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2 **The Effect of Seawater Spatial density heterogeneity on Storm Surges**  
3 **— A Case Study of Typhoon "In-Fa" and Hangzhou Bay and Its**  
4 **Adjacent Waters**

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13

14 **Abstract:** Previous studies have shown that considering spatial density heterogeneity can improve the  
15 simulation capability and accuracy of water level, including storm surges, but the underlying dynamical  
16 mechanisms remain unclear. In this study, we conducted two contrasting models by FVCOM— a  
17 heterogeneous ocean model and a homogeneous ocean model to simulate water levels in Hangzhou Bay during  
18 the passage of Typhoon In-Fa in July 2021. We analyzed the spatiotemporal distribution characteristics of  
19 storm surges, and investigated carefully the mechanisms of storm surges by which spatial density  
20 heterogeneity. The simulation results show that both models perform comparably and with high accuracy in  
21 simulating astronomical tides and storm surges outside Hangzhou Bay. However, there are significant  
22 differences emerge in simulating storm surges inside Hangzhou Bay, where the heterogeneous ocean model  
23 substantially outperforms the homogeneous ocean model, with a maximum error of less than 0.2 m compared  
24 to 0.9 m for the latter, approximately 25% of the maximum storm surge. Based on the magnitude and  
25 duration of storm surge rise and fall, the interior and exterior of Hangzhou Bay exhibit two distinctly  
26 different situations: the interior is characterized by large surge amplitudes with durations exceeding 2 days,  
27 while the exterior shows smaller amplitudes with durations of approximately half a day. Dynamical analysis  
28 reveals that in regions where the storm surge duration exceeds the inertial period (e.g., the interior of  
29 Hangzhou Bay), the part of the storm surge affected by density can be expressed as a coupled form of the  
30 product of the density Laplacian and a quadratic term of the storm surge. In regions where the density  
31 Laplacian is negative (i.e., the density surface is convex upward), the part of the storm surge affected by  
32 density is negative during the surge rise phase, thereby reducing the overestimation bias of the homogeneous  
33 model; during the surge fall phase, it becomes positive, reducing the underestimation bias. In regions where



34 **the density Laplacian is positive, the above effects are reversed.**

35 **Keywords: storm surge; spatial density heterogeneity; density–storm surge coupling**

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## 38 **1 Introduction**

39 Storm surges are one of the most destructive marine hazards, which can contribute to more than  
40 95% of the total economic losses caused by marine disasters in coastal areas (China Marine Disaster  
41 Bulletin, Ministry of Natural Resources, 2024). It is important for people to understand the  
42 mechanisms of storm surges and then make accurate forecasts. Before the 1970s, many studies  
43 focused on the shallow water equation theory (Doodson, 1956; Heaps, 1969) to explain the  
44 phenomenon of storm surges. From the 1980s to present, benefiting from the progress of technology,  
45 more and more in-depth investigations of storm surges have been conducted through numerical model  
46 simulations. These studies pay more attention to the main effects of typhoons, astronomical tides, and  
47 wind waves on storm surge forecast accuracy, and have gotten many detailed conclusions (Yu et al.,  
48 2020). However, few studies consider the spatial density heterogeneity's effects on storm surges. In  
49 recent years, there are some studies which have observed that spatial density heterogeneity can  
50 significantly affect storm surge forecast accuracy. Among them, there are three dynamical  
51 mechanisms are considered to account for: coastal trapped waves, bottom friction, and pressure  
52 gradient force. Each factor can affect storm surges in different ways, but need more in-depth  
53 investigation.

54 Certain coastal trapped waves may exert an influence on storm surges within regions  
55 characterized by strong water density stratification, such as certain low-latitude coastal zones and  
56 adjacent shelf margins. In weakly stratified, wide-shelf, high-latitude waters, where  $Bu \ll 1$ , coastal  
57 trapped waves approximate barotropic shelf waves; conversely, in strongly stratified, narrow-shelf,  
58 near-equatorial waters, where  $Bu \gg 1$ , coastal trapped waves more closely resemble internal Kelvin  
59 waves. Some relevant studies have observed that storm surges are also affected in regions where  
60 coastal trapped waves propagate prominently. Kodaira et al. (2016) made a comparative study using  
61 barotropic and baroclinic models to investigate storm surges at 257 tide gauge stations worldwide in  
62 2014. The results showed that the baroclinic model improved storm surge forecast accuracy at nearly  
63 all stations, but the degree of improvement varied. The greatest improvement was found near the East



64 London station on the southeastern coast of Africa, reaching about 10 cm, other stations'  
65 improvements are in the range of 3 cm to 4 cm. The authors pointed out that it may be the topography  
66 change the storm surge forecast accuracy in the baroclinic model. In the area where the continental  
67 shelf rapidly narrows in southern Africa, the amplitude of coastal trapped waves increases  
68 significantly, leading to a significant change in storm surge forecast accuracy. Wang et al. (2022)  
69 made a comparative study using barotropic and baroclinic models to examine total water levels,  
70 astronomical tides, and residual water levels at 211 global tide gauge stations from 2019 to 2021  
71 across multiple timescales (seasonal, inter-seasonal, sub-seasonal, and weather-scale). The results  
72 showed that the baroclinic model improved simulation accuracy for all stations relative to the  
73 barotropic model. In the eastern Indian Ocean, where the greatest improvement in extreme residual  
74 water levels at the weather timescale has been found, reaching up to 42 cm. In the area, the authors  
75 also attributed the changes of storm surges to coastal trapped waves. Scale analysis pointed out that  
76 the Burger number in this region is much greater than 1, and the associated waves have a longer  
77 critical period affected by the slanted coastline— both factors favor the formation of coastal trapped  
78 waves in the baroclinic model, and then the storm surge is also influenced.

79 Bottom friction is also considered to alter the storm surge forecast accuracy due to the density  
80 stratification. In the study by Kodaira et al. (2016), an notable improvement in storm surge forecast  
81 accuracy was found at the Naha station in Japan, which was directly in the path of the typhoon. The  
82 comparative experiments showed that the storm surge height in the baroclinic model was greater than  
83 that in the barotropic model, and the authors suggested that the bottom friction affected the accuracy  
84 of storm surge, but there was no clear evidence. Orton et al. (2012) made sensitivity numerical  
85 experiments to simulate two storm surge events in the Hudson River estuary and analyzed five  
86 physical processes influencing storm surges. The results took an order of the extent of precision  
87 variation resulting from the above physical processes, and the effect of density variation was second  
88 and the atmospheric forcing was first. The result showed that the storm surge forecast accuracy  
89 improved by 1%–5% in the lower river and offshore areas, and up to 13% in the upper river. The  
90 authors thought that the stratified water column and the weakened turbulent mixing caused the  
91 accuracy change. This finding is similar to the work of Muller, who researched the mechanisms



92 underlying seasonal variations in tidal amplitude on shallow continental shelves. Through analytical  
93 solutions and numerical modeling, Muller concluded that stratification of water column can modify  
94 the eddy viscosity profile and bottom friction, thereby affecting tidal transport and causing changes  
95 in tidal amplitude.

96 The pressure gradient force caused by spatial density heterogeneity is also an important factor  
97 affecting storm surge forecast accuracy. Ye et al. (2020) made a comparative study using barotropic  
98 and baroclinic models to study storm surges during a typhoon event. The results showed that the  
99 baroclinic model had higher accuracy. Besides, the difference in water levels between the two models  
100 was relatively small during the storm surge rising, but became obviously larger during the storm surge  
101 falling. The baroclinic model reduced the error of the barotropic model by 14%. The authors thought  
102 that the large-scale baroclinic effect of the Gulf Stream may reduce the error. Pringle et al. (2019)  
103 made a comparative experiment using barotropic and baroclinic models to research the influence of  
104 the density on storm surges around Puerto Rico and the U.S. Virgin Islands. The results showed that  
105 the baroclinic model had the smallest errors. At most tide gauge stations, storm surge peaks in the  
106 baroclinic model were 10–20 cm taller than the barotropic model, what's more, water levels in the  
107 baroclinic model were 10–15 cm lower than the levels before the typhoon. The authors thought that  
108 this water level depression may be related to the pressure gradient force because of deep-water  
109 upwelling in that region.

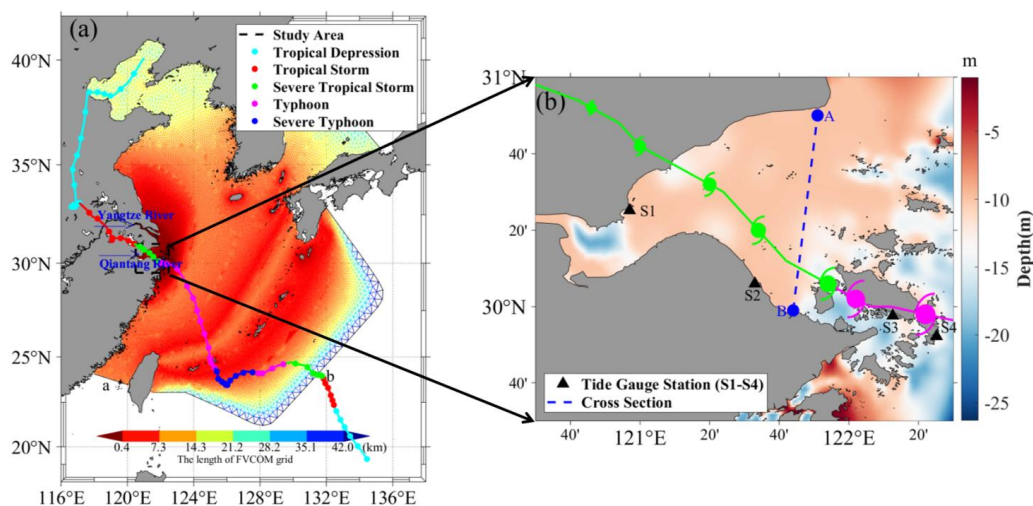
110 In summary, spatial density heterogeneity can obviously affect the capability and accuracy of  
111 numerical models in simulating storm surges. However, the reasons why there were the accuracy  
112 change were unclear. The above studies just gave the conjecture, such as coastal trapped waves, the  
113 bottom friction, and large-scale ocean circulation, which need more rigorous analysis. According to  
114 this research background, the present study takes Typhoon In-Fa as a case study and selects Hangzhou  
115 Bay as the study area, to investigate the dynamical mechanisms by which spatial density  
116 heterogeneity affects storm surges.

