify the elastodynamic rupture component of the solver, rather than the rate-and-state friction implementation. For completeness, the verification results for both the fully implicit and explicit components of the framework are provided in the Supplementary Material.

The computational domain is a square region measuring 400 m × 400 m, representing a horizontal cross-section of a fractured reservoir subjected to anisotropic in-situ stresses. As illustrated in Figure 4, the pre-existing fault is represented in the XFEM discretization as a lower-dimensional, 1D line segment embedded in the 2D porous domain. Its end points are located at (100 m, 101 m) to (300 m, 301 m), giving a total fault length of about 282.8 m. The fault is orientated at 45° counterclockwise from the horizontal x-axis. A fluid injection well is positioned near the fault center at (188 m, 212 m) to promote fault activation through pore-pressure diffusion and poroelastic stress redistribution.

All boundaries are mechanically constrained: the left and right boundaries are fixed in the x-direction, while displacements along the bottom and top boundaries are constrained in the y-direction. Uniform compressive in-situ stresses are applied within the domain, with $\sigma_x = -60$ MPa and $\sigma_y = -20$ MPa, where the negative sign denotes compression. Together with an initial friction coefficient of $f_0 = 0.6$, this stress state places the fault close to critical failure. The pore pressure boundary condition is set to $P = 0$ MPa (drained condition) along the right edge, while the remaining boundaries are hydraulically impermeable. The initial pore pressure throughout the domain is set to $P_0 = 0$ MPa. The initial slip velocity along the fault is assumed to be equal to the reference slip velocity, V^*. During the simulations, the slip velocity is not allowed to decrease below this reference value, in order to avoid numerical singularities associated with the logarithmic dependence of the rate-and-state friction law at very small slip velocities. Figure 5 presents the initial fault conditions after mechanical equilibrium is reached. Specifically, Figure 5a shows the distributions of shear stress, effective normal stress, and Coulomb ratio along the fault, while Figure 5b shows the corresponding aperture distribution along the fault. Note that initial fault aperture before applying the mechanical loading w_0 is set to 0.001 m.

After the model reaches mechanical equilibrium under the prescribed initial stress conditions, fluid injection is initiated at a constant rate of $5 \times 10^{-4}\ m^3(s.m)^{-1}$ from the injection well at $t = 0$ s and continues throughout the simulation period (36 h). This injection induces a progressive increase in pore pressure and associated poroelastic deformation in the vicinity of the fault. The entire domain is discretized using 40,000 bilinear quadrilateral elements (200 × 200 mesh). Rock, fault, and fluid properties are summarized in Table 1.

4 Results and discussion

4.1 Fully implicit solution

In this section, adaptive time stepping within the implicit solution framework is examined using three different minimum time-step sizes: 0.001 s, 0.0001 s, and 0.00001 s. The minimum time step represents the lower bound below which the adaptive algorithm is not allowed to further reduce the time increment. This analysis is motivated by the fact that, although the implicit Newmark scheme adopted in this study is stable for the selected integration parameters, stability alone does not guarantee

sufficient temporal accuracy. In particular, during rupture nucleation and dynamic propagation, rapid variations in slip velocity, stress redistribution, and seismic response require adequate temporal resolution. Therefore, the effect of the prescribed minimum time-step size is investigated to assess its influence on the accuracy of the simulated dynamic rupture response. In all simulations, the maximum allowable time-step size is fixed at 3600 s.

Table 1. Material properties

Type	Parameter	Value
Rock mass	Poisson's ratio	0.3
	Young's modulus (GPa)	50
	Intrinsic permeability (m^2)	2×10^{-15}
	Biot coefficient	0.75
	Rock density (kg/m^3)	2500
	Porosity	0.1
Injection fluid	Fluid density (kg/m^3)	1000
	Fluid viscosity (Pa s)	0.001
	Fluid compressibility (Pa^{-1})	5×10^{-9}
Fault	Reference friction coefficient	0.6
	Reference slip velocity (m/s)	10^{-9}
	Reference state variable (s)	10^5
	Characteristic slip distance (mm)	0.1
	Rate-and-state parameters	$a = 0.01,\ b = 0.02$
	Initial normal stiffness (GPa/m)	20
	Initial shear stiffness (GPa/m)	20
	Maximum normal displacement (mm)	-1

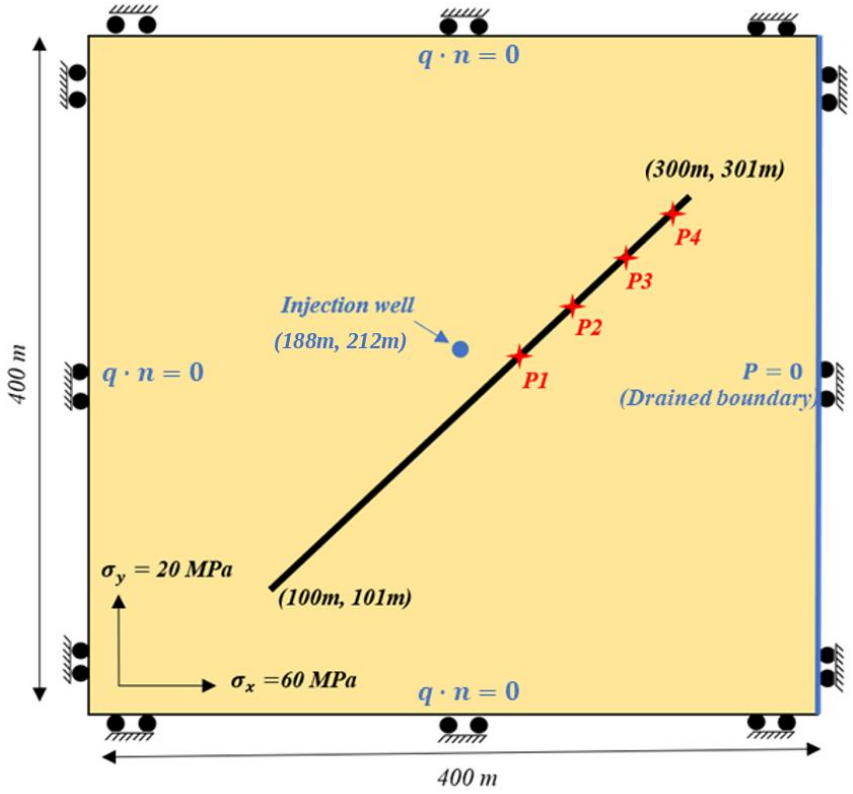


Figure 4: Schematic of a single-phase fluid injection scenario in a homogeneous porous formation containing a fault. The injection well is located at coordinates (188 m, 212 m), and the fault spans from (100 m, 101 m) to (300 m, 301 m). Four observation points along upper part of fault (denoted P1–P4) are used to monitor fault response. The reservoir is subjected to anisotropic in-situ compressive stresses of $\sigma_x = 60$ MPa and $\sigma_y = 20$ MPa, applied as initial stress conditions throughout the domain rather than as boundary tractions. All external boundaries are mechanically constrained by roller boundary conditions, allowing tangential displacement while preventing normal motion. The pore-pressure boundary condition is set to $P = 0$ MPa (drained condition) along the right boundary, while the remaining boundaries are hydraulically impermeable.

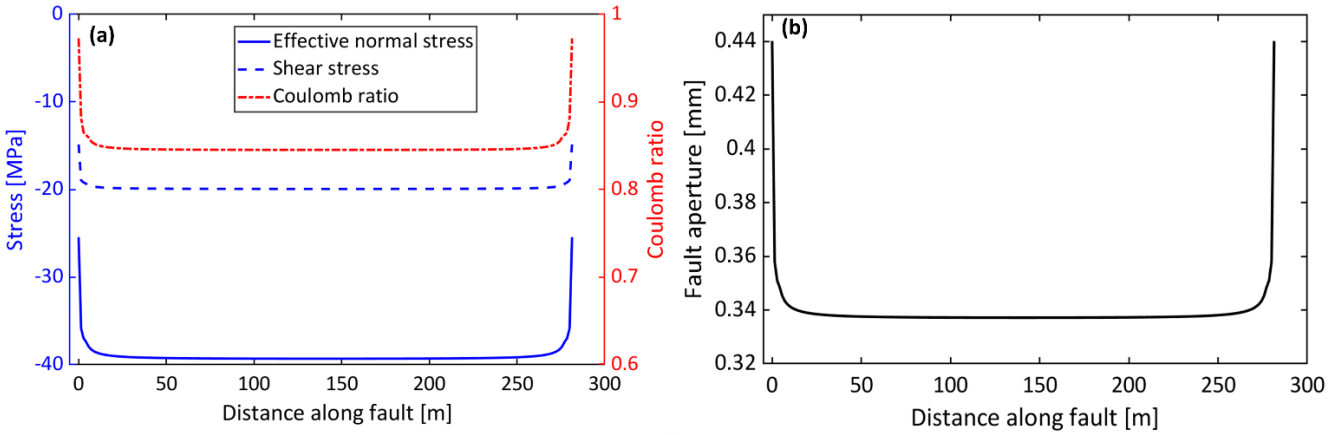


Figure 5. Initial fault conditions used in the simulations after reaching mechanical equilibrium: (a) distributions of effective normal stress, shear stress, and Coulomb ratio along the fault; and (b) corresponding initial fault aperture distribution. These equilibrated values are used as the starting state for the subsequent dynamic simulations.

Figure 6 presents the simulation results for the three minimum time-step sizes. The right column corresponds to 0.001 s, the middle column to 0.0001 s, and the left column to 0.00001 s. The results indicate that a minimum time step of 0.001 s provides insufficient temporal resolution during the dynamic rupture phase. In contrast, reducing the minimum time step to 0.0001 s and 0.00001 s leads to increased slip accumulation, higher peak slip rates, and earlier termination of the seismic phase. Despite these differences, the upper and lower bounds of the friction coefficient and shear stress remain largely unchanged across the three cases.

Figure 7 compares the different approaches in terms of average slip, seismic moment, moment magnitude, and rupture speed normalized by the shear wave velocity. Larger minimum time-step sizes result in lower average slip, smaller seismic moments, and lower rupture speed (Fig. 7a, c-d). However, the differences in moment magnitude are relatively minor due to its logarithmic nature (Fig. 7-b). The results obtained with minimum time steps of 0.0001 s and 0.00001 s are similar, indicating that 0.0001 s provides an optimal compromise between numerical accuracy and computational efficiency. Although slight differences in nucleation time are observed among the three time-step sizes, these differences are on the order of milliseconds and are therefore negligible relative to the overall simulation timescale.

