Comments on the manuscript

“Hybrid implicit–explicit XFEM simulation of injection-induced seismicity: resolving multi-scale rupture nucleation and dynamics”

The authors propose an implicit-explicit (IMEX) time-integration scheme to resolve different timescales in a fully coupled hydromechanical finite element model. The results look promising, especially because of the potential speedup of calculations. However, many details and definitions are missing, therefore improvements are needed before publication. Overall, I thus recommend major modifications of the manuscript before resubmission.

1. Line 13: Not all implicit time integration schemes are A-stable, hence your statement about all implicit schemes being unconditionally stable is not correct.
2. Line 14: Shouldn't it depend in the maximal time step? Later you define a lower bound for the time step, but that is not clear when reading this first part, please reformulate.
3. Line 46: Should be “well-suited”.
4. Equation (3): What does it mean that $\mathbf{u}, p \in \Gamma_d$? I thought these are functions of $\mathbf{x}$ and t.
5. Equation (4): Please define $\boldsymbol{\sigma}$ and $\boldsymbol{t}$. Furthermore, it would generally improve the readibility if you could use different names for the test functions, somthing like $\boldsymbol{v}$ and q instead of $\delta\boldsymbol{u}$ and δp.
6. Line 105: Give a precise mathematical definition of $\bar{\boldsymbol{t}}$.
7. Line 106: Give a precise mathematical definition of Γ_t.
8. Equation (5): Please define $\boldsymbol{q}$ and Γ_w and derive it in more detail.
9. Lines 111-112: It sounds better to me to multiply Eq. (3) by the test function than vice versa.
10. Line 112: What does it mean for a domain to be discontinuous? Probably you mean that a function on this domain is discontinuous?
11. Equation (6): Please derive it in more detail.
12. Figure 1: Please align the angles/orientations of all letters/symbols in the figure.
13. Line 119: Please change ”cap omega to the prime” to a formula.
14. Equation (7): Please define $\langle\cdot\rangle$ and $[\cdot]$. Also, why is there a w^3 while in (6) there only was w^2?
15. Line 135: Give a reference or explanation for the formula in (10).
16. Equation (12): What does the notation with the

$$r = \dots \begin{cases} = 0 \text{ slip condition} \\ < 0 \text{ stick condition} \end{cases}$$

mean? If it should just say that if $r = 0$ it's a slip condition and if $r < 0$ it's a stick condition, I would write it like that in the text.

17. Line 152: "the above relation", cite an equation number (probably (12) or (13)) here.

18. Line 155: "Coulomb's friction law", cite equation (12), where you defined it, here.

19. Line 159: It should be Eq. (15) I think, please also check all preceeding equation numbers.

20. Equation (16): Please explain in more detail how to get from (15) to (16).

21. Equation (18): Give a reference or explanantion for this law and why you chose it.

22. Equation (19): This only holds if V is scalar, since you are considering 2-dimensional problems later it should also be correct for vector-valued V's.

23. Equation (20): I think the formula for V is missing, please add it or cite its equation number in case I overlooked it.

24. Equation (21): Should be N_{ui} instead of N_{uI} in the first sum. Also the meaning of $\mathbf{x}_J$ is not clear to me. And why do you use "$\equiv$" instead of "$=$" in the end?

25. Line 188: Should be "where $\mathcal{N}$ denotes the number of all nodal points".

26. Lines 188-189: Explain formally what "enrichment by the Heaviside function" means, potentially this could also go into the appendix (depending on how complicated the definition is).

27. Line 196: Move the definition of φ before or directly after Eq. (21).

28. Line 199: What do you mean by "strong discontinuities", is it referring to the jump size?

29. Equation (24): The last summand appearing in line 201 here should be $[\![\mathbf{N}_u^{\text{tip}}(\mathbf{x})\bar{\mathbf{b}}(t)]\!]$ I think.

30. Equation (26): Same as in 29, also is r the same quantity as defined in Eq. (12)? If yes, please cite that equation.

31. Equation (29): Please define $\mathbf{N}_p^{\text{mod}}(\mathbf{x})\bar{\mathbf{d}}(t)$. Furthermore, you again use "$\equiv$" here which I do not fully understand, see 24.

32. Line 226: What is ϕ_i? Do you mean φ_i? In that case, please decide for one symbol.

33. Equation (31): Currently only the terms inside the first sum are time-dependent, please add all missing t's.

34. Equation (33): Please define $\mathbb{D}$.

35. Line 249: Mathematically it's the Dirac delta distribution (not function) since it is not a function.

36. Lines 273-274: Please move the paragraph about the stability parameters (lines 279-281) here so that it immediately explains the statements given here.

37. Lines 290-300: Why not considering time step n instead of $n+1$? That would save space and make the equations more readable.

38. Lines 298-299: What does a "more symmetric structure" mean? Give a precise definition or stick to symmetric or non-symmetric cases.

39. Line 304: Please define **E** formally and explain what an algorithmic tangent operator generally is and what it means for it to be consistent.
40. Equation (48): What happened with the sgn terms in (47)? Is K_T different from k_T? And please give a more detailed derivation of this equation, since it is not obvious to me how to derive it from (20) and (47).
41. Line 334: Please comment on which adaptive time stepping scheme you are using here.
42. Line 344: Please briefly comment on what happens if you increase the value of ξ. I assume $\xi < 1$ should still work, but it would be interesting to see up to which value the algorithm works well.
43. Line 351: Please give an explanantion on why you chose this switching criterium, maybe there is also some physical intuition behind?
44. Line 354: Please give a reference for the efficiency and robustness of the method.
45. Equation (59): Please write "$\min_e \ldots$".
46. Lines 346-375: Generally it would be helpful if you could mention whether your implicit and explicit schemes are using the same step size or not.
47. Line 393: What is "the cubic law"?
48. Lines 416-417: Since you mention it here, it would also be nice to show 2d plots of your simulations later, not only evaluations at four points.
49. Figure 4: In the caption some text is not bold, please fix that.
50. Lines 433-434: The statement on implicit schemes is not true, see 1.
51. Line 450: The typesetting after this line is weird (half empty page), this should be fixed before a potential publication.
52. Figure 5: Whe you say that you shift the time axis, does it mean that all "initial conditions" (i.e. at the shifted time $t = 0$) are qual or do they differ due to different evolution before? Please clarify that.
53. Figure 7: Please clarify if the time step sizes for explicit and implicit schemes are equal here or if you used adaptive implicit time steps as mentioned in line 334.
54. Line 490: Please change "slightly higher" to an objective value, in this case "around 25-30% higher".
55. Lines 550-551: Still not true, see 1 and 50.
56. Line 589: It sounds odd to me that the assembly of the Jacobian is affected by longer time steps, please give additional explanations here or relate it instead to solving times.
57. Lines 597-598: I don't see any columns in Figure A1, please fix this.

58. Figure A3: If I understand correctly you compare both implicit and IMEX schemes without using the slip tangent operator. Since this operator improves the speed of the fully implicit method, I think it would be more fair to compare implicit+operator with IMEX (with or without). It would be nice if you could add such tests to give a better understanding of the potential speedup one can expect.

59. Generally the "·" symbols you use to indicate multiplications are looking more like normal dots at the end of a sentence since they are very low, maybe it is possible to fix that.