## 117 **2 Data and Methods**



118        The study area is Hangzhou Bay, as shown in Fig. 1. This region is affected by typhoons  
119 frequently (Hou et al., 2011). In 2021, strong Typhoon In-Fa made landfall in Zhoushan and  
120 subsequently traversed Hangzhou Bay, exerting an important impact on the region. Fig. 1a shows the  
121 track and intensity variation of Typhoon In-Fa; Fig. 1b shows the specific spatial location and current  
122 field characteristics of the study area. The region is situated in northern Zhejiang Province on the  
123 southern flank of the Yangtze River Delta, Zhoushan Archipelago is to the east. The mouth of the bay  
124 is defined by the line connecting Lu chao gang (A) to Zhen hai (B), the width is approximately 98  
125 km, a mean water depth of approximately 10 m. The density field in this region is jointly influenced  
126 by the Yangtze River diluted water, the Taiwan Warm Current, and the Zhe-Min Coastal Current,  
127 different currents have different density, resulting in obvious spatial density heterogeneity. This  
128 makes the region an ideal study area for researching the influence of spatial density heterogeneity on  
129 storm surges.

130        There are two kinds of data, one is the observed data and the other is the simulated data. The  
131 observed data contains typhoon data and water level records from tide gauge stations. In addition,  
132 two storm surge simulation experiments are using the FVCOM forced by Typhoon In-Fa: a  
133 homogeneous ocean model (HM), in which seawater density keeps a constant value, and a  
134 heterogeneous ocean model (HT), in which density varies spatially. In the two experiments, only the  
135 density condition is different, and all other conditions remain unchanged.



136

137 Fig. 1a Model grid resolution and the track of Typhoon In-Fa; Fig. 1b water depth (color  
138 shading), tide gauge stations, track of Typhoon In-Fa, and transects.

## 139 2.1 Overview of Typhoon In-Fa

140 Information on the 2021 No. 6 strong Typhoon In-Fa was obtained from the China Typhoon  
141 Meteorological Network (<http://typhoon.nmc.cn>), including the track, maximum wind speed,  
142 translation speed, translation direction, and radius of maximum wind. The track and intensity of  
143 Typhoon In-Fa at different stages are shown in Fig. 1a. Tropical Storm In-Fa formed over the  
144 northwestern Pacific Ocean on July 16, 2021, and then developed into a tropical storm on July 18. At  
145 approximately 12:30 on July 25, it made landfall in Putuo District, Zhoushan City, the level of  
146 intensity is typhoon, which had a maximum wind speed of 38 m/s, traversed Hangzhou Bay along the  
147 northwestward direction. By 03:00 on July 28, it had weakened to a tropical depression with a wind  
148 speed of 15 m/s. During its passage through Hangzhou Bay, Typhoon In-Fa had a significant impact  
149 on water levels. Based on the basic parameters of Typhoon In-Fa, the typhoon wind field was  
150 produced by the method proposed by Emanuel (2004), in which the wind speed at the radius of  
151 maximum wind is given by the following formula:



$$152 \quad U(r) = \frac{R_m(2CU_m - U_t \sin(\theta))r + \frac{1}{2}R_m^2 fr}{R_m^2 + r^2} - \frac{fr}{2} \quad (1)$$

153 Where  $C = 0.736$  is the correction coefficient for the maximum wind speed  $U_m$ ,  $U_t$  is the translation  
154 speed,  $R_m$  is the radius of maximum wind,  $\theta$  is the translation direction, and  $f$  is the Coriolis  
155 parameter (Xuan et al., 2021).

## 156 2.2 Observed Water Level Data

157 In accordance with the track of Typhoon In-Fa through Hangzhou Bay, four tide gauge stations  
158 were selected for water level records: Ganpu (S1), Gaobeipu (S2), Changzhi (S3), and Shenjiamen  
159 (S4). Hourly water level records are used in this study. S1 is located in the innermost part of Hangzhou  
160 Bay, S2 is in the central portion of the bay, and S3 and S4 are located outside the bay. The four  
161 stations collectively provide a representative picture of water level variations in different sub-regions  
162 during the passage of Typhoon In-Fa. Detailed information on the water level data is provided in  
163 Section 3.1.

## 164 2.3 Numerical Model Description

165 We use the Finite Volume Community Ocean Model (FVCOM) to simulate storm surges.  
166 FVCOM employs the unstructured triangular grids that can flexibly adapt to complex coastline  
167 geometries, strengthen the resolution in coastal regions, and also accommodate shallow water  
168 environments such as estuaries and nearshore areas.

169 We take the HT model as an example to illustrate the characteristics of the model configuration.  
170 First, the water depth in the vertical direction is interpolated from the bathymetric data of the General  
171 Bathymetric Chart of the Oceans (GEBCO). A sigma coordinate system uniformly divided into 20  
172 layers is adopted in the vertical direction. In the horizontal direction, spherical coordinates are used,  
173 and the Surface-water Modeling System (SMS) are using to generate a high-resolution unstructured  
174 triangular grid. The horizontal grid resolution is approximately 1 km in the Yangtze River Estuary  
175 region and decreases to approximately 9 km in areas farther away. The model has three open



176 boundaries: the Taiwan Strait to the southwest (a), the northwestern Pacific Ocean to the southwest,  
177 south, and southeast (b), and the Japan Sea to the north (c) (not shown in the figure).

178 The HT model is used again as an example to describe the model forcing, open boundary  
179 conditions, and initial field settings. The forcing in HT includes tidal currents, river discharge, heat  
180 flux, and wind field. The open boundary conditions include tidal elevation and mean flow. The initial  
181 fields of temperature, salinity, and mean flow are interpolated from the HYCOM data assimilation  
182 product (<https://www.hycom.org/>). Tidal currents are driven by tidal elevation at the open boundaries,  
183 which are obtained from Oregon State University Tidal Prediction Software (OTPS;  
184 <http://volkov.oce.orst.edu/tides>) global tidal model. Egbert et al. (1994) and Egbert and Erofeeva  
185 (2002) calculated the amplitudes and phase of 11 single tides: M<sub>2</sub>, S<sub>2</sub>, N<sub>2</sub>, K<sub>2</sub>, K<sub>1</sub>, O<sub>1</sub>, P<sub>1</sub>, Q<sub>1</sub>, M<sub>4</sub>, MS<sub>4</sub>,  
186 and MN<sub>4</sub>. River discharge primarily considers the Yangtze River and Qiantang River, with Yangtze  
187 River discharge obtained from the Yangtze River Hydrology Network ([www.cjh.com.cn](http://www.cjh.com.cn)) and  
188 Qiantang River discharge from the Ministry of Water Resources  
189 (<http://www.mwr.gov.cn/sj/tjgb/zghlmsgb/>). The daily mean heat flux is from the objectively analyzed  
190 air-sea fluxes (OAFlux; Yu and Weller, 2007). The wind field consists of two parts data, one is the  
191 ERA-Interim reanalysis data, the online address is  
192 <https://www.ecmwf.int/en/forecasts/dataset/ecmwf-reanalysis-interim>), and the other is the typhoon  
193 wind field in Equation (1).

194 The grid configuration of HM is the same as that of HT. In the HM, temperature and salinity are  
195 set to constant values of 20°C and 30 PSU, with all other settings remaining the same. Both models  
196 simulate water levels, temperature fields, salinity fields, and current fields from May 1 to July 31,  
197 2021, and the output data is given every half hour.

## 198 **3 Results**

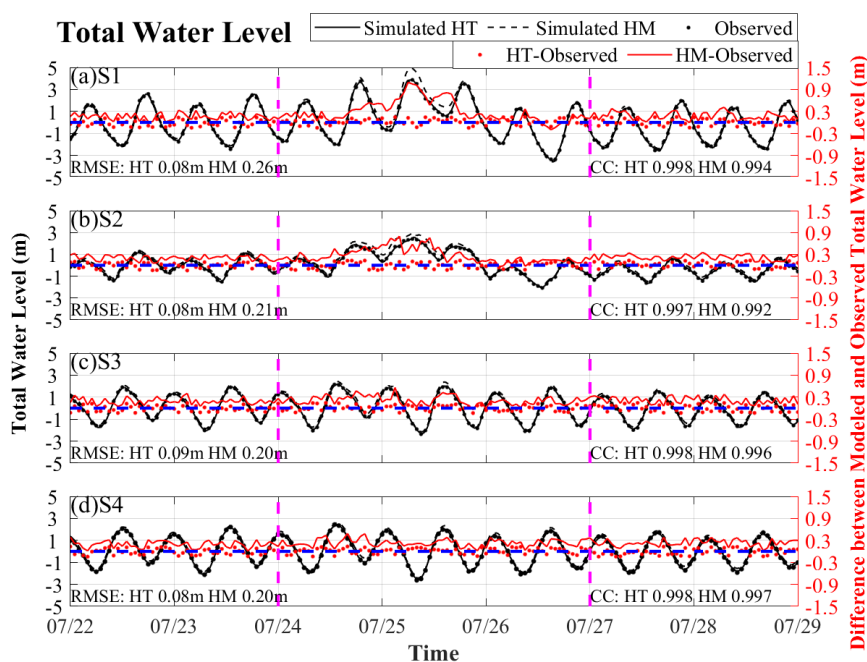
### 199 **3.1 Comparison of Water Level Simulation Results between HM and HT**

200 Based on the above configurations and the governing dynamical equations of FVCOM, we can  
201 get the water levels during the typhoon period. Since we take tidal currents as an important open



202 boundary condition, the simulated water levels were the total water levels, which contains  
203 astronomical tides. We use the `t_tide` toolbox to extract the storm surge signal. Complex processes  
204 such as tide–surge interaction are not considered in this study.

205 Firstly, we compare the total water levels in the two simulated models. The results are shown in  
206 Fig. 2. The root mean square error (RMSE) and correlation coefficient (CC) are chosen to evaluate  
207 the simulation capability of the two models. RMSE measures the magnitude of the deviation between  
208 simulated and observed values, CC measures the degree of correlation between simulated and  
209 observed values, the two values are used to assess simulation accuracy. In Fig. 2, the mean RMSE  
210 across the four stations is 0.08 m for HT and 0.22 m for HM, meaning that HM has considerably  
211 larger deviations from observed water levels. The mean CC across four stations is 0.998 for HT and  
212 0.995 for HM; both values are relatively high, very similar to the observed water level, but HT  
213 remains slightly superior. Overall, HT demonstrates stronger capability and higher accuracy than HM  
214 in simulating total water levels.