For the fully implicit simulations analyzed above, the time-step size is controlled adaptively using Eq. (55), which ensures that slip at each time step remains smaller than a prescribed fraction of the characteristic slip distance (Lapusta and Liu, 2009; Lapusta et al., 2000). This adaptive procedure is essential for resolving pre-dynamic processes such as nucleation, accelerating slip, and localization, and helps prevent important transient fault responses from being missed or delayed before the onset of fully dynamic rupture. Once fully dynamic rupture develops, however, accurate resolution of rupture propagation and the associated wave field requires further reduction of the time-step size. The results show that, even within an implicit solution framework, the minimum time step required to adequately resolve the dynamic phase is of the same order as the CFL-restricted explicit time step (Eq. 61), which is approximately 0.00023 s in the present case. In addition, Lapusta & Liu (2009) proposed a minimum time-step estimate for achieving sufficient resolution of dynamic rupture propagation in elastodynamic simulations:

$$\Delta t_{min} = \gamma \cdot \min_{e} \left(\frac{l^e}{c_s} \right) \tag{64}$$

where γ is constant, taken here as $1/3$ following Day et al. (2005), and c_s represents the shear wave speed. Using the material properties and mesh size adopted in the present simulations, this estimate gives $\Delta t_{min} = 0.00024$ s. This value is also consistent with the minimum time-step size required in our fully implicit simulations to obtain a well-resolved dynamic rupture response. This agreement supports our finding that, although the employed fully implicit scheme is not conditionally stable in

the same way as the explicit scheme, resolving fully dynamic rupture and associated wave propagation still requires a very small time step comparable to that used in the explicit phase. However, achieving this numerical resolution during dynamic rupture comes at the cost of increased computational expense, with the CPU time increasing from approximately 4.5 h for $\Delta t_{min} = 0.001$ s to about 10.9 h for $\Delta t_{min} = 0.0001$ s. The computational cost of this approach for different minimum time-step sizes, with and without inclusion of the slip tangent operator during Jacobian assembly, is analyzed in detail in Appendix A.

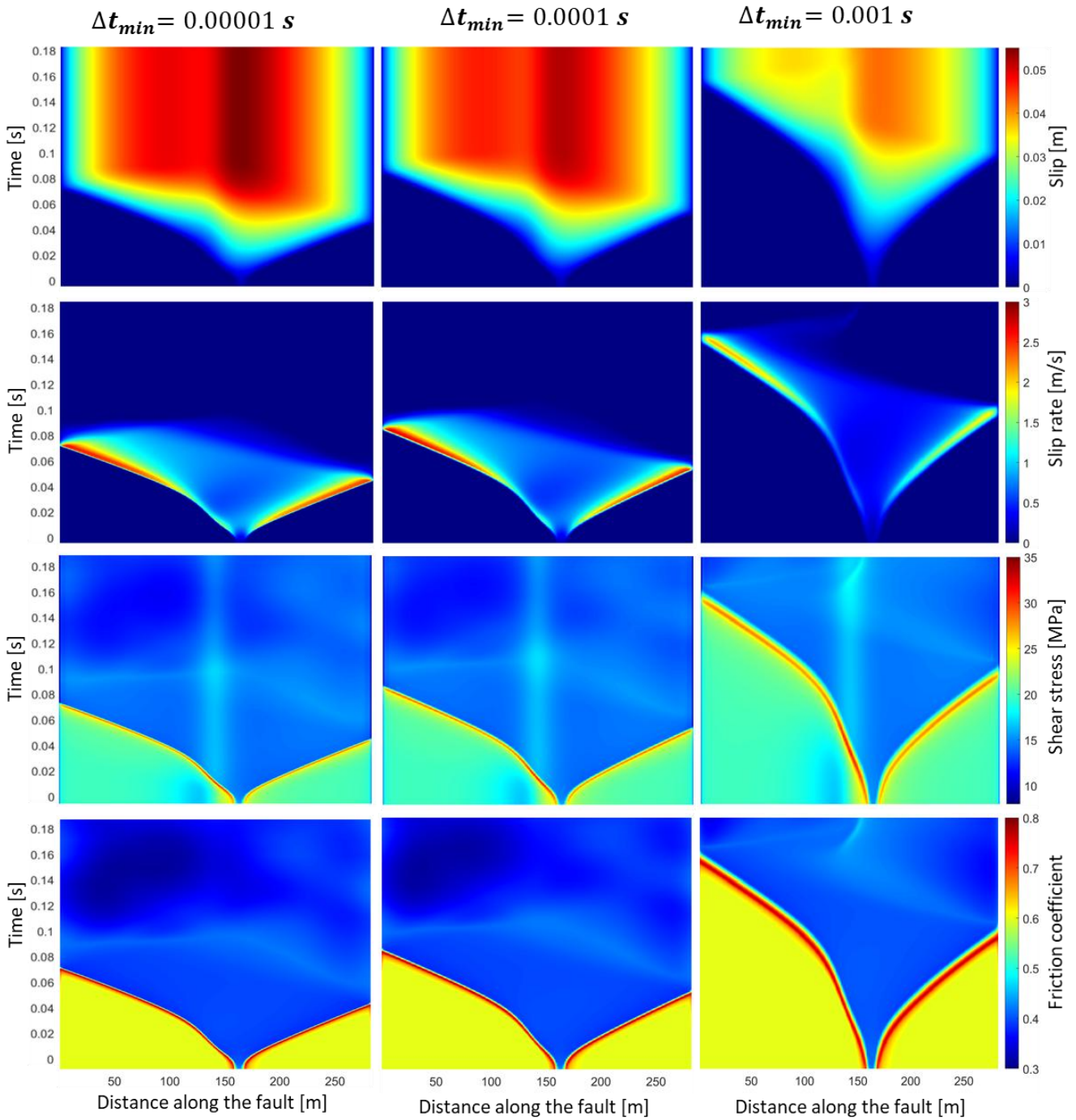


Figure 6: Space–time evolution of fault slip, slip rate, shear stress, and friction coefficient for three minimum time-step sizes: $\Delta t_{min} = 0.00001\ s$ (left), $\Delta t_{min} = 0.0001\ s$ (middle), and $\Delta t_{min} = 0.001\ s$ (right). For each simulation, the time axis is shifted independently such that $t = 0$ corresponds to the instant when the maximum slip velocity on the fault first exceeds 0.1 m/s. Therefore, the shifted time origin is used only to align the onset of dynamic rupture for comparison; the field variables at $t = 0$ are not imposed to be identical and may differ due to differences in the preceding quasi-static/pre-dynamic evolution. The results highlight the influence of temporal resolution on rupture nucleation and the subsequent dynamic fault response.

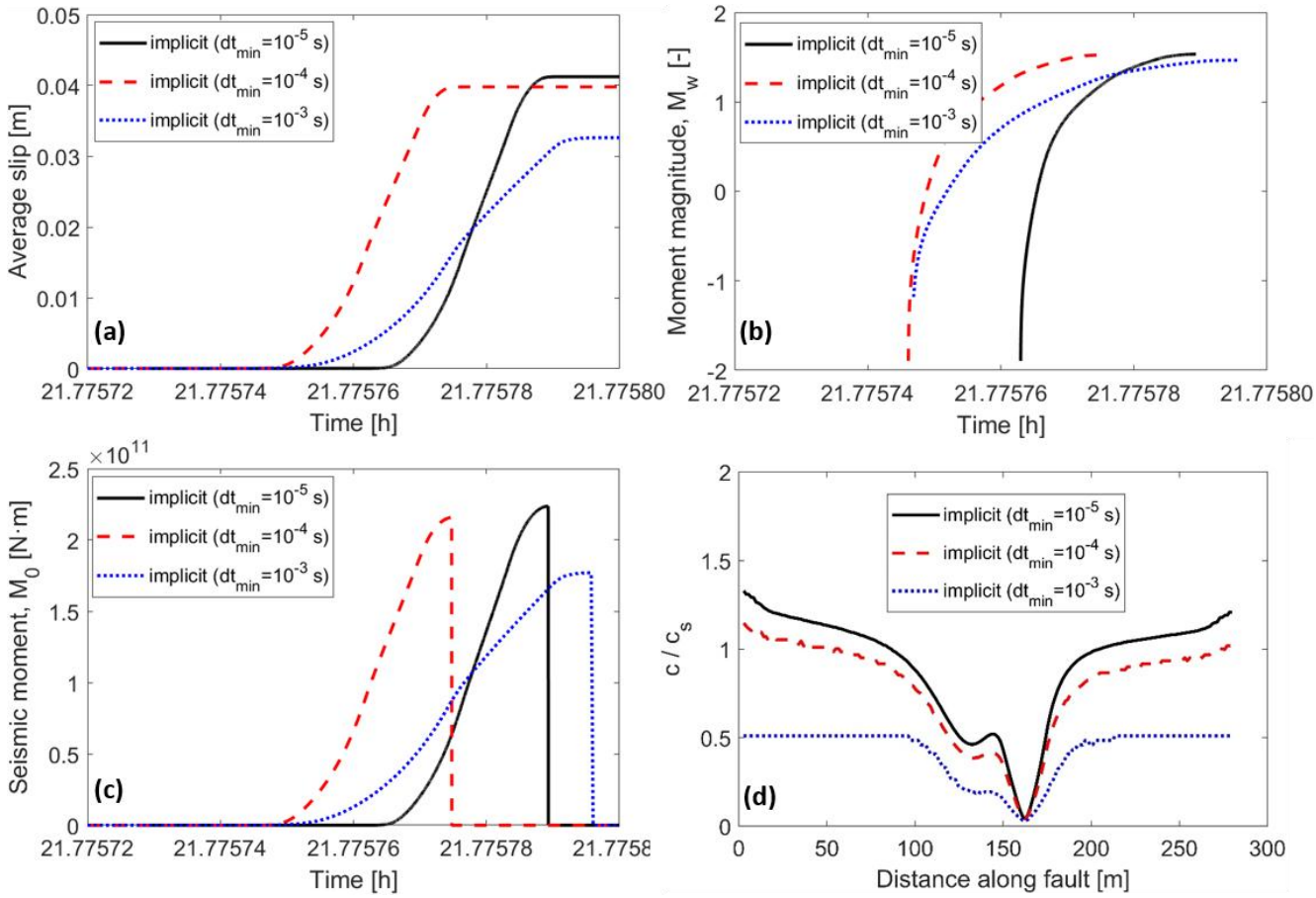


Figure 7: Influence of the minimum time-step size on event evolution for fully implicit simulations. Shown are the time histories of average fault slip (a), moment magnitude (b), seismic moment (c), and the spatial distribution of rupture speed normalized by the shear-wave velocity (d) for $\Delta t_{min} = 0.00001$ s, $\Delta t_{min} = 0.0001$ s, and $\Delta t_{min} = 0.001$ s. In panels (a–c), time is measured from the start of fluid injection. Differences highlight the sensitivity of rupture timing and magnitude estimates to temporal resolution during dynamic instability.