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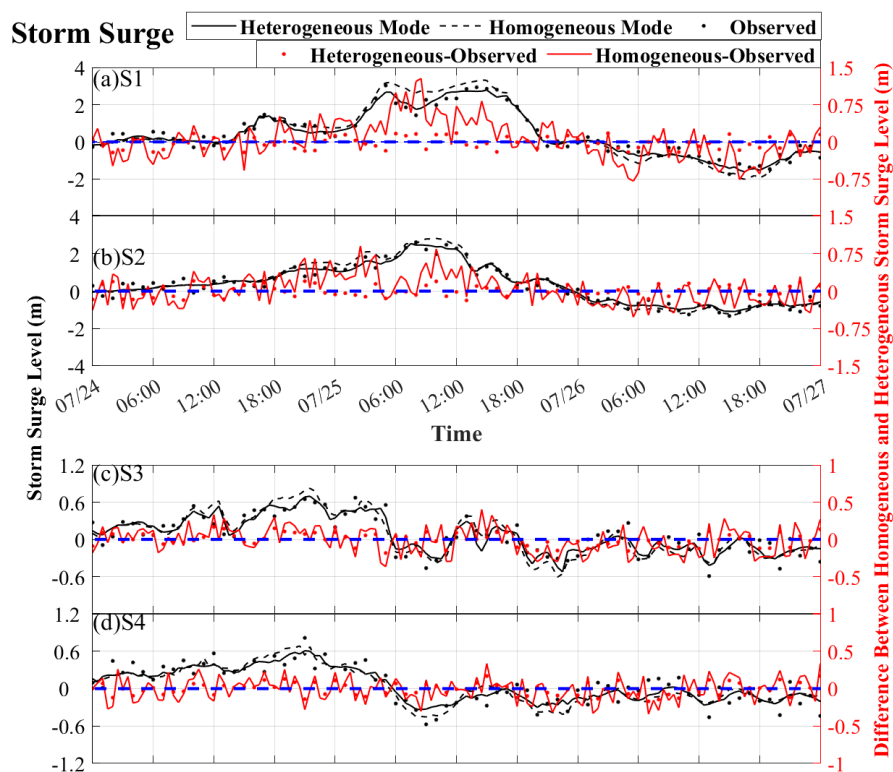
Fig. 2 Comparison of simulated and observed total water levels.

217 According to the different stages of Typhoon In-Fa's passage through Hangzhou Bay, we also  
218 compared the differences in astronomical tides and storm surges between the two models. Before July  
219 24 and after July 27 are the periods for describing the astronomical tide . As shown in Fig. 2, before  
220 and after the typhoon, the astronomical tides are dominated by semi-diurnal tides with an amplitude  
221 of approximately 2 m. The deviation between HT model and the observed values is small, at  
222 approximately 0 m, whereas HM model displays considerably larger deviations, at approximately  
223 0.25 m. The accuracy of astronomical tides in HT is improved by approximately 12.5% relative to  
224 HM.

225 After eliminating the astronomical tide from the total water level, the storm surges are shown in  
226 Fig. 3. There are different characteristics of surges. We divided them into two parts, one at S1-S2  
227 and the other at S3-S4. At stations S1–S2, the surge rises up to nearly 4 m, after the peak, water levels  
228 decrease rapidly, and a storm surge depression is observed during the residual period, the level is  
229 approximately -2m. The overall duration of the surge rise and fall lasts 2–3 days. The deviation  
230 between HT-simulated and observed storm surges is small, whereas the deviation between HT-  
231 simulated and observed storm surges is larger, with the maximum discrepancy approaching 1.5 m.  
232 The storm surge forecast accuracy in HT is improved by approximately 38% relative to HM.

233 At stations S3–S4, the surge curves are of a mixed type, showing both oscillatory patterns and  
234 small-amplitude variations, with the values range from -0.6 m to 0.6 m, obviously smaller than those  
235 at S1–S2. The overall duration of the surge rise and fall is approximately 1–2 days. The deviation  
236 between HT-simulated and observed storm surges is small, while HM-simulated storm surges deviate  
237 considerably from the observations, with the discrepancy displaying a significant oscillatory pattern  
238 ranging from -0.2 m to 0.2 m. The storm surge forecast accuracy in HT is improved by approximately  
239 30% relative to HM.

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Fig. 3 Comparison of simulated and observed storm surges.

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In summary, the HT model shows superior simulation capability and accuracy compared to the

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HM model for all water level types, including total water levels, astronomical tides, and storm surges.

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At different stations, for astronomical tides, the results are similar across all four stations, with the

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mean deviation of HM-simulated values are approximately 0.3 m; for storm surges, there are obvious

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differences between stations S1–S2 and S3–S4: at S1–S2, the amplitudes are large during the rising

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time and falling time, and deviations of HM-simulated values from observations are correspondingly

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large, whereas at S3–S4, the amplitudes are small in the whole surging time, and the deviations of

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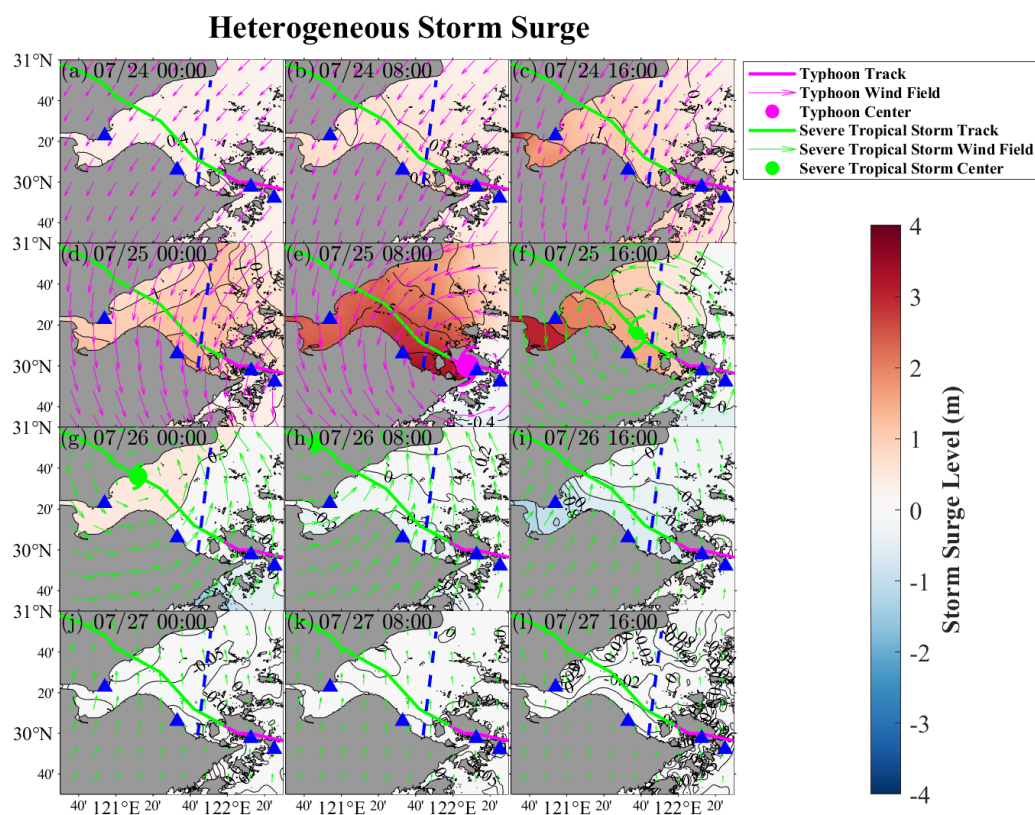
HM-simulated values from observations are also relatively small.

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### 3.2 Spatiotemporal Distribution Characteristics of Storm Surges



252 Since HT models has more simulation accuracy, we choose the HT's results to represent the  
253 spatiotemporal variation characteristics of storm surges across the entire study area.



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255 Fig. 4 Spatiotemporal variations of storm surges.

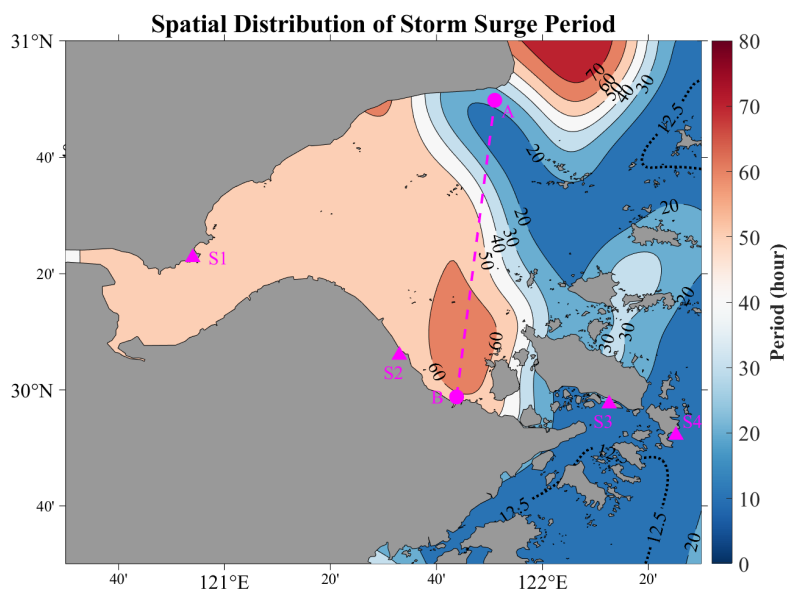
256 Fig. 4 shows the spatial distribution of storm surges per 8 hours from July 24 to July 27, including  
257 the initial surge rise, peak, and subsequent surge fall. The surge rise process unfolds as follows: a  
258 slight water level rise of less than 1 m appears across the entire study area at 08:00 on July 24; by  
259 00:00 on July 25, the surge height increases with values exceed 1 m to the west of the bay mouth; by  
260 08:00 on July 25, the surge reaches its maximum, with water levels inside Hangzhou Bay exceeding



261 3 m, while at 16:00 the elevated surge inside the bay is maintained while the surge outside decreases;  
262 by 00:00 on July 26, a notable water level drop is already evident; by 16:00 on July 26, a slight surge  
263 depression appears to the west of the bay mouth, with a maximum depression of  $-0.4$  m inside  
264 Hangzhou Bay; by 00:00 on July 27, water levels have largely returned to normal. In terms of the  
265 spatial distribution of the surge, the bay and estuarine areas near stations S1 and S2 experience large  
266 surge rise and fall amplitudes, while the archipelago waters near stations S3 and S4 exhibit smaller  
267 surge rise with no surge depression observed.

268 In terms of typhoon intensity and track, the maximum surge occurs when the typhoon is at its  
269 greatest intensity and is located to the left of the typhoon track. For the typhoon's wind field, the  
270 primary surge rise period inside Hangzhou Bay corresponds to onshore winds from the northeast,  
271 while the primary surge fall period corresponds to offshore winds from the southwest. Overall, the  
272 spatial variations are governed by typhoon intensity, track, wind direction, and the topographic  
273 characteristics of different sub-regions.

274 According to the above results, the duration of the storm surge's rising and falling phases is  
275 approximately 2–3 days. So we take a wavelet analysis to determine the dominant periods of storm  
276 surges across the entire study area, as shown in Fig. 5.



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Fig. 5 Spatial distribution of storm surge periods across the study area.

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Fig. 5 shows the spatial distribution of storm surge periods across the entire study area. There is a bay mouth transect AB as the boundary, the dominant period inside Hangzhou Bay to the west is 50 h or above, while to the east it is 20 h or below, with the period increasing gradually from east to west in the vicinity of the bay mouth. The inertial frequency across the study area is approximately  $7.3 \times 10^{-5} s^{-1}$ , relative to which the storm surge frequency inside Hangzhou Bay is approximately  $3.5 \times 10^{-5} s^{-1}$ , classifying it as a low-frequency storm surge, while the storm surge frequency outside Hangzhou Bay is approximately  $8.7 \times 10^{-5} s^{-1}$ , classifying it as a high-frequency storm surge. Based on the above results, storm surges around stations S1-S2 has a low-frequency, and the rising and falling amplitudes are large and the differences between HM-simulated and observed storm surges are also large. Storm surges around stations S3-S4 exhibit high-frequency characteristics, with small rising and falling amplitudes and small differences between HM-simulated and observed storm surges.

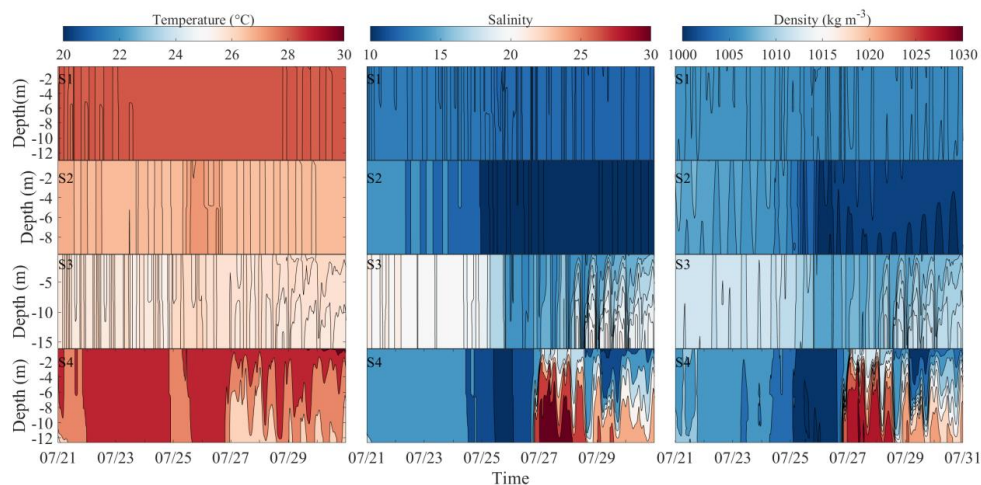


290 **3.3 Spatiotemporal Distribution Characteristics of Temperature, Salinity, and Density**

291 Since observational data on temperature, salinity, and density in Hangzhou Bay and its adjacent  
292 waters during the passage of Typhoon In-Fa were not available, the physical quantities simulated by  
293 the heterogeneous ocean model (HT) are used to represent the real values. The model directly  
294 simulates the spatiotemporal distributions of temperature and salinity, while density is calculated  
295 using the Thermodynamic Equation of Seawater - 2010 (TEOS-10), which is the standard  
296 recommended by the international oceanographic community. In this process, the model-simulated  
297 temperature, given as potential temperature, and salinity, given as practical salinity, are first converted  
298 to conservative temperature and absolute salinity, respectively, before density is computed using the  
299 Gibbs function.

300 We first examine the spatiotemporal distribution characteristics of temperature, salinity, and  
301 density in the vertical direction at individual stations.

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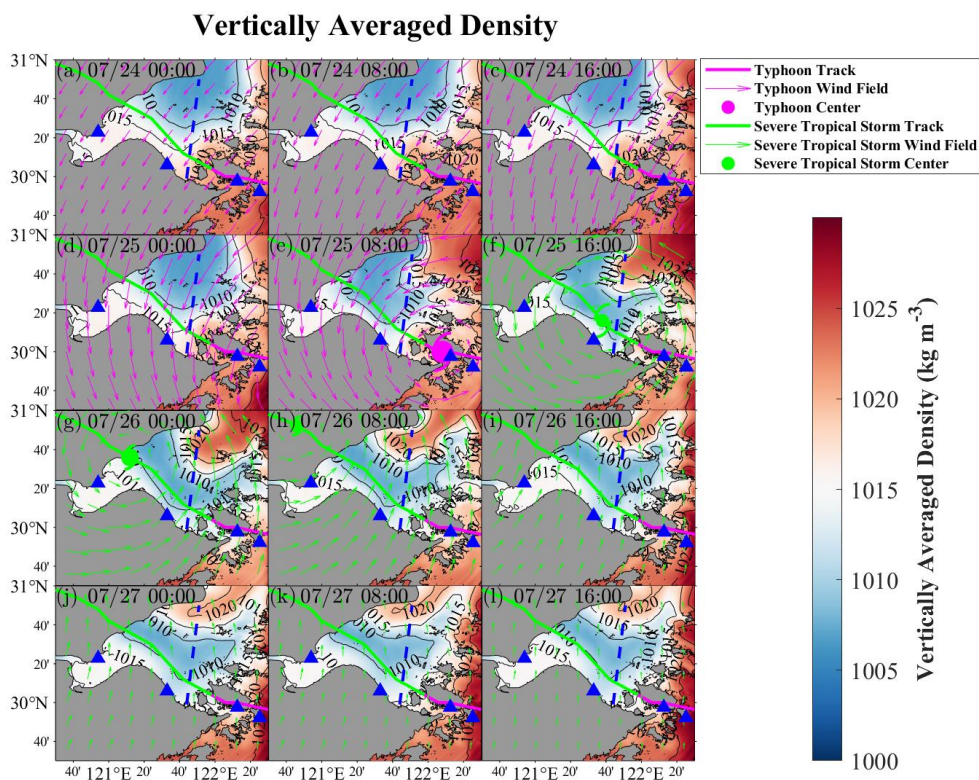
304 Fig. 6 Temporal evolution of vertical profiles of temperature, salinity, and density at stations  
305 S1–S4.

306 The density variations are similar to the salinity variations. The water depths at the four stations



307 are similar and small, with a mean depth of approximately 12 m. At stations S1–S2, temperature,  
 308 salinity, and density are well mixed in the vertical direction. Specifically, there are no obvious changes  
 309 at S1 and S2 before or after the typhoon. At stations S3–S4, the water column remains vertically well  
 310 mixed before and during the typhoon, but the stratification develops during the surge falling period.

311 In general, at all four stations, temperature, salinity, and density are vertically well mixed both  
 312 before and during the typhoon, so we use the depth-averaged values to represent the characteristics  
 313 of the whole water column at each station. The following section presents the density variation  
 314 characteristics of the entire study area when the typhoon passed through this area.



315  
 316 Fig. 7 Spatial distribution characteristics of depth-averaged density during the passage of the  
 317 typhoon.

318 During the passage of the typhoon, the primary density changes occur outside Hangzhou Bay



319 and in the bay mouth region, while there is no notable density change in the interior or at the head of  
320 the bay. As the typhoon gradually moves northwestward, a high-density water tongue gradually  
321 develops in the bay mouth region and expands northwestward from 00:00 on July 24 to 00:00 on July  
322 26, reaching its maximum areal extent at around 00:00 on July 26. After 00:00 on July 26, the areal  
323 extent of the water tongue gradually diminishes. Overall, the spatial distribution of density in this  
324 region displays a high–low–high pattern throughout the typhoon passage.

### 325 **3.4 Analysis of the Mechanisms by Which Spatial density heterogeneity Influences Storm**

#### 326 **Surges**

327 As demonstrated above, the HT model achieves significantly higher storm surge simulation  
328 accuracy than the HM model. To investigate the reasons for this improvement, the following analysis  
329 will use the govern equations about the water level and current field variations to progressively and  
330 clearly elucidate the mechanisms how the spatial density heterogeneity influences storm surges.

331 According to fundamental ocean dynamics theory, the spatiotemporal variations of water levels  
332 are essentially driven by the convergence and divergence of the current field. The continuity equation  
333 can precisely describe this progress. The current field itself is affected by all kinds of dynamical  
334 factors, including the Coriolis force, pressure gradient force, wind stress, surface wind force, bottom  
335 friction force, all of which can be comprehensively summarized by the momentum equation. On this  
336 basis, our goal is that to find the quantitative relationship between A and B by combing the continuity  
337 equation and momentum equations.

338 Because the change of the water level is associated with the whole column, the depth-integrated  
339 form of the equations in the vertical direction is required. Here the water level refers to the total water  
340 level variation at a given spatial point at a given time.

$$341 \quad D(x, y, t) = H(x, y) + \eta(x, y, t) \quad (2)$$

342 Correspondingly, the depth-integrated velocity is defined as follows. Here, the velocity also  
343 refers to the total velocity at a given spatial point at a given time, which includes tidal currents as well  
344 as other flow components:



$$345 \quad \vec{U} = \int_{-H}^{\eta} \vec{u}(x, y, z, t) dz \quad (3)$$

346 So, the continuity equation and momentum equation in vector form are given as follows:

$$347 \quad \frac{\partial \eta}{\partial t} + \nabla \cdot \vec{U} = 0 \quad (4)$$

$$348 \quad \frac{\partial \vec{U}}{\partial t} + \left( \frac{\vec{U}}{D} \cdot \nabla \right) \vec{U} + f \hat{k} \times \vec{U} = -gD \nabla \eta - \frac{g}{\rho_0} \left[ \int_{-H}^{\eta} \nabla \left( \int_z^{\eta} \rho' dz' \right) dz \right] + \vec{\tau}_s - \vec{\tau}_b \quad (5)$$

348 To describe the individual terms in the momentum equation clearly, we use the following  
 349 simplifications and notations. In the depth-integrated form, the advection term represents a nonlinear  
 350 coupling between the vertically averaged velocity and the depth-integrated velocity. Here, the  
 351 combined coupling term is treated as a nonlinear stress term:

$$352 \quad \vec{N} = \left( \frac{\vec{U}}{D} \cdot \nabla \right) \vec{U} \quad (6)$$

353 in the homogeneous ocean model, the pressure gradient force caused by sea surface height  
 354 variations is given as follows:

$$355 \quad \vec{P} = gD \nabla \eta \quad (7)$$

356 in the heterogeneous ocean, the pressure gradient force caused by spatial density heterogeneity  
 357 is given as follows:

$$358 \quad \vec{P}' = \frac{g}{\rho_0} \left[ \int_{-H}^{\eta} \nabla \left( \int_z^{\eta} \rho' dz' \right) dz \right] \quad (8)$$

359 the sea surface wind stress is given as follows:

$$360 \quad \vec{\tau}_s = C_{ds} |U_w| \vec{U}_w \cdot \rho_{air} / \rho_0 \quad (9)$$

361 the bottom friction force is given as follows:

$$362 \quad \vec{\tau}_b = C_{db} |U_b| \vec{U}_b \quad (10)$$

363 where  $g = 9.8 m/s^2$  is the gravitational acceleration;  $U_w$  is the wind speed and  $U_b$  is the bottom  
 364 layer current velocity; the wind stress coefficient  $C_{ds}$  is 0.0012 (Bernier and Thompson, 2015), the  
 365 bottom friction coefficient is 0.0025 (Chen et al., 2003),  $\rho_{air} = 1 kg/m^3$  is the air density, and the