4.2 Hybrid solution approach

In the proposed solution strategy, the implicit solver efficiently resolves the quasi-static and nucleation phases, while the explicit solver captures the ensuing dynamic fault slip and rupture propagation in a computationally efficient manner. The transition between implicit and explicit solvers is controlled by a maximum slip-velocity threshold attained on any fault element. An important practical question is therefore the choice of an optimal slip-velocity threshold that minimizes the total computational cost without degrading numerical accuracy.

To assess the computational efficiency of the proposed hybrid time-integration strategy, we report the total CPU time required to complete each simulation and compare it with the corresponding fully implicit solution. In the fully implicit approach, a minimum time step of $\Delta t_{min} = 0.0001$ s is prescribed; below this value, the adaptive time-stepping scheme does not further reduce the time step during the seismic phase. For the hybrid solution approach, the same minimum time step of $\Delta t_{min} = 0.0001$ s, which is smaller than the maximum allowable value imposed by the CFL condition ($\Delta t_{cr} = 0.00023$ s).

Figure 8 shows that the hybrid strategy consistently reduces the computational cost relative to the fully implicit approach, with CPU-time reductions of approximately 58–71%, depending on the selected slip-velocity threshold V_{th}. In the fully implicit approach, the time-step size is controlled adaptively throughout the entire simulation, with the prescribed minimum time step defining the lower bound of the adaptive algorithm. In the hybrid approach, the implicit phase uses the same adaptive time-stepping strategy as the fully implicit simulation, whereas the explicit phase is advanced using a fixed time-step size to satisfy the CFL condition.

The threshold is varied within the range $V_{th} \in [5 \times 10^{-4}, 10^{-1}]$ m/s. Below this range, the explicit phase is activated too early and therefore dominates the overall runtime, making the hybrid approach inefficient. Conversely, for thresholds above this range, the contribution of the explicit solver to the total computational efficiency diminishes; as observed for $V_{th} > 0.1$ m/s, the relative CPU-time reduction no longer improves and begins to decrease. The largest computational savings are obtained for intermediate thresholds, with a maximum reduction of about 71% at $V_{th} = 0.01$ m/s, indicating an optimal balance between implicit and explicit solver contributions. A more detailed comparison of the hybrid solution approach, with and without inclusion of the slip tangent operator during the implicit phase, together with its comparison against the fully implicit approach in terms of computational cost, is provided in Appendix B.

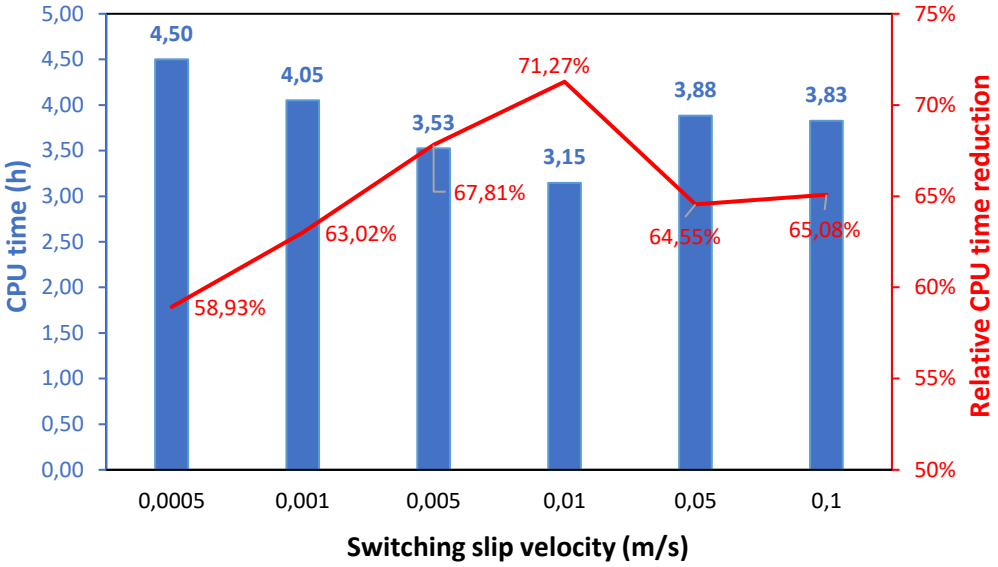


Figure 8: Computational performance of the hybrid solution strategy as a function of the slip-velocity threshold used to switch solvers. Shown are the total CPU time (blue bars) and the corresponding CPU-time reduction relative to a fully implicit simulation (red line).

Figure 9 compares the temporal evolution of the fault response at four observation points along the fault (P1–P4) obtained using the fully implicit and hybrid solution strategies. In the simulations using the hybrid approach, the transition from implicit to explicit time integration is triggered when the local slip velocity at any fault element exceeds 0.01 m/s.

The slip-rate histories (Fig. 9a) show that both approaches capture the onset, propagation, and overall sequence of dynamic rupture consistently across all observation points. The primary differences between the two approaches are confined to the peak slip velocities, which are around 25-30% higher in the fully implicit simulations. Despite these differences, the timing of rupture arrival and the relative ordering of peak slip rates along the fault remain consistent. The corresponding shear stress evolution (Fig. 9b) exhibits similar stress accumulation prior to instability and rapid stress drop during rupture for both schemes, indicating that the transition between implicit and explicit solvers does not introduce spurious stress artifacts. The accumulated plastic slip (Fig. 9c) demonstrates close agreement between the two approaches, with final slip magnitudes at all observation points remaining consistent despite the different integration schemes. Similarly, the evolution of Coulomb failure stress (ΔCFS) (Fig. 9d) shows comparable timing and amplitude of instability, confirming that the hybrid solution strategy reproduces the key failure conditions resolved by the fully implicit solver. The friction coefficient evolution (Fig. 9e) highlights the expected rapid weakening during dynamic slip, followed by post-rupture recovery, with both approaches exhibiting nearly identical trends. Finally, the state variable evolution (Fig. 9f) shows consistent rapid evolution during rupture and gradual post-seismic recovery, indicating that the rate-and-state frictional response is preserved across the solver transition.

Overall, the figure demonstrates that the hybrid solution approach, using a slip-velocity threshold of 0.01 m/s for solver switching, accurately reproduces the spatial and temporal evolution of key fault variables obtained with a fully implicit formulation, while enabling substantial reductions in computational cost.

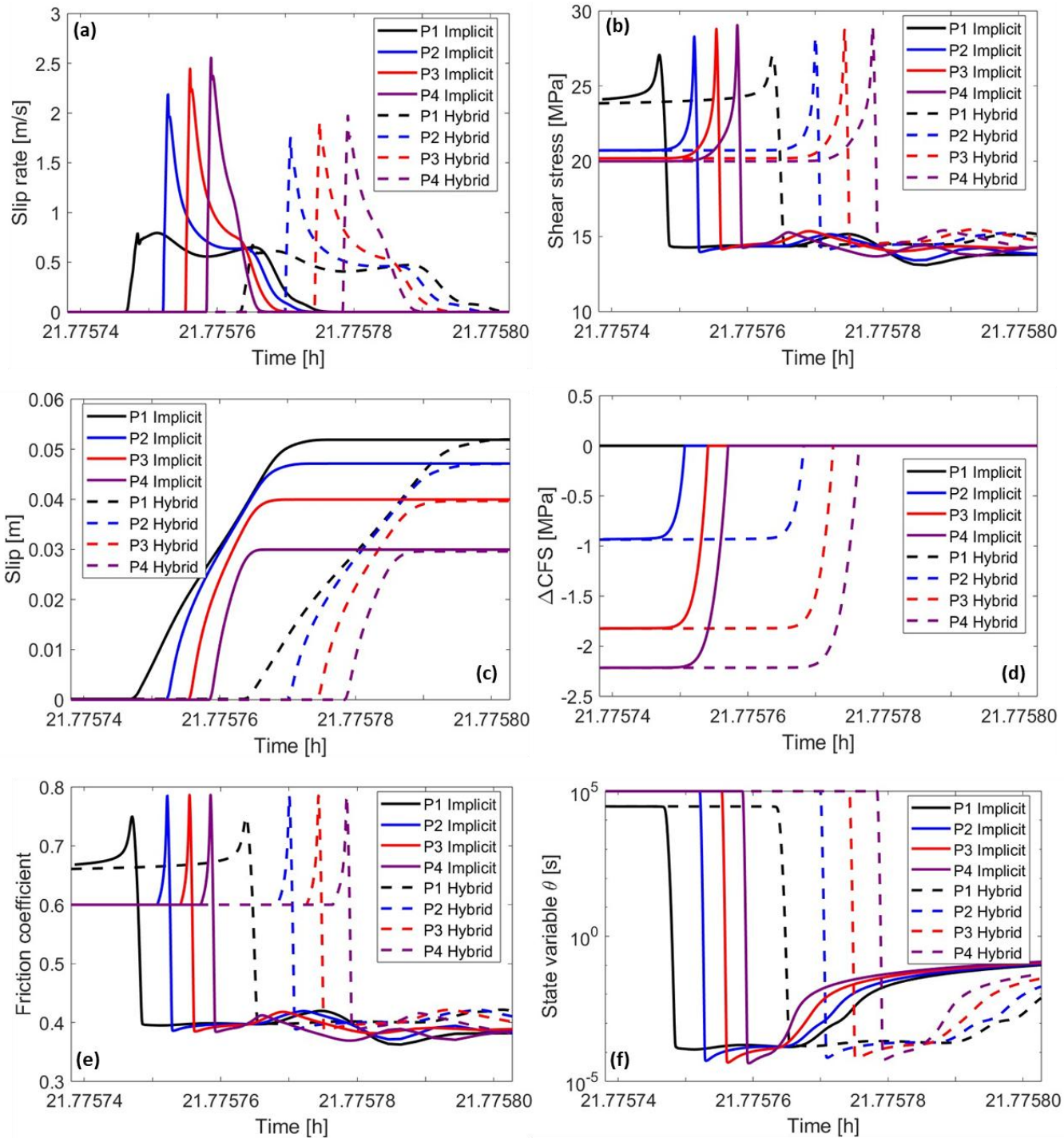


Figure 9: Comparison of fault response at four observation points along the fault (P1–P4) obtained with the fully implicit and hybrid solution strategies. Shown are the time histories of slip rate (a), shear stress (b), accumulated plastic slip (c), Coulomb failure stress change (d), friction coefficient (e), and state variable (f). Time is measured from the start of fluid injection. Solid lines denote the implicit phase, while dashed lines indicate the explicit phase following the solver switch, triggered when the slip-velocity threshold of 0.01 m/s is exceeded. Both approaches capture the rupture sequence consistently, with minor differences primarily confined to peak slip velocities

Figure 10 compares key event-scale seismicity metrics obtained with the fully implicit solution and the hybrid approach. The temporal evolution of average slip, moment magnitude, and seismic moment shows that both approaches reproduce the overall rupture evolution and final event size with close agreement (Fig. 10, a-c), with only minor differences in rupture timing. The most pronounced difference is observed in the normalized rupture speed for which the fully implicit solution predicts

systematically higher rupture velocities than the hybrid solution approach (Fig. 10d). Despite this difference, the spatial pattern of rupture propagation remains consistent between the two methods. Overall, these results indicate that the hybrid formulation preserves the essential characteristics of fault rupture and seismic moment release resolved by the fully implicit solver, while introducing only limited deviations in rupture speed and enabling substantial computational savings.