366 seawater density in the wind stress calculation takes a reference value  $\rho_0 = 1020 \text{ kg/m}^3$ . After

367 undergoing these treatments, the simplified momentum equation is given as follows:

$$368 \quad \frac{\partial \vec{U}}{\partial t} + f\hat{k} \times \vec{U} = -\vec{P} - \vec{P}' - \vec{N} + \vec{\tau}_s + \vec{\tau}_b \quad (11)$$

369 take the divergence of Equation (11):

$$370 \quad \frac{\partial}{\partial t} (\nabla \cdot \vec{U}) - f(\nabla \times \vec{U}) = -\nabla \cdot (\vec{P} + \vec{P}' + \vec{N} - \vec{\tau}_s + \vec{\tau}_b) \quad (12)$$

371 taking the curl of Equation (11):

$$372 \quad \frac{\partial}{\partial t} (\nabla \times \vec{U}) + f(\nabla \cdot \vec{U}) = -\nabla \times (\vec{P} + \vec{P}' + \vec{N} - \vec{\tau}_s + \vec{\tau}_b) \quad (13)$$

373  $\frac{\partial(12)}{\partial t} + f \times (13)$  eliminating the vorticity term:  $(\nabla \times \vec{U})$ :

$$\frac{\partial^2}{\partial t^2} (\nabla \cdot \vec{U}) - f \frac{\partial (\nabla \times \vec{U})}{\partial t} = - \frac{\partial [\nabla \cdot (\vec{P} + \vec{P}' + \vec{N} - \vec{\tau}_s + \vec{\tau}_b)]}{\partial t} \quad (14)$$

$$374 \quad f^2 (\nabla \cdot \vec{U}) + f \frac{\partial (\nabla \times \vec{U})}{\partial t} = -f [\nabla \times (\vec{P} + \vec{P}' + \vec{N} - \vec{\tau}_s + \vec{\tau}_b)] \quad (15)$$

$$\frac{\partial^2}{\partial t^2} (\nabla \cdot \vec{U}) + f^2 (\nabla \cdot \vec{U}) = -f [\nabla \times (\vec{P} + \vec{P}' + \vec{N} - \vec{\tau}_s + \vec{\tau}_b)] - \frac{\partial [\nabla \cdot (\vec{P} + \vec{P}' + \vec{N} - \vec{\tau}_s + \vec{\tau}_b)]}{\partial t} \quad (16)$$

375 eliminating the divergence term using the continuity equation:  $(\nabla \cdot \vec{U})$ , the final equation governing

376 water level variations is obtained as:

$$377 \quad f^2 \frac{\partial \eta}{\partial t} + \frac{\partial^3 \eta}{\partial t^3} = f [\nabla \times (\vec{P} + \vec{P}' + \vec{N} - \vec{\tau}_s + \vec{\tau}_b)] + \frac{\partial [\nabla \cdot (\vec{P} + \vec{P}' + \vec{N} - \vec{\tau}_s + \vec{\tau}_b)]}{\partial t} \quad (17)$$

378 Equation (17) is the governing dynamical equation describing the relationships between water  
379 level variations and various dynamical forcing factors. The left-hand side shows water level variations  
380 at different timescales, while the right-hand side shows several dynamical factors at different  
381 timescales in the form of vorticity and divergence. Since this study focuses on storm surge variations,  
382 and as described in Section 2.1, the storm surge is obtained by subtracting the harmonically predicted  
383 astronomical tide from the total water level at the corresponding time step. Accordingly, harmonic  
384 analysis is also applied to each term in Equation (17) in order to remove the influence of astronomical  
385 tides.



386 We firstly analyze the storm surge variations represented on the left-hand side of Equation (17).  
 387  $f^2 \frac{\partial \eta}{\partial t}$  represents the variation of storm surge influenced by the Earth's rotation effect, while  $\frac{\partial^3 \eta}{\partial t^3}$   
 388 represents the variation of storm surge governed by its own period. Based on the previous analysis,  
 389 the dominant period of storm surges inside Hangzhou Bay exceeds 50 h, the storm surge variations  
 390 are primarily dominated by  $f^2 \frac{\partial \eta}{\partial t}$ .

391 We next analyze the right-hand side of Equation (17). Since this equation includes the pressure  
 392 gradient force generated by spatial density heterogeneity, it effectively represents the dynamical  
 393 equation under the HT model. To combine with HM model, Equation (17) is rewritten as follows:

$$394 \left( f^2 \frac{\partial \eta}{\partial t} + \frac{\partial^3 \eta}{\partial t^3} \right)_{HT} = \left\{ f [\nabla \times (\vec{P} + \vec{N} - \vec{\tau}_s + \vec{\tau}_b)] + \frac{\partial [\nabla \cdot (\vec{P} + \vec{N} - \vec{\tau}_s + \vec{\tau}_b)]}{\partial t} \right\}_{HT} + \left[ (\nabla \times \vec{P}') + \frac{\partial (\nabla \cdot \vec{P}')}{\partial t} \right]_{HT} \quad (18)$$

395 The first part term on the right-hand side means that these terms are not directly influenced by density,  
 396 which are combined grouped together; the second part term means the direct influence of density  
 397 variations on storm surges and is accordingly grouped together. The corresponding dynamical  
 398 equation under the HM model is given as follows:

$$399 \left( f^2 \frac{\partial \eta}{\partial t} + \frac{\partial^3 \eta}{\partial t^3} \right)_{HM} = \left\{ f [\nabla \times (\vec{P} + \vec{N} - \vec{\tau}_s + \vec{\tau}_b)] + \frac{\partial [\nabla \cdot (\vec{P} + \vec{N} - \vec{\tau}_s + \vec{\tau}_b)]}{\partial t} \right\}_{HM} \quad (19)$$

400 We use the HT model as the reference model, as it provides a more accurate representation of the  
 401 actual conditions due to its higher accuracy. The corresponding terms in the governing equations of  
 402 the HM and HT models are then subtracted from each other to quantify the storm surge bias and the  
 403 biases in each dynamical mechanism attributable to density variations, denoted by the subscript MT:

$$404 \left( f^2 \frac{\partial \eta}{\partial t} + \frac{\partial^3 \eta}{\partial t^3} \right)_{MT} = \left\{ - \left[ (\nabla \times \vec{P}') + \frac{\partial (\nabla \cdot \vec{P}')}{\partial t} \right] \right\}_{MT} + \left\{ f [\nabla \times (\vec{P} + \vec{N} + \vec{\tau}_b)] + \frac{\partial [\nabla \cdot (\vec{P} + \vec{N} + \vec{\tau}_b)]}{\partial t} \right\}_{MT} \quad (20)$$

405 Since the wind stress is identical in both models, it does not appear in the bias equation. The first term  
 406 of the dynamical mechanism on the right-hand side of Equation (20) is regarded as the direct influence  
 407 of density, denoted as  $Q_{MT\_Direct}$ , where the negative sign has no physical meaning; the second  
 408 dynamical mechanism term is regarded as the indirect influence of density, denoted as  $Q_{MT\_Indirect}$ .



409 The simplified form is as follows:

$$410 \quad \begin{cases} Q_{MT\_Direct} = \left\{ - \left[ (\nabla \times \vec{P}') + \frac{\partial(\nabla \cdot \vec{P}')}{\partial t} \right] \right\}_{MT} \\ Q_{MT\_Indirect} = \left\{ f \left[ \nabla \times (\vec{P} + \vec{N} + \vec{\tau}_b) \right] + \frac{\partial \left[ \nabla \cdot (\vec{P} + \vec{N} + \vec{\tau}_b) \right]}{\partial t} \right\}_{MT} \end{cases} \quad (21)$$

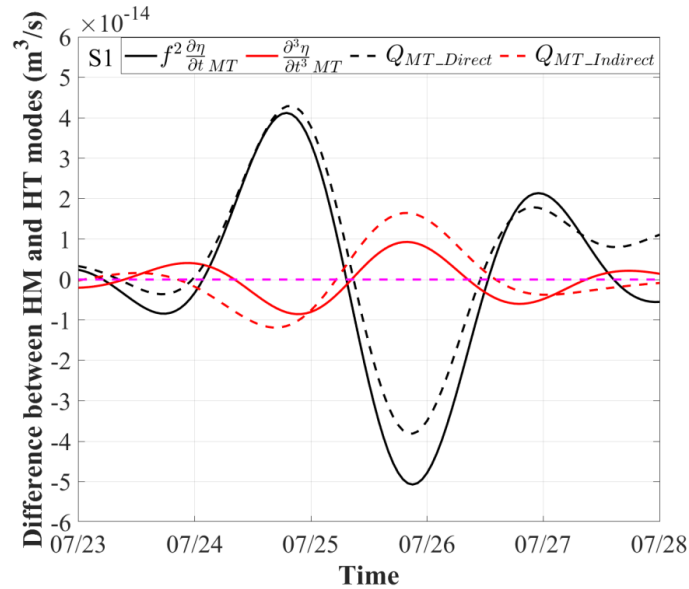
411 Equation (20) is then summarized as follows:

$$412 \quad \left( f^2 \frac{\partial \eta}{\partial t} \right)_{MT} + \left( \frac{\partial^3 \eta}{\partial t^3} \right)_{MT} = Q_{MT\_Direct} + Q_{MT\_Indirect} \quad (22)$$

413 Next, we take station S1 as an example to analyze the magnitudes of the individual terms in  
414 Equation (22). In Section 3.1, the results indicate that storm surges inside Hangzhou Bay are  
415 dominated by low-frequency components, so a low-pass filter is applied to each term in Equation  
416 (22). The cutoff frequency of the low-pass filter is determined based on the dominant period at station  
417 S1 of 50 h, giving  $f_{50} = \frac{1/50}{1/2} = 0.04$ , the low-pass filtered results are shown in the figure below.

418 The storm surge difference between HM and HT is predominantly governed by the term  $f^2 \frac{\partial \eta}{\partial t}$ .  
419 Among the dynamical mechanism terms, the one whose temporal variation and magnitude are most  
420 consistent with  $f^2 \frac{\partial \eta}{\partial t}$  is  $Q_{MT\_Direct}$ , showing that the pressure gradient force directly generated by  
421 spatial density heterogeneity can produce substantial storm surge variations. It is also evident from  
422 the figure that density variations exert an indirect influence on storm surges as well, though the  
423 magnitude of this indirect contribution is relatively small and will not be further analyzed.