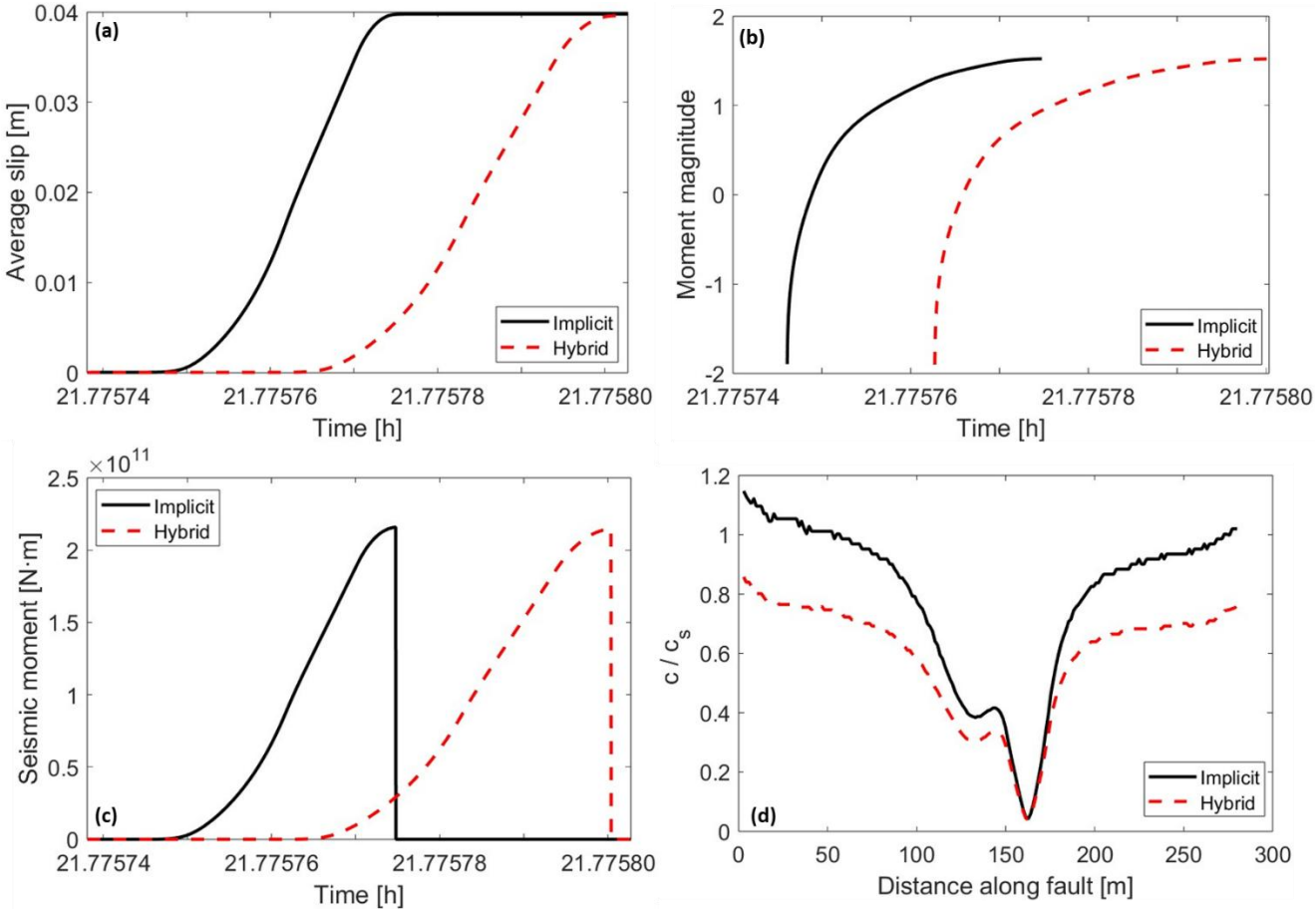


Figure 10: Comparison of event-scale seismicity metrics obtained with the fully implicit and hybrid solution strategies. Shown are the temporal evolutions of average slip (a), moment magnitude (b), and seismic moment (c), together with the spatial distribution of normalized rupture speed along the fault (d). In panels (a–c), time is measured from the start of fluid injection The hybrid solution approach reproduces the overall rupture evolution and final event size with only minor differences in timing, while the fully implicit solution predicts systematically higher rupture speeds.

Figure 11 compares the computed synthetic particle-motion evolutions at the location of the injection well obtained with the fully implicit and hybrid approaches. The acceleration time histories (Fig. 11, a-b) show that the hybrid simulations exhibit slightly higher peak accelerations than the fully implicit solution, reflecting the reduced numerical damping associated with explicit integration. In contrast, the velocity records (Fig. 11, c-d) indicate that the fully implicit solution yields higher peak velocities, while the hybrid solution results remain slightly attenuated. A small delay in wave arrival time is observed for the hybrid simulations relative to the implicit solution, and this delay increases modestly with increasing slip-velocity threshold; however, the absolute timing differences remain negligible. The displacement evolution (Fig. 11, e-f) is nearly identical for all three approaches, with all methods converging to the same final values in both directions. Overall, these results indicate that the hybrid strategy preserves the essential dynamic response and final deformation while introducing only minor differences in transient wave characteristics. Spatial distributions of acceleration, velocity, and displacement over the computational domain during the seismic events, simulated using both the fully implicit and hybrid solution approaches, are provided in Appendix C.

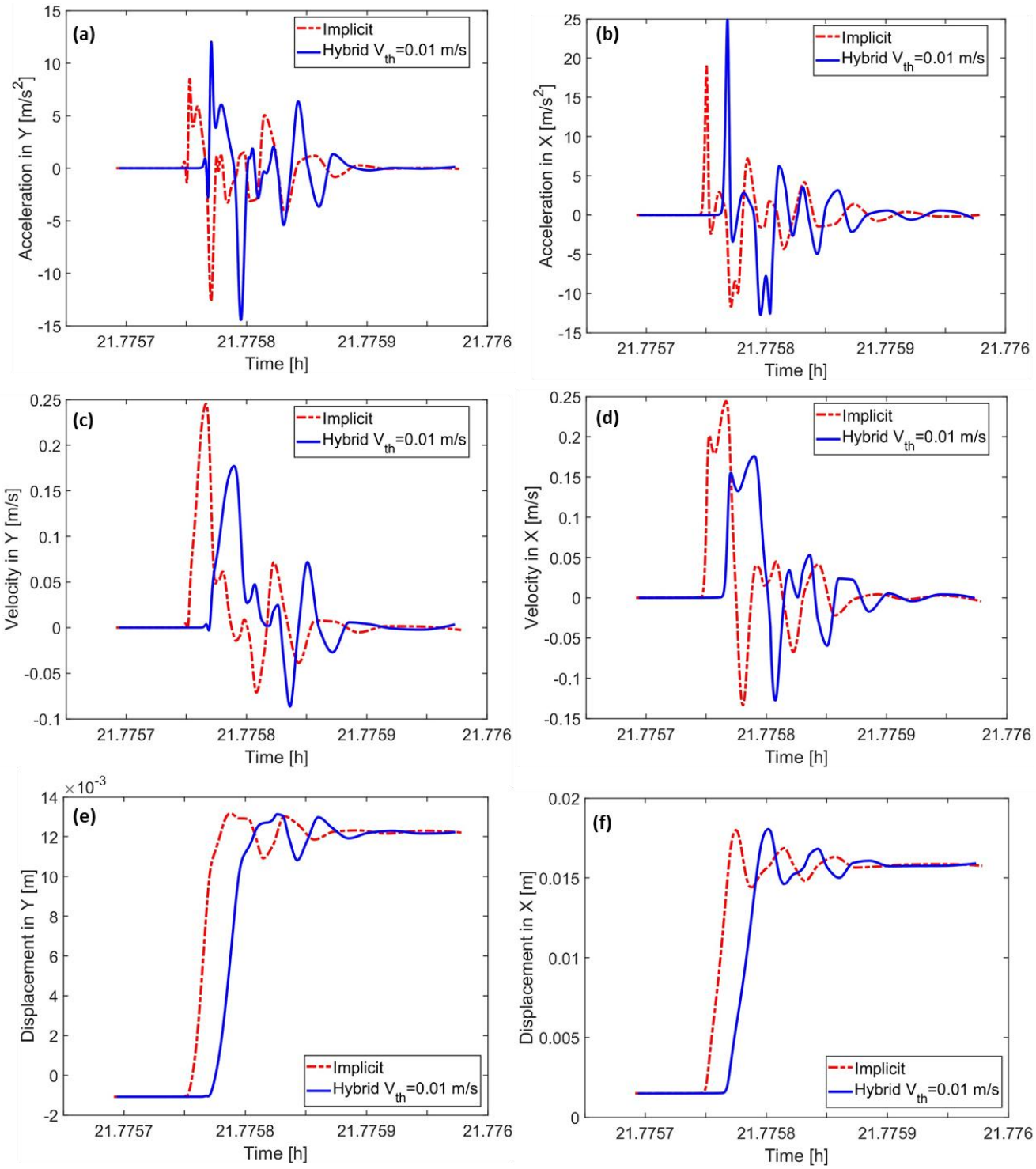


Figure 11: comparison of acceleration, velocity, and displacement time histories at the location of the injection well obtained with the fully implicit and hybrid approaches. Time is measured from the start of fluid injection. Hybrid simulations exhibit slightly higher peak accelerations, while the implicit solution yields higher peak velocities; all approaches converge to the same final displacement, with only minor delays in wave arrival for hybrid solution strategy.