424



425

426 Fig. 8 Differences in storm surges and individual dynamical mechanism terms between HM and  
427 HT.

### 428 3.5 Diagnostic Equation for Density-Related Storm Surges

429 Based on the above results, the subsequent analysis in this study will focus on the storm surges  
430 generated by  $Q_{MT\_Direct}$ . The storm surge difference between the two models is primarily attributable  
431 to the pressure gradient force  $\vec{P}'$ , generated by density variations. This mechanism is summarized  
432 in Equation (23), and the storm surge variation generated by the divergence and vorticity of  $\vec{P}'$  are  
433 denoted as  $\eta'_{\rho'}$ , referred to hereafter as the density-related storm surge.

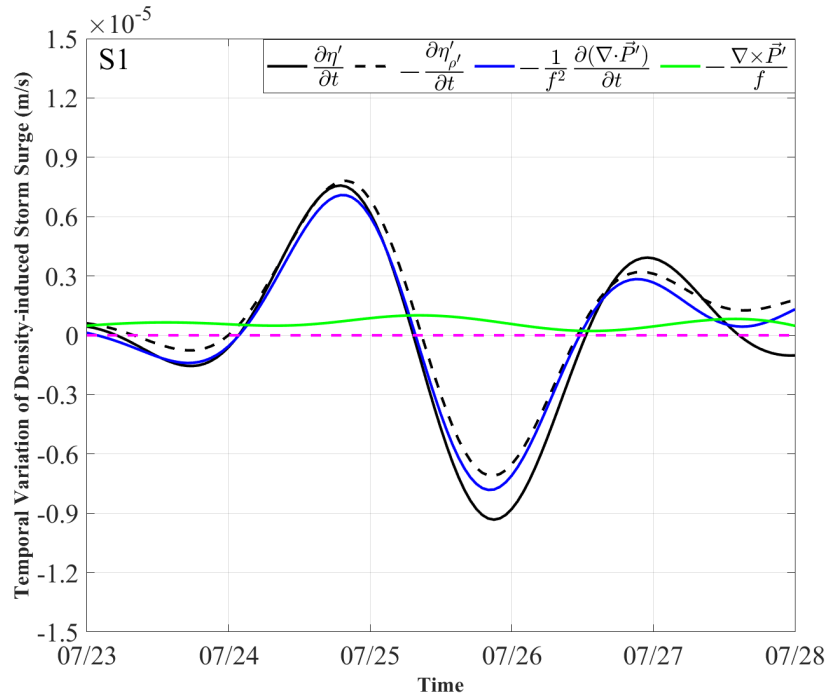
$$434 \quad \frac{\partial \eta'_{\rho'}}{\partial t} = \frac{\nabla \times \vec{P}'}{f} + \frac{1}{f^2} \frac{\partial (\nabla \cdot \vec{P}')}{\partial t} \quad (23)$$

435 The difference between the storm surges simulated by HM and HT is denoted as  $\eta'$ :

$$436 \quad \eta' = \eta_{HM} - \eta_{HT} \quad (24)$$



437 Taking station S1 as an example, the time derivative  $\frac{\partial \eta'}{\partial t}$  is compared with the individual terms  
 438 of Equation (23) to further analyze the underlying causes.



439

440 Fig. 9 Comparison of  $\frac{\partial \eta'}{\partial t}$  and  $\frac{\partial \eta'_{\rho'}}{\partial t}$ .

441 Fig. 9 shows the comparison between  $\frac{\partial \eta'}{\partial t}$  and  $\frac{\partial \eta'_{\rho'}}{\partial t}$ , as well as the individual terms of  
 442 Equation (23) at station S1, where the negative sign carries no physical meaning. The time variation

443 of the density-related storm surge  $\frac{\partial \eta'_{\rho'}}{\partial t}$  is obtained by summing the divergence term  $\frac{1}{f^2} \frac{\partial(\nabla \cdot \vec{P}')}{\partial t}$

444 and the vorticity term  $\frac{\nabla \times \vec{P}'}{f}$  of the density-related pressure gradient force. As can be seen from

445 the figure, the variations of  $\frac{\partial \eta'}{\partial t}$  and  $-\frac{\partial \eta'_{\rho'}}{\partial t}$  are broadly consistent, and it can be further inferred

446 that  $\frac{\partial \eta'_{\rho'}}{\partial t}$  is primarily driven by  $\frac{1}{f^2} \frac{\partial(\nabla \cdot \vec{P}')}{\partial t}$ , while  $\frac{\nabla \times \vec{P}'}{f}$  is negligible.



447 Given that the mean water depth of the study area is only 10 m, and considering the form of  $\vec{P}'$ , the  
 448 vertical variation of density is neglected, yielding the following simplified expression:

$$\begin{aligned}
 \vec{P}' &= \frac{g}{\rho_0} \left[ \int_{-H}^{\eta} \nabla \left( \int_z^{\eta} \rho' dz' \right) dz \right] \\
 &= \frac{g}{\rho_0} \left\{ \int_{-H}^{\eta} \nabla [\rho'(\eta - z)] dz \right\} \\
 &= \frac{g}{\rho_0} \left\{ \int_{-H}^{\eta} (\eta - z) \nabla \rho' dz \right\} + \frac{g}{\rho_0} \left\{ \int_{-H}^{\eta} \rho' \nabla (\eta - z) dz \right\} \\
 &= \frac{g \nabla \rho'}{\rho_0} \int_{-H}^{\eta} (\eta - z) dz + \frac{g \rho'}{\rho_0} \int_{-H}^{\eta} \nabla (\eta - z) dz \\
 449 &= \frac{g \left( \frac{1}{2} \eta^2 + \eta H + \frac{1}{2} H^2 \right) \nabla \rho'}{\rho_0} + \frac{g (\rho' \eta + \rho' H) \nabla \eta}{\rho_0} \tag{25}
 \end{aligned}$$

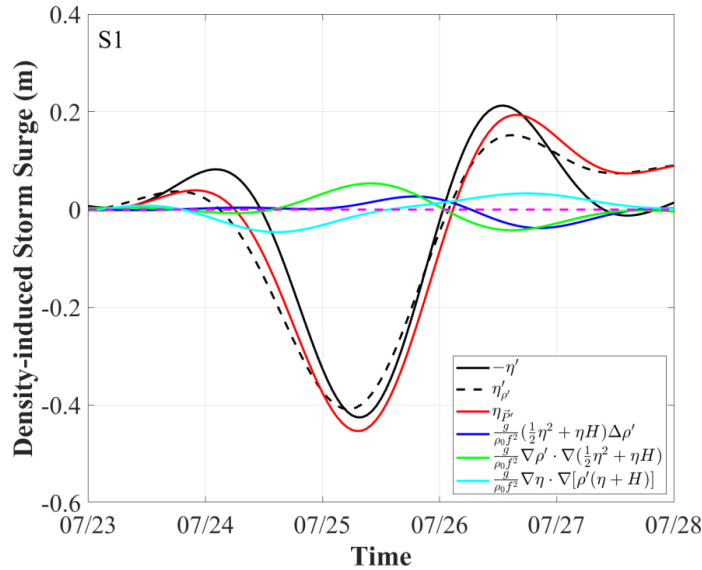
450 Neglecting  $\frac{\nabla \times \vec{P}'}{f}$ , equation (23) can be rewritten as follows:

$$\begin{aligned}
 451 \quad \frac{\partial \eta'_{\rho'}}{\partial t} &= \frac{1}{f^2} \frac{\partial (\nabla \cdot \vec{P}')}{\partial t} = \frac{g}{\rho_0 f^2} \frac{\partial}{\partial t} \left\{ \nabla \cdot \left[ \left( \frac{1}{2} \eta^2 + \eta H + \frac{1}{2} H^2 \right) \nabla \rho' \right] \right\} + \frac{g}{\rho_0 f^2} \frac{\partial}{\partial t} \left\{ \nabla \cdot [\rho'(\eta + H) \nabla \eta] \right\} \\
 &= \frac{g}{\rho_0 f^2} \frac{\partial}{\partial t} \left[ \left( \frac{1}{2} \eta^2 + \eta H \right) \Delta \rho' \right] + \frac{g}{\rho_0 f^2} \frac{\partial}{\partial t} \left( \frac{H^2}{2} \Delta \rho' \right) + \frac{g}{\rho_0 f^2} \frac{\partial}{\partial t} \left[ \nabla \rho' \cdot \nabla \left( \frac{1}{2} \eta^2 + \eta H \right) \right] \\
 &\quad + \frac{g}{\rho_0 f^2} \frac{\partial}{\partial t} \left( \nabla \rho' \cdot \frac{\nabla H^2}{2} \right) + \frac{g}{\rho_0 f^2} \frac{\partial}{\partial t} [\rho'(\eta + H) \Delta \eta] + \frac{g}{\rho_0 f^2} \frac{\partial}{\partial t} \left\{ \nabla \eta \cdot [\nabla \rho'(\eta + H)] \right\} \tag{26}
 \end{aligned}$$

452 Since the water depth H is time-independent and the density anomaly  $\rho'$  does not vary with time  
 453 either, the time derivative terms,  $\frac{g}{\rho_0 f^2} \frac{\partial}{\partial t} \left( \frac{H^2}{2} \Delta \rho' \right)$  and  $\frac{g}{\rho_0 f^2} \frac{\partial}{\partial t} \left( \nabla \rho' \cdot \frac{\nabla H^2}{2} \right)$ , involving H and  $\rho'$   
 454 in Equation (21) are both zero. Neglecting these time-invariant terms, the expression for the density-  
 455 related storm surge  $\eta'_{\rho'}$  is given as follows:

$$\begin{aligned}
 456 \quad \eta'_{\rho'} &= \nabla \cdot \vec{P}' = \frac{g}{\rho_0 f^2} \left[ \left( \frac{1}{2} \eta^2 + \eta H \right) \Delta \rho' \right] + \frac{g}{\rho_0 f^2} \left[ \nabla \rho' \cdot \nabla \left( \frac{1}{2} \eta^2 + \eta H \right) \right] \\
 &\quad + \frac{g}{\rho_0 f^2} [\rho'(\eta + H) \Delta \eta] + \frac{g}{\rho_0 f^2} \left\{ \nabla \eta \cdot \nabla [\rho'(\eta + H)] \right\} \tag{27}
 \end{aligned}$$

457 The magnitudes of the individual terms in Equation (22) are calculated based on the model-simulated  
 458 values as follows:



459

460

Fig. 10 Comparison of  $\eta'$  and  $\eta'_{\rho'}$

461 As shown in Fig. 8,  $\eta'$  and  $-\eta'_{\rho'}$  exhibit broadly consistent temporal variations, with

462  $\frac{g}{\rho_0 f^2} \left[ \left( \frac{1}{2} \eta^2 + \eta H \right) \Delta \rho' \right]$  playing the dominant role in determining  $\eta'_{\rho'}$ , namely the coupling term

463 between the storm surge, water depth, and the Laplacian of the density anomaly.

464 The following analysis examines the overall characteristics of density-related storm surges

465 across the entire study area. Based on the spatial distribution of storm surge periods discussed in

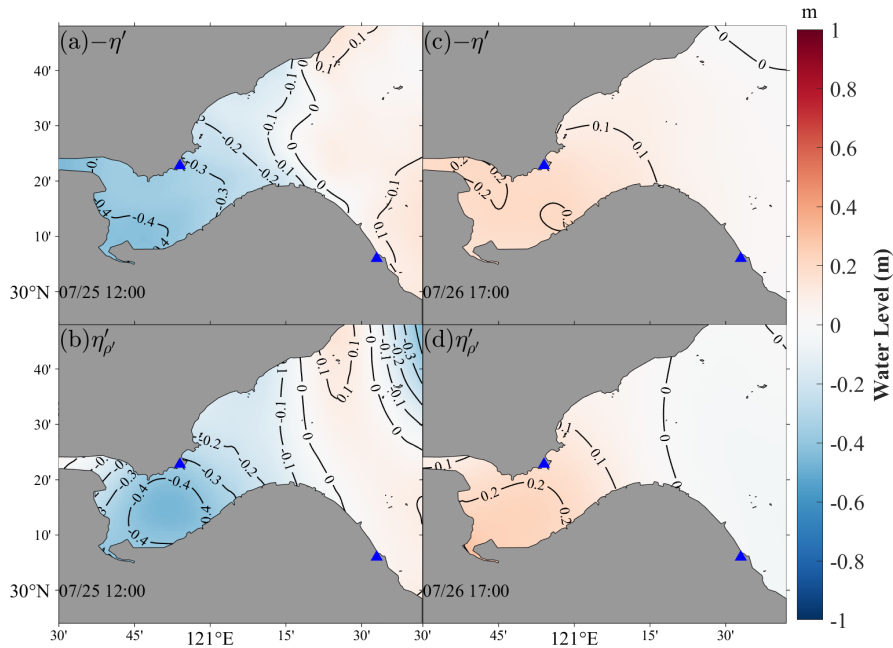
466 Section 3.1, low-frequency storm surges are predominantly distributed within the interior of

467 Hangzhou Bay (120.5°E–121.7°E, 29.9°N–30.8°N). Using the storm surge time series at station S1

468 as a reference, the moments of maximum surge rise and maximum surge fall are selected for

469 comparison of  $\eta'$  and  $\eta'_{\rho'}$ , the corresponding spatial distributions.

470



471

472 Fig. 11 Spatial distributions at the moments of maximum storm surge rise and maximum storm  
473 surge fall.

474 Fig. 11 shows the spatial distributions of  $\eta'$  and  $\eta'_{\rho'}$  at the moments of maximum storm surge  
475 rise and maximum storm surge fall. Overall, the values of  $\eta'$  and  $\eta'_{\rho'}$  are in close agreement within  
476 the interior of Hangzhou Bay at both moments. However, slight discrepancies exist in localized areas:  
477 at the moment of maximum surge rise,  $\eta'_{\rho'}$  is relatively elevated in the exterior of Hangzhou Bay near  
478 the northern bay mouth region, while at the moment of maximum surge fall,  $\eta'_{\rho'}$  is relatively elevated  
479 in the exterior of Hangzhou Bay near the southern bay mouth region.

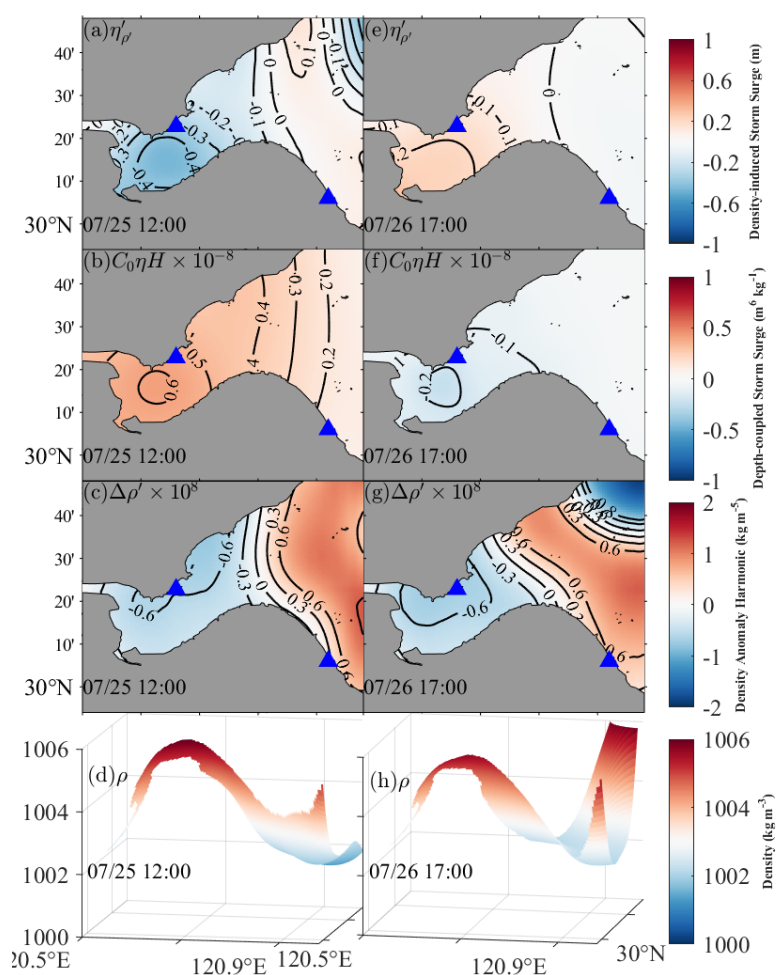
480 Based on the above results, the dominant term in  $\eta'_{\rho'}$  is  $\frac{g}{\rho_0 f^2} \left[ \left( \frac{1}{2} \eta'^2 + \eta' H \right) \Delta \rho' \right]$ , in which the  
481 primary variable of  $\frac{g}{\rho_0 f^2}$  is the Coriolis parameter  $f$ . Since the study area is located near 30°N with a  
482 relatively small latitudinal extent, the spatial variation of  $f$  is neglected, and  $f$  is uniformly



483 computed using the value at 30°N throughout the entire domain. Consequently,  $\frac{g}{\rho_0 f^2}$  reduces to a  
484 constant, denoted as  $C_0$ , with a value of approximately  $1.84 \times 10^6$ . Given that the mean water depth  
485  $H$  of the study area is 10 m and the storm surge height is approximately 2–3 m, the contribution of  
486  $\frac{1}{2}\eta^2$  is neglected, retaining only  $\eta H$ , and the storm surge  $\eta$  is replaced by  $\eta_{HM}$  from the  
487 homogeneous model for computation. Based on the above analysis, Equation (28) is defined as the  
488 diagnostic equation, in which the symbol  $\approx$  indicates that the two sides are not strictly equal, but  
489 rather that the spatial distribution of  $\eta'_{\rho}$  is approximately inferred from the storm surge–water depth  
490 coupling term  $C_0\eta H$ , the spatial distribution of density  $\rho$ , and the Laplacian of the density anomaly  
491  $\Delta\rho'$ . In this expression, the water depth term  $H$  in  $C_0\eta H$  is time-invariant, so the temporal variation  
492 of this term depends solely on the storm surge  $\eta$ ; the Laplacian of the density anomaly  $\Delta\rho'$  is the  
493 second-order spatial derivative of the density surface, whose spatial variation is inversely related to  
494 the undulation trend of the density surface itself. Therefore, the spatiotemporal variation of the  
495 density-related storm surge  $\eta'_{\rho}$  can be approximately determined from the spatial distribution of  
496 density and the spatiotemporal characteristics of storm surges within the study area. The specific  
497 diagnostic procedure is described with reference to Fig. 9 and its caption.

$$498 \quad \eta'_{\rho} \approx C_0\eta H\Delta\rho' \quad (28)$$

499



500

501 Fig. 12 Spatial distributions of  $\eta'_{\rho}$ ,  $C_0\eta H$ ,  $\Delta\rho'$  and  $\rho$  during the periods of storm surge rise

502

and fall.

503

As indicated by Equation (28), the density-related storm surge results from the coupled effect of  
 504 density and storm surge. Taking the surge rise period as an example, panels (a)–(d) of Fig. 12 are  
 505 analyzed. Panel (a) shows the spatial distribution of the density-related storm surge as described in  
 506 Figure 11; during the typhoon surge rising, it is negative inside Hangzhou Bay and positive outside.  
 507 According to Equation (28), panel (a) is given by multiplying panel (b) and panel (c); panel (b) shows  
 508 the storm surge rise superimposed on the water depth, with a pronounced water level increase inside



509 Hangzhou Bay; panel (c) shows the Laplacian of the density anomaly across the entire region, which  
510 exhibits a negative interior and positive exterior pattern. Multiplying the two yields, we can get the  
511 density-related storm surge shown in panel (a). Further analysis reveals that the spatial distribution  
512 of the Laplacian of the density anomaly is fundamentally due to the density distribution in the  
513 Hangzhou Bay region, which is characterized by higher density inside and lower density outside the  
514 bay. During the surge fall period, panels (e)–(h) similarly satisfy the above distribution pattern. In  
515 addition, a large influx of seawater into the exterior of Hangzhou Bay in the later stage results in a  
516 high–low–high density distribution across the entire region, though this does not affect the variations  
517 inside Hangzhou Bay.

518 In summary, the diagnostic equation provides a concise and intuitive means of determining the  
519 spatial distribution characteristics of density-related storm surges based on the spatial features of  
520 storm surges and density variations. This indicates that the differences between the HT and HM  
521 models arise not from density variations directly, but from the coupling effect between density and  
522 storm surges. This leads to an interesting conclusion.

## 523 **4 Discussion**

### 524 **4.1 Analysis of the Influence of Spatial density heterogeneity Based on Water Level Type and** 525 **Timescale**

526 Spatial density heterogeneity influences the simulation accuracy of different water level  
527 components (total water levels, astronomical tides, and storm surges) at different timescales to  
528 varying degrees. This phenomenon has been observed in both the present study and several previous  
529 studies, and will be discussed sequentially below.

530 In terms of different water level types, spatial density heterogeneity improves more accuracy on  
531 storm surge than that of astronomical tide. As shown in Section 3.1, the simulation bias for storm  
532 surges in the homogeneous model is far greater than that for astronomical tides. Wang et al. (2021)  
533 similarly compared the accuracy improvements for different water levels, including the total water  
534 levels, astronomical tides, and residual water levels, the results showed that the improvement in  
535 residual water level accuracy far exceeded that of astronomical tides: 94% of the 211 tide gauge  
536 stations showed residual water level accuracy improvements of 31%–37%, while 78% of stations



537 showed astronomical tide accuracy improvements of 13%–18%. Pringle et al. (2019) also compared  
538 accuracy improvements for total water levels and residual water levels; they concluded that the total  
539 water levels increased from 87% to 93%, while that for the residual water levels increased from 54%  
540 to 85%. Although the present study involves a limited number of stations and need more extensive  
541 statistical analysis, the combined evidence from these studies consistently indicates that storm surges  
542 are more strongly influenced by spatial density heterogeneity.