4.3 Sensitivity of the hybrid solution to mesh resolution and explicit-phase parameters

To further investigate the differences between the fully implicit and hybrid solution strategies, we first assessed the mesh sensitivity of the hybrid solution and subsequently examined the effects of the switching slip-velocity threshold and the minimum time-step size used during the explicit phase. Hybrid simulations were performed using five mesh resolutions to determine the extent to which the predicted response depends on spatial discretization (Fig. 12). The analysis focuses on slip rate because, among the variables presented in Fig. 9, it exhibits the most noticeable difference between the fully implicit and hybrid solutions. As shown in Fig. 12, mesh refinement produces only a modest increase in the peak slip rate, with a coarsest-

to-finest variation of 6.99%, even for the finest mesh containing 255×255 elements. The small difference between the results obtained using the 200×200 and 255×255 meshes indicates that the hybrid solution is effectively mesh-converged at the 200×200 resolution adopted in the main simulations. Further mesh refinement would therefore substantially increase the computational cost while providing only a negligible improvement in the predicted response. These findings suggest that the remaining differences between the fully implicit and hybrid solutions are primarily attributable to the time-integration strategy, particularly the switching slip-velocity threshold and the explicit time-step size.

Figure 13 presents the sensitivity of the slip-rate evolution measured at monitoring point P4 during the seismic event to these two numerical parameters. Figure 13a compares the fully implicit solution with hybrid simulations in which the explicit time-step size is systematically reduced from 2×10^{-4} s to 5×10^{-6} s with fixed switching velocity threshold of 0.01 m/s. Reducing the explicit time-step size produces a slight increase in the peak slip rate and shifts the slip-rate profile toward earlier times, closer to the fully implicit response. However, even when the explicit time-step size is reduced by 1.6 orders of magnitude, the hybrid and fully implicit solutions do not coincide exactly. In particular, for minimum time step sizes below 1×10^{-5} s, no significant further change is observed in either the peak slip rate or the temporal shift of the slip-rate profile. Figure 13b shows the sensitivity of the slip-rate response at point P4 to the switching velocity threshold V_{th} for a fixed explicit time-step size of 1×10^{-4} s. The results indicate that decreasing V_{th} shifts the slip-rate profile toward the fully implicit solution. The closest agreement among the tested thresholds is obtained for $V_{th} = 0.0005$ m/s. However, for $V_{th} \geq 0.0005$ m/s, the slip-rate curves show only minor differences in peak timing and temporal evolution, suggesting that the response becomes nearly insensitive to further increases in the switching threshold.

Overall, the results show that reducing the explicit time-step size and lowering the switching threshold improve the agreement between the hybrid and fully implicit solutions. However, an exact match is not achieved within the tested parameter range. Closer agreement would require substantially smaller explicit time steps and/or lower switching thresholds, which would significantly increase the computational cost. The remaining discrepancy is also expected to partly reflect the different numerical characteristics of the central-difference explicit update compared with the fully implicit time-integration scheme.

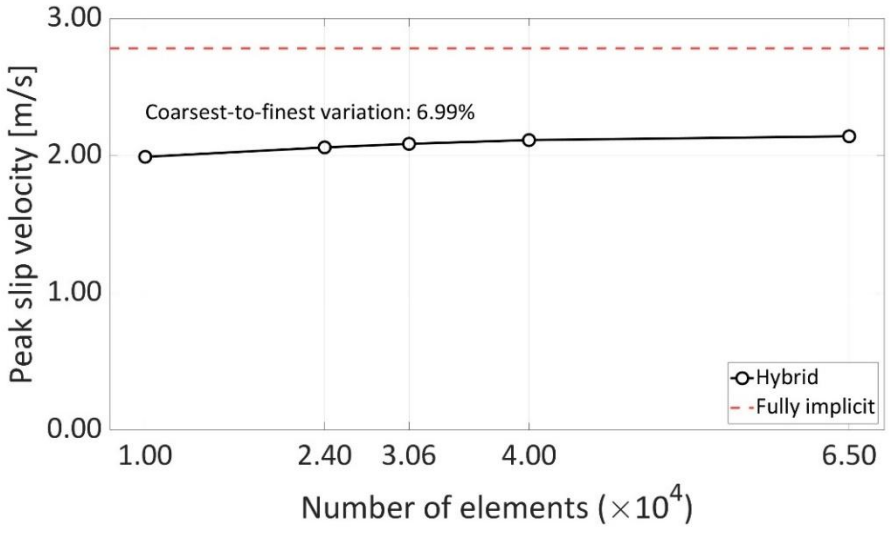


Figure 12. Mesh-sensitivity analysis of the peak slip velocity predicted by the hybrid solution strategy for mesh resolutions ranging from 100×100 to 255×255. The peak slip velocity increases slightly with mesh refinement, with a coarsest-to-finest variation of 6.99%. The dashed line indicates the corresponding fully implicit solution obtained using the 200×200 mesh.

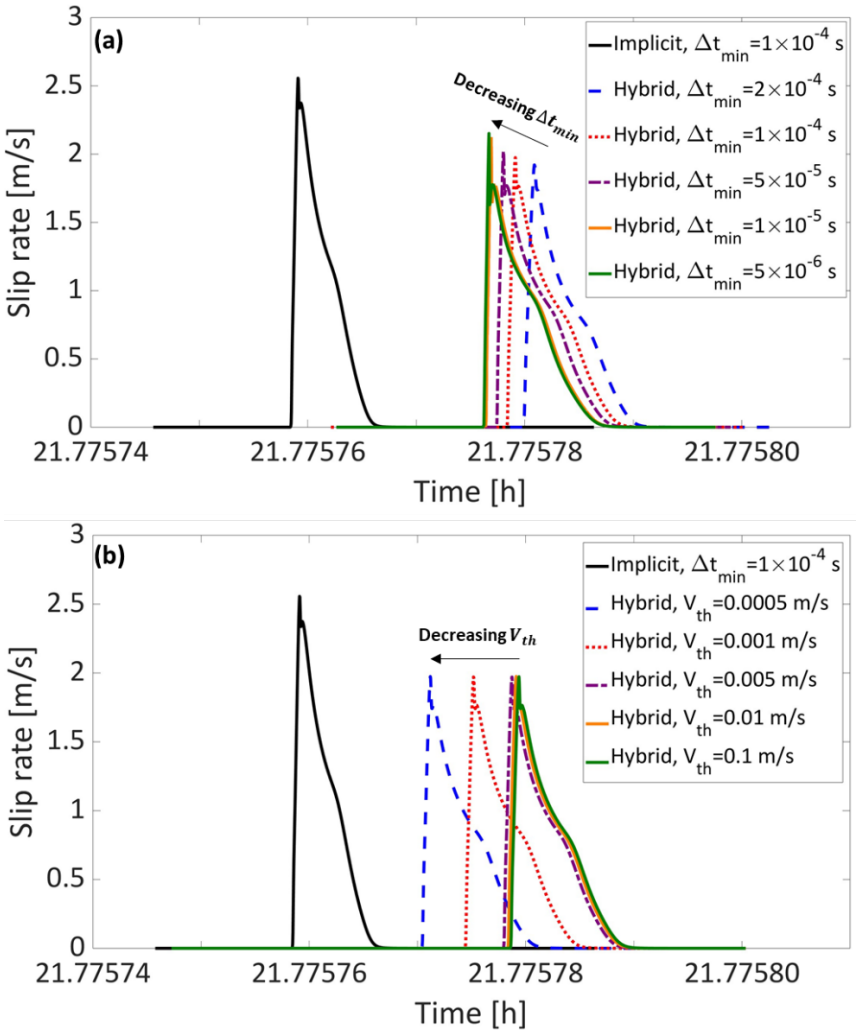


Figure 13. Sensitivity of the slip-rate response at point P4 during the seismic event to the numerical parameters of the hybrid implicit–explicit scheme. (a) Comparison between the fully implicit solution and hybrid solutions obtained using different minimum time-step sizes during the explicit phase for a fixed switching criterion. (b) Comparison between the fully implicit solution and hybrid solutions obtained using different switching slip-velocity thresholds for a fixed explicit time-step size.

5 Conclusion

This study introduced a hybrid time-integration strategy for simulating injection-induced fault reactivation within a fully coupled hydromechanical XFEM framework. The proposed approach directly addresses the fundamental challenge posed by the extreme separation of time scales inherent to fluid-driven seismicity, ranging from slow reservoir pressurization to rapid fault nucleation and dynamic rupture.

Using a fully implicit poroelastodynamic formulation as a reference solution, we demonstrated that although Newmark implicit scheme can be unconditionally stable, its accuracy during rupture nucleation and propagation is strongly controlled by the minimum time-step size. Insufficient temporal resolution leads to delayed instability, underestimated slip rates, and reduced seismic moments. Our results further showed that the minimum time step required to accurately capture rupture dynamics is of the same order as the critical time step dictated by the CFL condition, highlighting an intrinsic limitation of purely implicit approaches for dynamic faulting problems.

To overcome this limitation, the proposed hybrid strategy advances the system implicitly during quasi-static loading and nucleation phases, and automatically transitions to an explicit dynamic solver once a prescribed slip-velocity threshold is exceeded. Systematic numerical experiments demonstrated that this adaptive solver switching preserves the accuracy of key fault response metrics, including slip evolution, shear stress, frictional weakening, Coulomb failure stress, and seismic moment, while substantially reducing computational cost. Depending on the selected slip-velocity threshold, the hybrid

approach achieved CPU-time reductions of approximately 60–70% relative to the fully implicit formulation, with an optimal balance between efficiency and accuracy obtained for intermediate threshold values.

Comparison of fault-scale and event-scale response revealed that the hybrid formulation reproduces rupture timing, spatial propagation patterns, and final deformation with close agreement to the fully implicit solution. Differences are primarily confined to peak slip velocities and rupture speeds, which are slightly higher in the fully implicit case, reflecting the different numerical characteristics of implemented approaches. Comparisons further showed that both approaches converge to identical final displacements, with only minor differences in transient wave characteristics.

An important practical outcome of this study is the demonstration that, within the hybrid solution framework, omission of the slip tangent operator during the implicit phase does not significantly degrade computational performance or solution accuracy. This finding is particularly relevant for large-scale or three-dimensional simulations, where construction and evaluation of complex frictional Jacobian can become prohibitively expensive.

Overall, the proposed XFEM framework combined with the hybrid implicit–explicit time-integration strategy provides an efficient numerical approach for simulating injection-induced reactivation of a prescribed pre-existing fault. The results demonstrate that using implicit integration during slowly evolving deformation and explicit dynamics during rapid rupture can resolve multi-scale fault-slip processes while reducing computational cost for the present two-dimensional homogeneous benchmark problem. The observed computational savings indicate the potential of the proposed hybrid strategy for larger-scale simulations.

6 Limitations and future work

The present study is limited to a moderate-size two-dimensional benchmark problem with a prescribed pre-existing fault in a homogeneous medium. Therefore, the reported CPU-time reduction should be interpreted as a relative efficiency improvement for the numerical setup considered here, rather than as a formal demonstration of scalability or operational readiness. A comprehensive scalability assessment would require simulations with progressively larger meshes and degrees of freedom, together with detailed reporting of assembly time, solver time, nonlinear iterations, time-step counts, memory use, preconditioner performance, and parallel efficiency. Such an analysis is beyond the scope of the present work.

Future work could focus on extending the framework toward larger and more realistic problems, including three-dimensional fault systems, heterogeneous poroelastic media, thermo-hydro-mechanical coupling, more complex frictional and permeability evolution laws, and fault-network interactions. Further developments should also include the formulation of a physics-based, non-dimensional switching criterion that can improve computational efficiency while remaining applicable across different geological conditions. In addition, systematic mesh-refinement and scaling studies, implementation of efficient parallel solvers and preconditioners, and validation against field-scale observations are needed. Such extensions are necessary before the proposed framework can be considered for operational induced-seismicity forecasting in subsurface energy applications.

7 Appendices

Appendix A: Effect of slip tangent operator in fully implicit solution approach

From a computational perspective, we compared two prescribed minimum time-step sizes, ($\Delta t_{min} = 0.0001\ s$ and $\Delta t_{min} = 0.001\ s$), in terms of total CPU time and the number of nonlinear iterations required for convergence. The comparison was performed both with and without the consistent slip tangent operator developed in Section 2.5.1, Eq. (51). It should be emphasized that the time-step size does not directly affect the assembly of the Jacobian matrix itself. Rather, the key issue is whether the consistent tangent contribution associated with the nonlinear frictional slip law is included in the Jacobian. When

this contribution is omitted, the resulting Jacobian is approximate, and the nonlinear convergence becomes more sensitive to the time-step size, especially during dynamic rupture when slip velocity, stress, and frictional state evolve rapidly.

For the case in which a balance between computational cost and temporal resolution is required, ($\Delta t_{min} = 0.0001\ s$), the fully implicit scheme including the slip tangent operator required 10.95 h of CPU time. Omitting the operator increased the runtime to 15.25 h, corresponding to an approximately 39% increase in CPU time, or equivalently a 28% reduction when the slip tangent is included (Fig. A1a). This indicates that, for sufficiently small minimum time-step sizes, the omission of the slip tangent operator does not lead to a prohibitive increase in computational cost, because the nonlinear changes within each increment remain relatively limited. This observation is relevant for more complex friction laws, for which deriving and implementing a fully consistent tangent operator may be considerably more difficult. In contrast, when computational efficiency is prioritized over temporal resolution, ($\Delta t_{min} = 0.001\ s$), omitting the slip tangent operator results in an approximately 68% increase in runtime (Fig. A1b). In this case, the larger time increments lead to stronger nonlinear variations within each step, and the approximate Jacobian requires a higher number of nonlinear iterations to achieve convergence. Therefore, the benefit of including the slip tangent operator is not related to a direct effect of the time-step size on Jacobian assembly, but rather to improved nonlinear convergence and reduced total solving time, particularly when larger time steps are used during dynamic rupture.

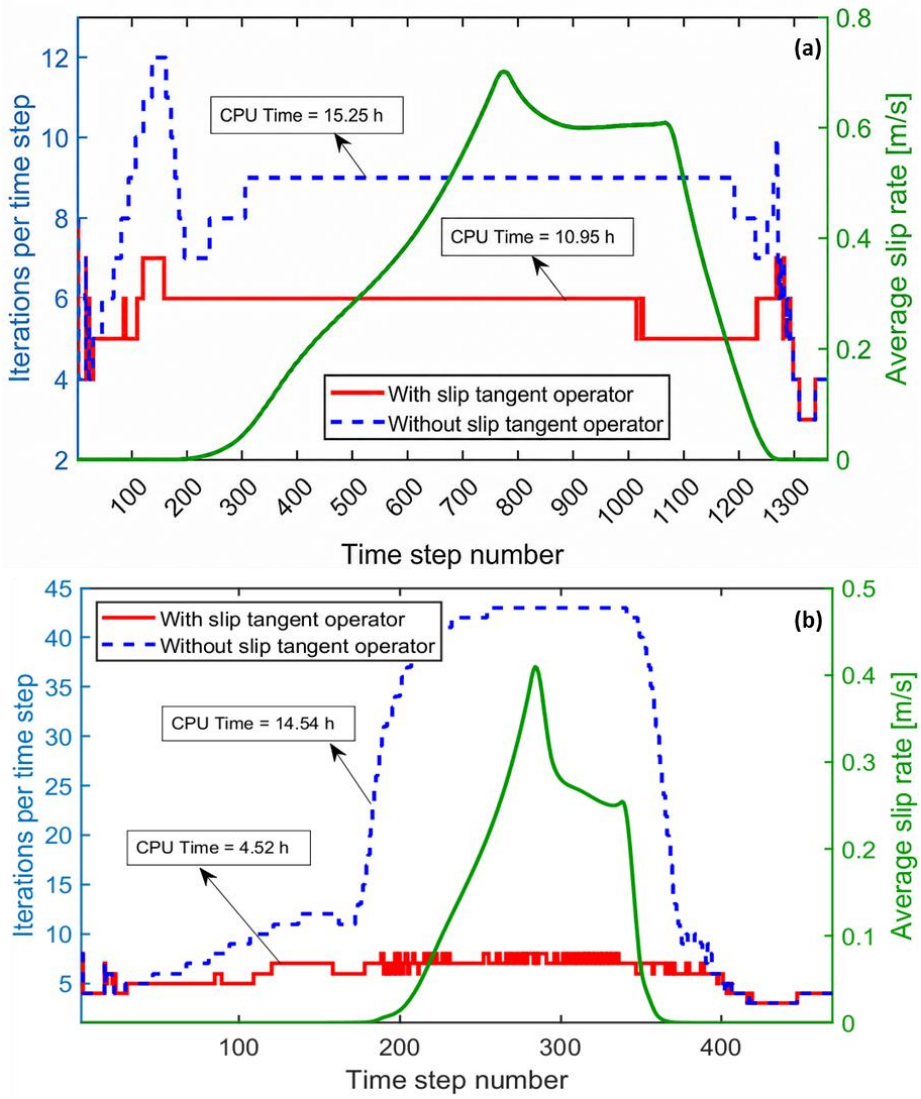


Figure A1: Solver performance with and without the slip tangent operator for two minimum time-step sizes. Shown are the Newton iterations per time step for $\Delta t_{min} = 0.0001$ s (a) and $\Delta t_{min} = 0.001$ s (b), together with the average slip rate. Inclusion of the slip tangent operator reduces iteration counts and total CPU time, with the strongest efficiency gain observed for the larger time step.

For two previous cases, Fig. A2 also compares Newton–Raphson convergence behavior for different slip velocities. The top row (Fig. A2, a-b) corresponds to $\Delta t_{min} = 0.0001\ s$ and the bottom row (Fig. A2, c-d) to $\Delta t_{min} = 0.001\ s$. Results obtained with the slip tangent operator are shown in the left column, while those without the slip tangent operator are shown in the right column. The evolution of the residual norm with iteration number demonstrates that inclusion of the slip tangent operator systematically improves convergence, reducing the number of iterations required to reach a given tolerance, with the effect becoming particularly pronounced for larger time steps and higher slip velocities.