543 In terms of different timescales of water levels, spatial density heterogeneity improves the  
544 accuracy of long-period water levels to a greater extent than short-period water levels. As shown in  
545 Section 3.2, the storm surge periods in this study exceed 50 h, while astronomical tidal periods range  
546 from 12 to 24 h, and the degree of accuracy improvement is greater for storm surges than for  
547 astronomical tides. Similarly, Wang et al. also found that the degree of accuracy improvement for  
548 residual water levels decreases progressively from seasonal, to inter-seasonal, sub-seasonal, and  
549 weather timescales. Pringle et al. made a power spectral density analysis of sea level variations  
550 recorded at 6-minute intervals, and the results indicated that as frequency increases, the discrepancy  
551 between the baroclinic model and observations increases progressively across the semi-annual,  
552 monthly/fortnightly, diurnal ( $K_1$ ,  $O_1$ ), semi-diurnal ( $M_2$ ,  $S_2$ ), and super-tidal periods. Although only  
553 two timescales are considered in the present study, limiting the number of comparative stages, the  
554 combined evidence from these two studies consistently indicates that spatial density heterogeneity  
555 exerts a more pronounced influence on low-frequency water level variations.

#### 556 **4.2 Discussion on the Mechanisms by Which Spatial density heterogeneity Influences Storm** 557 **Surges**

558 In examining the influence of spatial density heterogeneity on storm surges on storm surges,  
559 many previous studies have approached the problem primarily from the perspective of  
560 phenomenological characteristics, whereas the present study focuses on the dynamical mechanisms  
561 of seawater. Compared to the former approach, the methodology and findings of this study are more  
562 rigorous and well-founded, while also offering novelty. Not only does this study provide a detailed



563 and comprehensive explanation of how spatial density heterogeneity influences storm surges, but it  
564 also derives a diagnostic relationship between density and storm surges, which is of significant  
565 importance for deepening the understanding of storm surge dynamics and related ocean dynamical  
566 processes.

567 Spatial density heterogeneity reflects the spatial variability of seawater density, which tends to  
568 give rise to distinctive water mass structures and environmental backgrounds, such as vertical density  
569 stratification, horizontal circulation, and fronts. Many studies have summarized and analogized these  
570 phenomenological characteristics to briefly identify the mechanisms by which Spatial density  
571 heterogeneity influences storm surges, broadly categorizing them into three types: coastal trapped  
572 waves, bottom friction, and horizontal pressure gradient force. The present study also identifies the  
573 influence of the horizontal pressure gradient force from the perspective of ocean dynamical  
574 mechanisms, though the specific mechanism differs from those identified in prior studies. Overall,  
575 spatial density heterogeneity can influence storm surges through multiple pathways, and each case  
576 warrants individual discussion.

577 The regions where coastal trapped waves occur frequently precisely those where storm surge  
578 forecast accuracy improves. This phenomenon has been observed in both Kodaira et al. (2016) and  
579 Wang et al.(2022), however, the specific mechanisms require further analysis. Regions where coastal  
580 trapped waves commonly occur tend to exhibit pronounced stratification, and there are two possible  
581 pathways existing through which density stratification influence storm surge forecast accuracy:  
582 density stratification may affect storm surge forecast accuracy indirectly by exciting coastal trapped  
583 waves, or it may exert a direct influence independently. If the reason were the former, future research  
584 should focus on analyzing the relationship between coastal trapped waves and storm surges, which  
585 also involves the coupling processes and mechanisms between water levels at different  
586 spatiotemporal scales. If the latter, future research should focus on the relationship between the  
587 density stratification and the storm surge. Based on the content of Kodaira et al. (2016) and Wang et  
588 al.(2022), coastal trapped waves typically have spatial scales of hundreds to thousands of kilometers,  
589 with temporal scales of variation reaching up to several months. In contrast, storm surges, which are  
590 of widespread concern, typically occur in nearshore regions and are strongly influenced by typhoon



591 tracks, the water levels change dramatically over short time periods and small spatial extents, with  
592 maximum spatial scales of only a few hundred kilometers. The generation mechanisms of these two  
593 types of water level variations are inherently different. Therefore, the role of spatial density  
594 heterogeneity in influencing water level accuracy may operate either through coastal trapped waves  
595 or through other processes, and the specific mechanisms remain to be clarified.

596 The mechanisms by which bottom friction influences storm surges are also unclear. Both Orton  
597 et al. (2012) and Kodaira et al.(2016) have observed that density stratification affects storm surge  
598 forecast accuracy. Orton et al. thought that the changes in storm surge forecast accuracy are simply  
599 due to the effects of bottom friction ,which has been found in Muller's work on the transport  
600 mechanisms and amplitude variations of tides under density stratification. However, the evolution of  
601 storm surges differs from that of tides, and the specific mechanisms warrant separate discussion.  
602 Muller's study investigated the influence of density stratification on tidal transport and amplitude  
603 using a combination of dynamical equations and numerical modeling. He concluded that the eddy  
604 viscosity coefficient, pycnocline depth, and bottom friction exert a combined influence. To focus  
605 specifically on the effect of bottom friction, he neglected the advection, he cared more about the  
606 influence of the Coriolis force; in the Northern Hemisphere, only the clockwise tidal current  
607 component has its transport magnitude altered by density stratification. Under stratified conditions,  
608 the eddy viscosity coefficient above the pycnocline is relatively low and current velocities are  
609 relatively large, reaching a peak at the pycnocline itself, while strong shear develops below the  
610 pycnocline and current velocities decay more rapidly. When water depth is large, the velocity  
611 attenuation below the pycnocline dominates, resulting in overall transport that is smaller than under  
612 well-mixed conditions; when water depth is small, the larger velocities above the pycnocline  
613 dominate, resulting in overall transport that exceeds that under well-mixed conditions. Since the  
614 pycnocline depth varies seasonally, tidal transport and amplitude also vary accordingly. It is worth  
615 noting that Muller's study also addresses the influence of surface wind stress on the eddy viscosity  
616 coefficient, finding that increasing surface wind speed leads to an increase in the eddy viscosity  
617 coefficient above the pycnocline and corresponding changes in current velocity. However, the



618 maximum wind stress considered in that study was only 4.5 m/s, which did not significantly affect  
619 the results, which means that the pycnocline depth is affected almost by the bottom friction. In storm  
620 surge research, wind stress is a critical factor and wind speeds are generally far in excess of 4.5 m/s;  
621 therefore, whether the conclusions drawn above remain valid when wind stress effects are the primary  
622 consideration requires further verification.

623 In this study, we draw a conclusion that the influence of the pressure gradient force generated  
624 by spatial density heterogeneity affect the storm surge forecast accuracy. While Ye et al.(2020) and  
625 Pringle et al.(2019) also noted the influence of the pressure gradient force, but the processes and  
626 mechanisms involved differ, leading to differences in the duration and magnitude of storm surge  
627 water level variations; the specific role of density variations therefore warrants careful distinction. In  
628 the present study, a key finding is that throughout the pre-, during-, and post-typhoon stages, the  
629 density field does not change in regions where storm surge forecast accuracy varies considerably,  
630 instead, it acts as a background field that couples with the storm surge to alter its accuracy. The  
631 duration of this influence is consistent with that of the storm surge itself, and its magnitude is jointly  
632 determined by spatial density heterogeneity and the storm surge. In contrast, Ye et al. and Pringle et  
633 al. focused on changes in the density field itself occurring during typhoon passage and their  
634 subsequent influence on storm surge forecast accuracy. Both studies strengthened the influence of the  
635 cold wake left by the typhoon, and Ye et al. thought of the influence of the large-scale Gulf Stream,  
636 the density variations of which continued to affect water levels for approximately 30 days after the  
637 storm. Pringle et al. similarly noted that density variations continued to influence water levels for  
638 approximately 15 days following the storm surge. However, the magnitude and the lasting time of  
639 water level changes induced by density variations still requires precise quantification. Overall,  
640 although the pressure gradient force is a relatively intuitive factor, the specific mechanisms must be  
641 determined in conjunction with the spatial distribution of density and the dynamical behavior of storm  
642 surges.

## 643 **5 Conclusions**



644 This study designed a homogeneous ocean model and a heterogeneous ocean model to investigate  
645 the influence of spatial density heterogeneity on storm surge characteristics in Hangzhou Bay and its  
646 adjacent waters during Typhoon In-Fa. Under the heterogeneous ocean model, the simulation  
647 accuracy of different water level types, including astronomical tides and storm surges, was improved,  
648 with a greater degree of improvement for storm surges. Furthermore, the influence of spatial density  
649 heterogeneity on water levels at different timescales also exhibits a systematic pattern, with a greater  
650 degree of accuracy improvement for long-period water levels (e.g., storm surges with periods  
651 exceeding 50 h) than for short-period water levels (e.g., astronomical tides with periods of 12–24 h).

652 This study analyzed the reasons for the improved storm surge forecast accuracy under the  
653 heterogeneous ocean model from the perspective of seawater dynamics. The governing equation for  
654 storm surge motion was derived, and by integrating the period characteristics with spectral analysis  
655 of storm surges, the mechanisms by which spatial density heterogeneity influences storm surges with  
656 frequencies below the inertial frequency were primarily discussed. The pressure gradient force  
657 generated by spatial density heterogeneity was identified as the principal cause of the accuracy  
658 improvement. Furthermore, a diagnostic relationship between density and storm surges was  
659 established, whereby spatial density heterogeneity acts as a background field that couples with storm  
660 surges to alter their accuracy.

661 This study also has several limitations. Regarding the variation characteristics of storm surges  
662 and other water levels under the heterogeneous ocean model, future work should incorporate  
663 additional typhoon cases and tide gauge records for statistical analysis in order to derive more  
664 representative conclusions. Regarding the reasons for the improved storm surge forecast accuracy  
665 under the heterogeneous ocean model, the high-frequency components remain unexplained, and the  
666 individual dynamical processes in the diagnostic analysis require precise quantification for magnitude  
667 for comparison; such quantification would render the findings more rigorous.

668 In summary, spatial density heterogeneity plays a significant role in improving water level  
669 forecast accuracy and advancing research on related scientific questions. Its influence is multifaceted  
670 and cannot be neglected, and the underlying mechanisms are diverse and require case-specific  
671 analysis, including but not limited to coastal trapped waves, bottom friction, and pressure gradient



672 force. Incorporating this factor in future numerical simulations would not only improve forecast  
673 accuracy and enhance the practical value of forecasting, but also provide important insights for  
674 research into related scientific problems, such as storm surge simulation, sea level prediction, cross-  
675 scale current transport, and cross-scale energy cascades in water level variations.

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