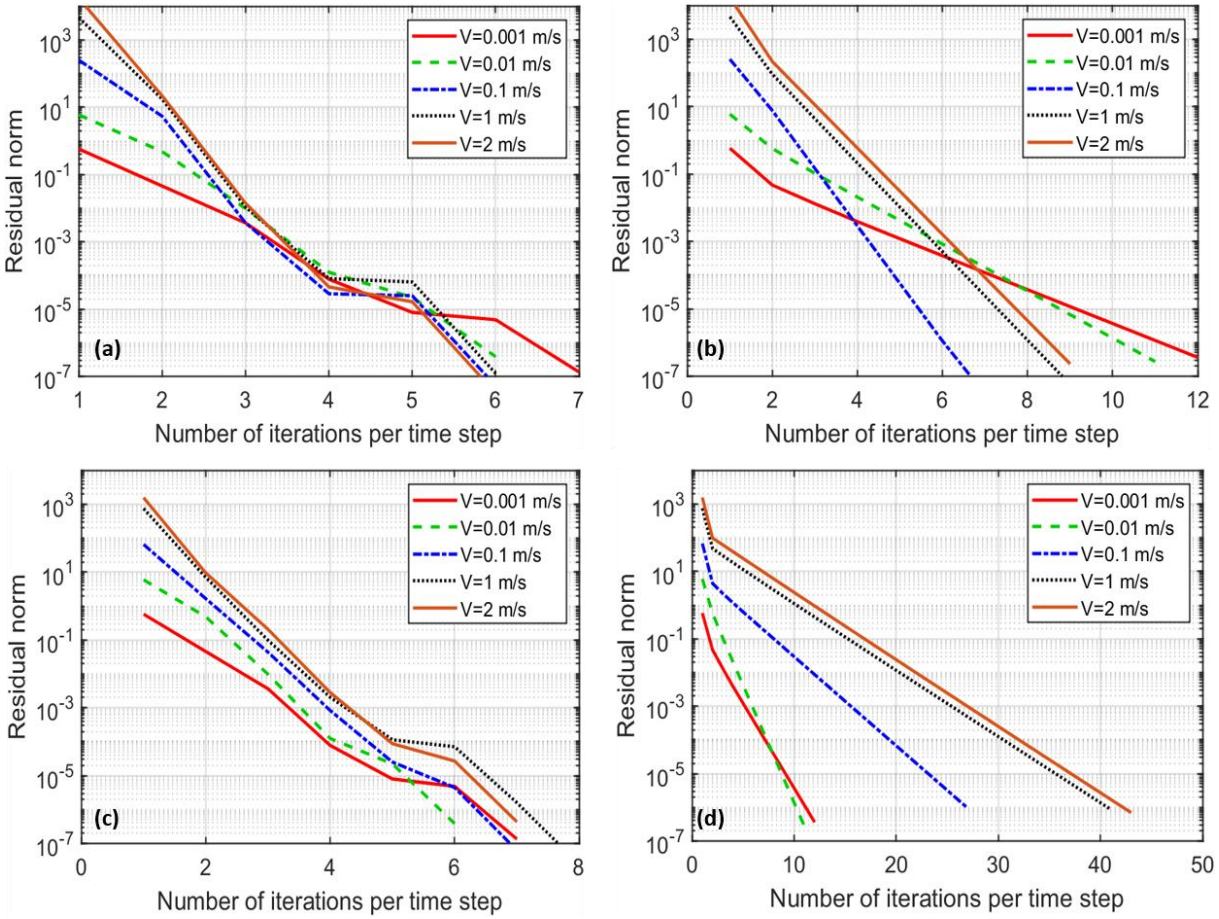


Figure A2: Newton–Raphson convergence behavior for different prescribed slip velocities and two minimum time-step sizes. Panels (a–b) correspond to $\Delta t_{min} = 0.0001$ s, while panels (c–d) correspond to $\Delta t_{min} = 0.001$ s. Results obtained with **the slip tangent operator are shown in the left column, and those** without **the slip tangent operator in the right column. The evolution of the residual norm with iteration number demonstrates that inclusion of the slip tangent operator consistently enhances convergence, reducing the number of iterations required to reach a given tolerance. This improvement becomes increasingly pronounced for larger time steps and higher slip velocities.**

Appendix B: Effect of slip tangent operator in hybrid solution approach

Figure A3 illustrates the computational efficiency of the hybrid solution strategy with and without including the slip tangent operator during the implicit phase, for a range of slip-velocity thresholds controlling the solver transition. A minimum time-step size of $\Delta t_{min} = 0.0001$ s is used for the implicit solver and also adopted as the explicit time step. Overall, for both approaches, the CPU time reaches its minimum at a switching velocity of 0.01 m/s (Fig. A3, a). Specifically, the CPU time decreases from 4.75 h and 4.5 h at a switching velocity of 0.0005 m/s to 3.76 h and 3.15 h at 0.01 m/s for the cases without and with the slip tangent operator, respectively. Interestingly, in contrast to the fully implicit approach, omitting the slip tangent operator within the hybrid solution framework does not lead to a substantial increase in computational cost. For example, excluding the slip tangent operator results in a minimum increase of approximately 5.5% and a maximum increase of approximately 31% in CPU time, occurring at switching velocities of 0.0005 m/s and 0.1 m/s, respectively. The comparison with the fully implicit scheme (Fig. A3, b) including the slip tangent operator is particularly significant. At a switching velocity of 0.01 m/s, the hybrid approach reduces the CPU time by 71.27% and 65.65% with and without the slip tangent operator,

respectively. These results indicate that, within the hybrid framework, the slip tangent operator can be neglected without severely compromising computational efficiency. This is particularly advantageous when the evaluation of the corresponding Jacobian terms becomes complex or computationally demanding.

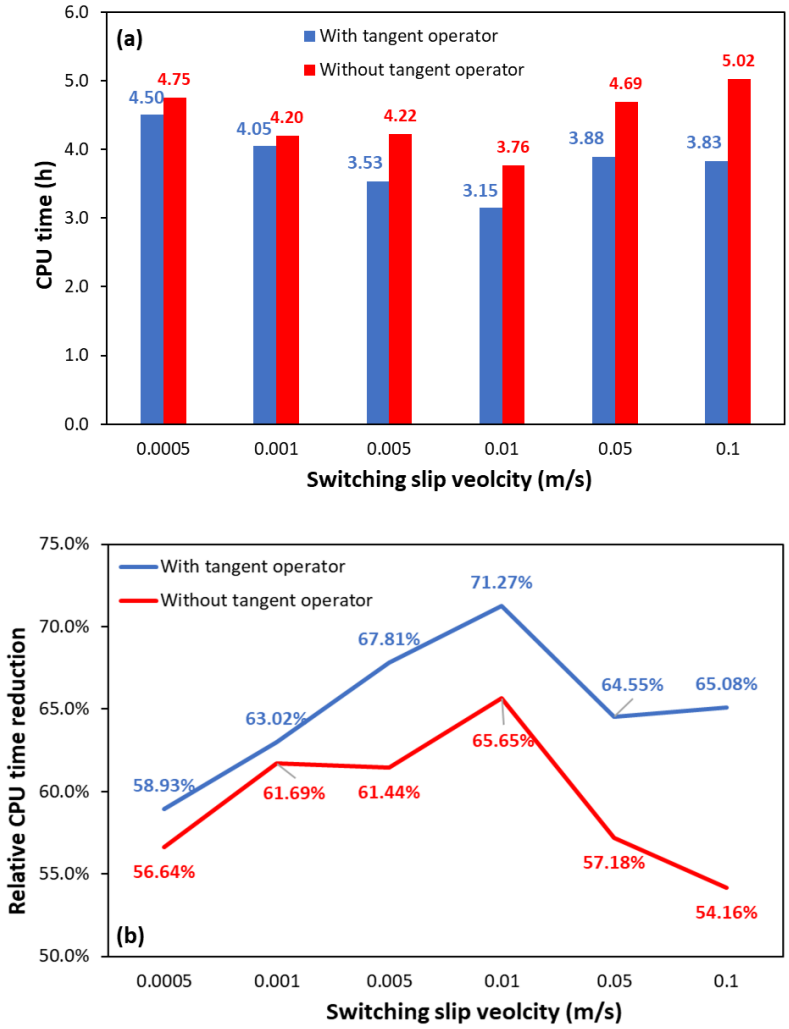


Figure B1: Computational performance of the hybrid solution strategy for different switching slip velocities, with and without inclusion of the slip tangent operator during the implicit phase. The minimum implicit time-step size is set to $\Delta t_{min} = 0.0001$ s, which is also used as the fixed explicit time step. (a) CPU time required by the hybrid solution-based simulations. For both formulations, the minimum CPU time is obtained at a switching slip velocity of 0.01 m/s. (b) Relative CPU time reduction of the hybrid strategy with respect to the fully implicit scheme including the slip tangent operator.

Appendix C: Acceleration, velocity, and displacement field during seismic events

Figures A4–A6 show the spatial distributions of acceleration, velocity, and displacement over the computational domain during the seismic events simulated using both the fully implicit and hybrid solution approaches. A seismic event is defined to begin when the slip velocity on any fault patch exceeds 0.1 m/s, and the time displayed above each snapshot is measured relative to this onset time. For each solution approach, the corresponding fields are presented at 25%, 50%, and 75% of the event duration.

This event-normalized comparison provides a more meaningful basis for evaluating the two approaches, because the hybrid solution exhibits a delay of a few milliseconds relative to the fully implicit solution. Therefore, comparing the two solutions at the same absolute time after injection would not be appropriate, particularly because the entire seismic event lasts only a few milliseconds. The results further support the findings presented in Fig. 11, showing that the hybrid solution approach reproduces the acceleration, velocity, and displacement distributions obtained with the fully implicit approach, as well as their spatial propagation around the fault during rupture. For the acceleration field (Fig. A4), the hybrid approach produces slightly higher values, whereas for the velocity field (Fig. A5), it gives slightly lower values, particularly near the rupture front.

However, as discussed previously, these differences are not significant. For the displacement field (Fig. A6), both approaches yield almost identical spatial distributions.

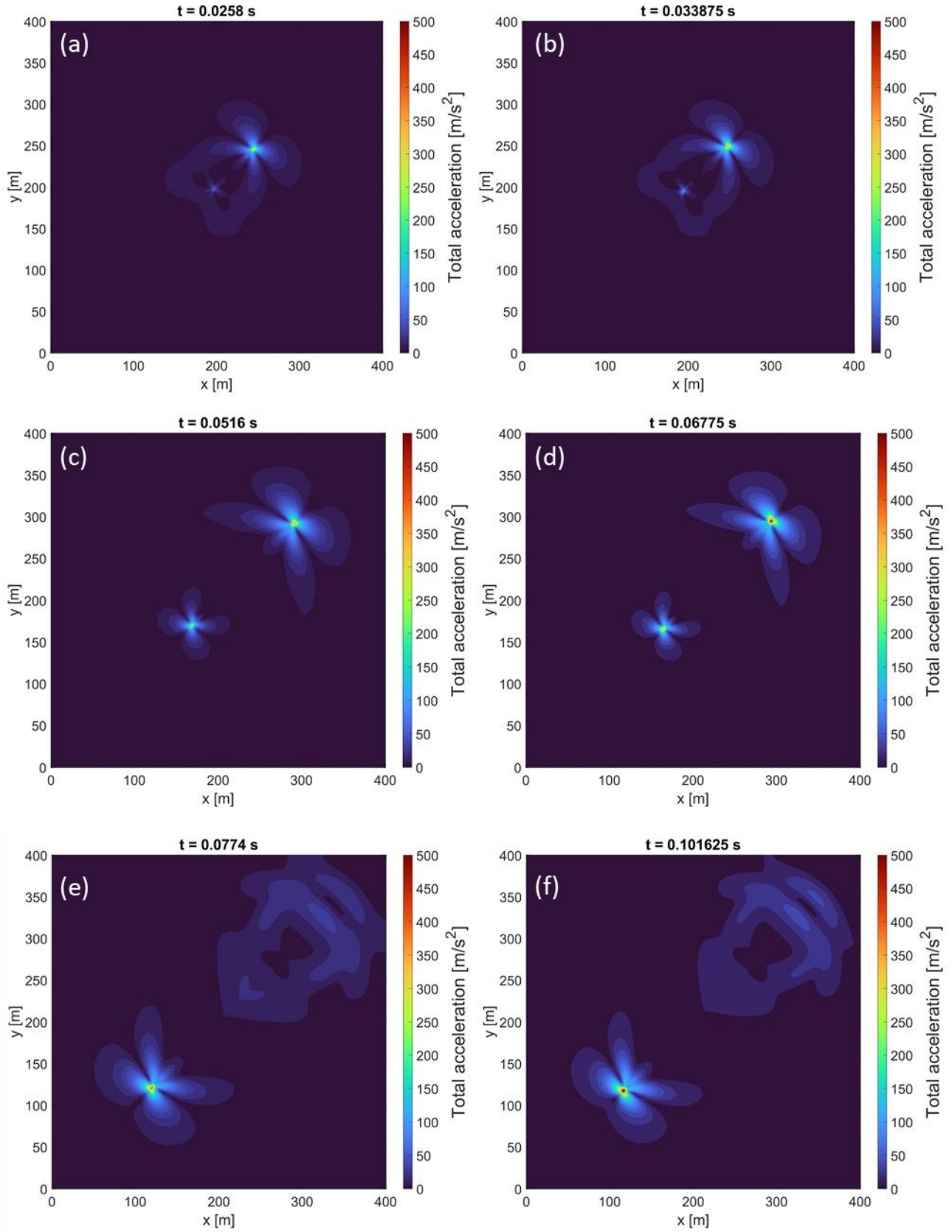


Figure C1. Spatial distribution of total acceleration during the seismic event obtained using the hybrid solution approach (left column: a, c, e) and the fully implicit solution approach (right column: b, d, f). A seismic event is defined to begin when the slip velocity on any fault patch exceeds 0.1 m/s, and the time displayed above each snapshot is measured relative to this onset time. For each solution approach, the acceleration field is shown at 25%, 50%, and 75% of the corresponding event duration, enabling an event-normalized comparison between the two approaches despite the small time delay between them.

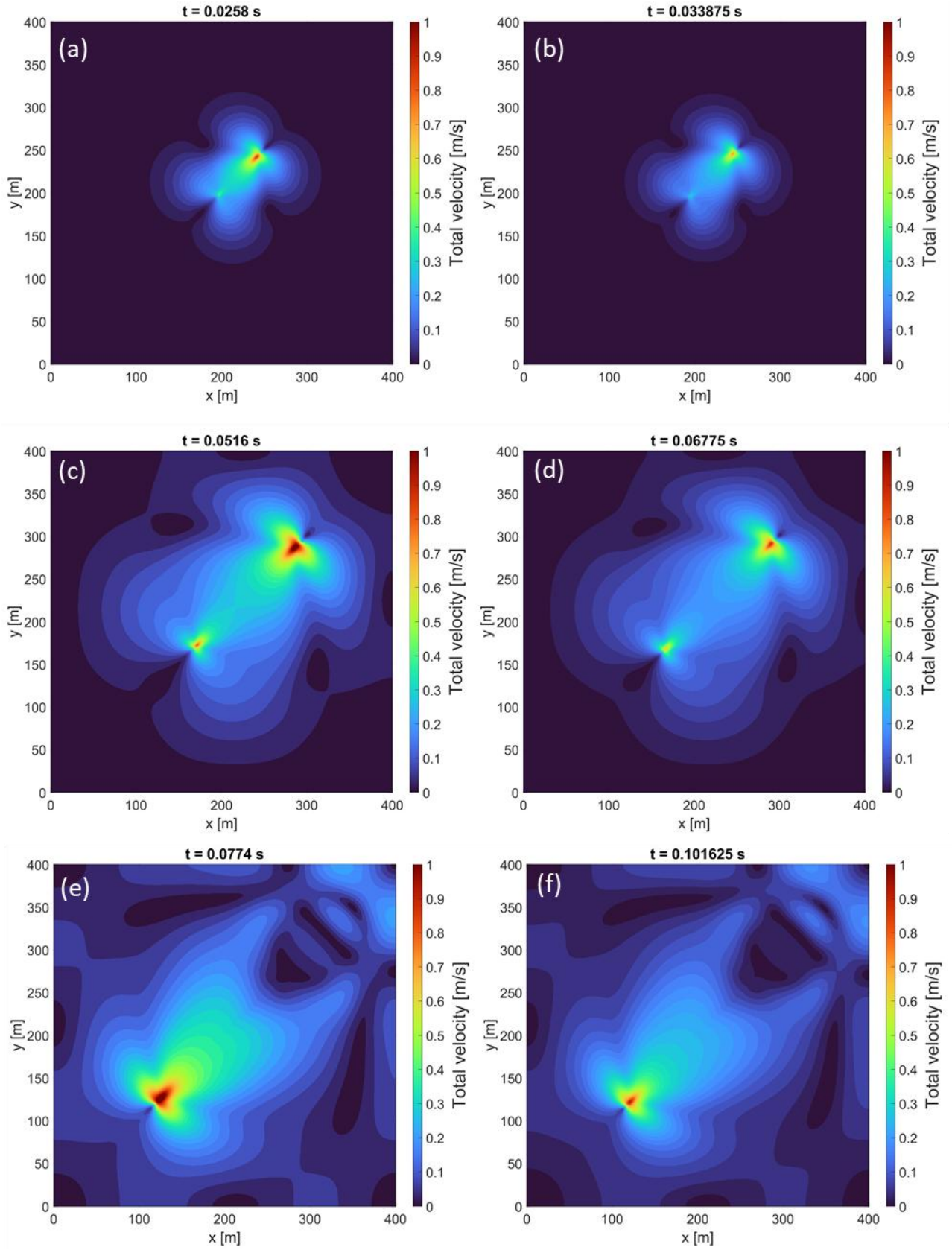


Figure C2. Spatial distribution of total velocity during the seismic event obtained using the hybrid solution approach (left column: a, c, e) and the fully implicit solution approach (right column: b, d, f). A seismic event is defined to begin when the slip velocity on any fault patch exceeds 0.1 m/s, and the time displayed above each snapshot is measured relative to this onset time. For each solution approach, the velocity field is shown at 25%, 50%, and 75% of the corresponding event duration, enabling an event-normalized comparison between the two approaches despite the small time delay between them.

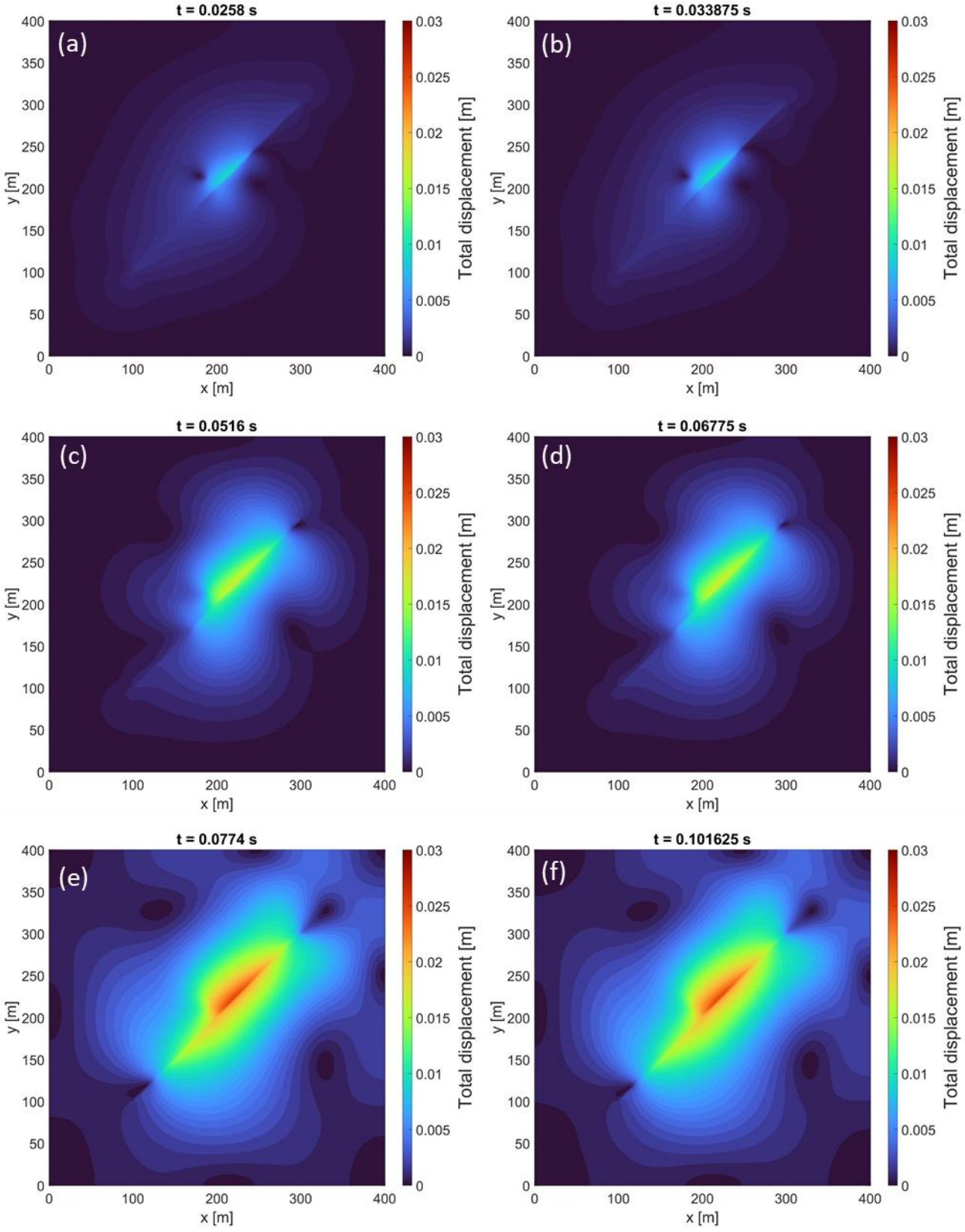


Figure C3. Spatial distribution of total displacement during the seismic event obtained using the hybrid solution approach (left column: a, c, e) and the fully implicit solution approach (right column: b, d, f). A seismic event is defined to begin when the slip velocity on any fault patch exceeds 0.1 m/s, and the time displayed above each snapshot is measured relative to this onset time. For each solution approach, the displacement field is shown at 25%, 50%, and 75% of the corresponding event duration, enabling an event-normalized comparison between the two approaches despite the small time delay between them.

Code availability

The source code used in this study, together with the input files and scripts required to reproduce the results, is publicly available via the following Zenodo repository: https://doi.org/10.5281/zenodo.19220256 (Sabah, 2026). The archived package includes (i) the full simulation/workflow code, (ii) example configuration/input files, and (iii) post-processing scripts for generating the figures and tables reported in this paper. Any additional data generated during the study are either included in the same repository or can be obtained from the corresponding author upon reasonable request.

Author contributions

All authors contributed to the discussion and interpretation of the results and to the review and editing of the manuscript. MS developed the code, designed the methodology, performed the simulations, and prepared the manuscript. MC contributed to the conceptualization and development of the methodology.

Competing interests

At least one of the (co-)authors is a member of the editorial board of Geoscientific Model Development.

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